IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN:2319-765X. Volume 9, Issue 6 (Jan. 2014), PP 114-118
www.iosrjournals.org

Diseased Prey with Harvesting Predator in Prey-Predator System – An Analytical Study

Madhusudanan.V[1], Gunasekaran .M[2,*], Vijaya. S[3]
Department of Mathematics
[1]S.A. Engineering College, Chennai-600 054;
[2]Sri SubramaniyaSwamy Government Arts College, Tiruttani-631 209
[3]Annamalai university, Chidambaram- 608 002

Abstract: *In this paper a Mathematical model is proposed and analysed to study a dynamical behaviour of exploited system consisting of two preys and a predator which is being harvested. It has been assumed susceptible and infected prey populations are predated by predated species. A local stability of the model has been carried out. It has been observed that the harvesting activity of the predator taken into the consideration. The population size of the prey decreased and naturally a stable equilibrium model becomes unstable.*
Keywords: *Prey-predator system, diseased prey, local stability, harvesting effort.*

I. Introduction

An explosion of the biological resources, the harvesting of population commonly practised in (fishery, forestry and wild life management). Mathematical models have been used extensively successfully to gain insight into scientific management of renewable sources like fish and forest.Lotka-voltera model is one of the earliest prey predator models which is based on sound mathematical logic [5]. It forms the basics of many models used to day in population dynamics. [2,3,4,9,12,13]. Mathematically the investigation of prey-predator models is simplified by the fact that they all two dimensional and the Poincare-Bedington theorem applies many investigators. Researchers has concentrated only persistence problem. In biological terms persistence states the asymptotically the density of each species remains above a positive bond independent of initial conditions, which means that species stay away from extinguish. Mathematically this may be stated in terms of behaviour of solution of models representing biological phenomena [6]. Harvesting has a strong impact on the dynamic evaluation of population [8]. Their result also suggested that the death rate of predator species able to control the chaotic dynamics.

A prey-predator model with diffusion and a supplementary resource for the prey in a two patch environment is studied by Dhar [7]. Stability analysis of a prey-predator model with time delay and harvesting are the subject of analysis Taha [11] and Martin [1].

Recently Sinha *et. al* [10] analysed two models to study the prey-predator dynamics under the simultaneous effect toxic diseases. In the model they have proposed model in which prey-population. It has assumed that the susceptible and infected has predated by predators. In this paper we have analysed the model within a harvesting predator for some situation.

II. Model

Let us assume that $x(t), y(t)$ are susceptible and infected parts of a prey population $z(t)$ represents the predator population which is feed by both susceptible and infected prey population with different predation rated. Assuming e as the co-efficient of biomass adopted by predator the mathematical model proposed study in following from

$$\frac{dx}{dt} = \theta - \beta xy - \alpha_1 xz - d_1 x \qquad (1)$$

$$\frac{dy}{dt} = \beta xy - \alpha_2 yz - d_2 y \qquad (2)$$

$$\frac{dz}{dt} = e\alpha_1 xz + e\alpha_2 yz - d_3 z - qEz \qquad (3)$$

Where

β : disease contact rate

θ : constant recruitment rate of susceptible prey rate

α_1, α_2 : Predation rate among the susceptible and infected prey population

d_1, d_2, d_3 : Natural death rate of respective population
q : catch per coefficient of predator
E : Harvesting effect
We shall use the initial condition

$x(0) = x_0 > 0,\ \ y(0) = y_0 > 0,\ \ z(0) = z_0 > 0$.

In order to prove the boundness of the solution we need the following lemma

Lemma 1: The set $\Omega = \{(x.y,z) : 0 \le x + y + z \le \frac{\theta}{\theta_1}\}$ where $\theta_1 = \min\{d_1, d_2, d_3 - qE\}$ is the region of attraction for all solution s initiating in the interior of the positive octant.

Proof: Let (x, y, z) be any solution with positive initial condition (x_0, y_0, z_0)

Now consider the function $w(t) = x(t) + y(t) + z(t)$

So that

$$\frac{dw}{dt} = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} = \theta - \theta_1$$

By usual comparison theorem we get following expression as $t \to \infty$

$$w(t) \le \frac{\theta}{\theta_1}$$

Thus $x(t) + y(t) + z(t) \le \frac{\theta}{\theta_1}$ this proves lemma

III. Equilibrium points

We now study the existence of fixed points of the system (1), (2) and (3), particularly we are interested in the interior fixed point to begin and we list all possible fixed points.

(i) $E_0 = (0,0,0)$ is trivial fixed point

(ii) $E_1 = \left(\frac{\theta}{d_1}, 0, 0\right)$

(iii) $E_2 = \left(\frac{d_1}{\beta}, \frac{\beta\theta - d_1 d_2}{\beta d_2}, 0\right)$

(iv) $E_3 = \left(\frac{d_3 + qE}{e\alpha_1}, \frac{-e\alpha_1\theta - d_1(d_3 + qE)}{\alpha_1(d_3 + qE)}, 0\right)$

v) The interior fixed point is $E_4(x^*, y^*, z^*)$ where

$$x^* = \frac{d_3 + qE - e\alpha_2 y^*}{e\alpha_1} \qquad y^* = \frac{d + qE}{e\alpha_2} + \frac{e\theta\alpha_1}{e\alpha_1 d_2 - e\alpha_2 d_1 - \beta(d_3 + qE)}$$

$$z^* = \frac{\beta d_3 - e\alpha_1 d_2 - \beta e\alpha_2 y^*}{e\alpha_1\alpha_2} \tag{4}$$

It is obvious that E_1, E_2, E_3, E_4 are boundary equilibrium point and E_4 is the interior equilibrium. The dynamical behaviour of the fixed points of the three dimensional system (1, 2, 3) can be studied by computation of the eigenvalues of the matrix of (1, 2, 3).the jacobian matrix J at the state variable (x, y, z) has the form

$$J = \begin{pmatrix} -\beta y - \alpha_1 z - d_1 & -\beta x & -\alpha_1 z \\ \beta y & \beta x - \alpha_2 z - d_2 & -\alpha_2 y \\ exz & e\alpha_2 z & e\alpha_1 x + e\alpha_2 y - (d_3 + qE) \end{pmatrix} \tag{5}$$

In order to study the stability of fixed points of the model we first give the following theorem.

Theorem: Let $p(\lambda)=\lambda^3+B\lambda^2+C\lambda+D$ be the roots of $p(\lambda)=0$. Then the following statements are true

a) If every root of the equation has absolute value less than one, then the fixed point of the System is locally asymptotically stable and fixed point is called a sink

b) If at least one of the roots of equation has absolute value greater than one then the fixed Point of the system is unstable and fixed point is called saddle

c) If every root of the equation has absolute value greater than one then the system is a source.

d) The fixed point of the system is called hyperbolic if no root of the equation has absolute value equal to one. If there exists a root of equation with absolute value equal to one then The fixed point is called non-hyperbolic.

IV. Dynamic behaviour of the model

Lemma 2: The boundary equilibrium point E_0 of the system (1) is a locally stable fixed point

Proof: By linearizing system (5) at E_0, one obtain the Jacobian

$$J(E_0)=\begin{pmatrix} -d_1 & 0 & 0 \\ 0 & -d_2 & 0 \\ 0 & 0 & -(d_3+qE) \end{pmatrix}$$

The eigenvalues of the matrix $J(E_0)$ are $\lambda_1=-d_1,\lambda_2=-d_2,\lambda_3=-(d_3+qE)$ All the eigenvalues are negative this clearly shows E_0 is locally stable.

Lemma: 3 The equilibrium point $E_1=\left(\frac{\theta}{d_1},0,0\right)$ is locally stable if $\beta\theta<d_1.d_2$ and $e\alpha_1\theta<d_1.d_3$ unstable otherwise

Proof: The Jacobian matrix $J(E_1)$ at the equilibrium point is

$$J(E_1)=\begin{pmatrix} -d_1 & \frac{-\beta\theta}{d_1} & 0 \\ 0 & \frac{\beta\theta-d_1.d_2}{d_1} & 0 \\ 0 & 0 & \frac{e\alpha_1\theta}{d_1}-(d_3+qE) \end{pmatrix}$$

Whose eigenvalues are $\lambda_1=-d_1,\lambda_2=\frac{\beta\theta-d_1.d_2}{d_1},\lambda_3=\frac{e\alpha_1\theta-d_1(d_3+qE)}{d_1}$ It is clear that equilibrium point is sink if $\beta\theta<d_1.d_2$ and $e\alpha_1\theta<d_1.d_3$ Also if $\beta\theta>d_1.d_2$ and $e\alpha_1\theta>d_1.d_3$ is unstable and fixed point is called saddle.

Lemma 4: when $\beta\theta>d_1.d_2$ and $\theta>\frac{d_1.d_3}{e\alpha_1}$ the equilibrium point $E_2=\left(\frac{d_2}{\beta},\frac{\beta\theta-d_1.d_2}{\beta d_2},0\right)$ for the system (1) is exists and if $d_3<[\frac{e\alpha d}{\beta}+e\alpha_2\frac{\beta\theta-d_1.d_2}{\beta d_2}]$ is locally stable

Proof: The Jacobian matrix $J(E_2)$ at the equilibrium point is

$$J(E_2)=\begin{pmatrix} -\frac{\beta\theta}{d_2} & -d_2 & \frac{-\alpha_1 d_2}{\beta} \\ \frac{\beta\theta-d_1.d_2}{d_2} & 0 & \frac{-\alpha_2(\beta\theta-d_1.d_2)}{\beta d_2} \\ 0 & 0 & \frac{e\alpha_1 d_2}{\beta}+\frac{e\alpha_2(\beta\theta-d_1.d_2)}{\beta d_2}-(d_3+qE) \end{pmatrix}$$

Which has the one of the eigen values is

$$\lambda_1=\frac{e\alpha_1 d_2}{\beta}+\frac{e\alpha_2(\beta\theta-d_1.d_2)}{\beta d_2}-(d_3+qE)$$

The other two eigenvalue are given by the roots of the quadratic equation

$$\lambda^2+\frac{\beta\theta}{d_2}\lambda+(\beta\theta-d_1d_2)$$

Solving we get

$$\lambda_2=\frac{-\beta\theta+\sqrt{\beta^2\theta^2-4d_2{}^2\beta\theta+4d_1d_2{}^3}}{2d_2}$$

$$\lambda_3=\frac{-\beta\theta-\sqrt{\beta^2\theta^2-4d_2{}^2\beta\theta+4d_1d_2{}^3}}{2d_2}$$

These two eigenvalues are attractive in x direction if $d_3<[\frac{e\alpha d}{\beta}+e\alpha_2\frac{\beta\theta-d_1.d_2}{\beta d_2}]$ and in this situation it shows a local stability.

Lemma 5: The equilibrium point $E_3=\left(\frac{d_3+qE}{e\alpha_1},0,\frac{\theta e\alpha_1-d_1(d_3+qE)}{e\alpha_1(d_3+qE)}\right)$ for the system (1) is exists if and only if $\theta e\alpha_1>d_1(d_3+qE)$ and it is locally stable .

Proof: The jacobian matrix $J(E_3)$ at the equilibrium point is

$$J(E_3)=\begin{pmatrix} \frac{-\alpha_1(\theta e\alpha_1-d_1(d_3+qE)}{e\alpha_1 d_3}-d_1 & \frac{-\beta(d_3+qE)}{e\alpha_1} & \frac{-\alpha_1(d_3+qE)}{e\alpha_1} \\ 0 & \frac{\beta(d_3+qE)^2-\alpha_1 e\alpha_2\theta+\alpha_2 d_1(d_3+qE)-e\alpha_1 d_1(d_3+qE)}{e\alpha_1(d_3+qE)} & 0 \\ \frac{(d_3+qE)(\theta e\alpha_1-d_1(d_3+qE)}{e\alpha_1{}^2 d_3} & \frac{\theta e\alpha_1-d_1(d_3+qE)}{\alpha_1 d_3} & 0 \end{pmatrix}$$

The eigen values of the matrix are $\lambda_1=\beta\overline{x}-\alpha_2\overline{z}-d_2$, and the other two eigen values as the root of the quadratic equation $\lambda^2+(\alpha_1\overline{z}+d_1)\lambda+e\alpha_1{}^2\overline{xz}=0$ by solving we get

$$\lambda=\frac{-(\alpha_1\overline{z}+d_1)+\sqrt{\alpha_1{}^2\overline{z}^2+d_1{}^2-2\alpha_1\overline{z}d_1-4e\alpha_1{}^2\overline{xz}}}{2}$$

and

$$\lambda=\frac{-(\alpha_1\overline{z}+d_1)-\sqrt{\alpha_1{}^2\overline{z}^2+d_1{}^2-2\alpha_1\overline{z}d_1-4e\alpha_1{}^2\overline{xz}}}{2}$$

Where

$$\overline{x} = \frac{d_3 + qE}{e\alpha_1} \quad \text{and} \quad \overline{z} = \frac{\theta e\alpha_1 - d_1(d_3 + qE)}{e\alpha_1(d_3 + qE)}$$

It is clear that the equilibrium point is stable and exists if $\theta e\alpha_1 > d_1(d_3 + qE)$.

Lemma 6: The interior equilibrium point $E_4(x^*, y^*, z^*)$ of the system (1) exists if and only if $d_3 + qE > e\alpha_2 x_2^\bullet, e\alpha_1 d_2 > e\alpha_2 d_1 + \beta(d_3 + qE)$ and $\beta d_3 > e\alpha_2 d_1 + \beta e\alpha_2 x_2^\bullet$ where $x^\bullet, y^\bullet, z^\bullet$ are given by equation (4).

Proof: The Eigen Values corresponding to this point given by the roots of the

Polynomial equation $\lambda^3 + A\lambda^2 + A_2\lambda + A_3 = 0$

Where

$$A_1 = \beta y^\bullet + \alpha_1 z^\bullet + d_1 \quad A_2 = \beta^2 x^\bullet y^\bullet + e\alpha_1{}^2 x^\bullet z^\bullet + e\alpha_2{}^2 y^\bullet z^\bullet \text{ and}$$

$$A_3 = e\alpha_2{}^2 y^\bullet z^\bullet(\beta y^\bullet + \alpha_1 z^\bullet + d_1)$$

Where

$$x^* = \frac{d_3 + qE - e\alpha_2 y^*}{e\alpha_1} > 0, \qquad y^* = \frac{d + qE}{e\alpha_2} + \frac{e\theta\alpha_1}{e\alpha_1 d_2 - e\alpha_2 d_1 - \beta(d_3 + qE)} > 0$$

and $$z^* = \frac{\beta d_3 - e\alpha_1 d_2 - \beta e\alpha_2 y^*}{e\alpha_1\alpha_2} > 0$$

Here it is clear that under the above condition $A_1 > 0$, $A_2 > 0$ and $A_3 > 0$. Thus from Hurwitz criteria it will be locally stable if $A_1 A_3 > A_2$.

V. Conclusion

Harvesting has a strong impact on the dynamic evaluation of a population subjected to it. Depending on the nature of the applied harvesting strategy, the long run stationary density of a population may be significantly smaller than the long run stationary density of the population in the absence of harvesting. Therefore while a population can in the absence of harvesting be free of extinction risk, harvesting can lead to the incorporation of positive extinction probability and therefore, to potential extinction in finite time. In this article, the model is analysed to study a dynamical behaviour of prey-predator system which is being harvested. Stability of the model has been carried out and the harvesting activity of the predator taken into consideration. The population size of prey decreased and naturally stable equilibrium model becomes unstable.

References:

[1]. A Martin and S. Ruan, Prey Predator Models with time delay and prey harvesting, J.Math. Biol 43 (2001), 247-267.
[2]. B. Dube and R.K.Upadhyay, Persistence and extinction of one prey and two predator system, J. Non Linear Anal. Appl : Model Control 9(4) (2004), 307-329.
[3]. D. Kesh, A.K.Sarkar and A.B. Roy, Persistence of two prey and predator system with ratio dependent predator influence. Math. Appl. Sci. 23 (2000), 347-356.
[4]. J.K. Hall, Ordinary differential equations second ed. Kriegor. Basel, (1980).
[5]. J.N. Kapur, Mathematical modeling in Biology and Medicines, Affiliated East West Press, (1981).
[6]. J.Zhang, Fanm and Y. Kung, Rabbits killing birds revisited. Math. Biosciences (2006), 100-123.
[7]. Joydip Dhar, A prey predator model with diffusion and a supplementary resource for the prey in a two patch environment. J.Math.Model. and Analysis 9(1) (2004), 9-24.
[8]. Manju Agarwal and Rachana Pathak, Harvesting Hopf Bifurcation and Chaos in three species food chain model with beddington –De anglises type functional response, *Journal of Global Research in Mathematical Archives*, volume 1,(2013).
[9]. P.A. Braza., A dominant predator, a predator and a prey. Math. Biosciand Engg. 5(1) (2008), 61.73.
[10]. Sudipa Sinha, O.P. Mishra and Joydip Dhar, Study of a prey predator dynamics under the simultaneous effect of Toxicant and disease. Non linearSci Appl. 1(2) (2008), 102-117.
[11]. Symsuddin Taha and Abu Hassan Malik, Stability Analysis of a prey predator model with time delay and constant rate of harvesting. Punjab Uni. Journ.Math. 40 (2008), 37-48.
[12]. T.K. Kar and Ashim Batabyal, Persistence and stability of two prey and one predator system, *Int. Jour. Engg. Sci. Tech*. 2(1) (2010), 174-190.
[13]. T.K. Kar and K.S. Chaudhary, Harvesting in a two prey one predator fishery, A Bio economic model. ANZIAM 45 (2004), 443-456.