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Irregular Intuitionistic fuzzy graph

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Abstract: *In this paper, some types of Irregular intuitionistic fuzzy graphs and properties of neighbourly irregular ,highly irregular intuitionistic fuzzy graphs are studied. Some results on totally Irregular intuitionistic fuzzy graphs are established.*
Keywords: *Intuitionistic fuzzy graph, degree, total degree, Intuitionistic fuzzy sub graph.*
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I. Introduction:

Atanassov [1] introduced the concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFGs). Research on the theory of intuitionistic fuzzy sets (IFSs) has been witnessing an exponential growth in Mathematics and its applications. This ranges from traditional Mathematics to Information Sciences. This leads to consider IFGs and their applications. R. Parvathy and M.G.Karunambigai's paper [5] introduced the concept of IFG and analyzed its components. A. Nagoor Gani and S.R. Latha[3] introduced Irregular fuzzy graphs and discussed some of its properties.

In this paper, some properties of Irregular Intuitionistic fuzzy graphs and neighbourly irregular intuitionistic fuzzy graphs are studied. Also Some results on totally Irregular intuitionistic fuzzy graphs are established.

II. Preliminary

Definition 2.1: A fuzzy graph $G = (\sigma,\mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$, where for all $u,v \in V$, we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 2.2: The fuzzy subgraph $H = (\tau, \rho)$ is called a fuzzy subgraph of $G = (\sigma,\mu)$ if $\tau(u) \leq \sigma(u)$ for all $u \in V$ and $\rho(u,v) \leq \mu(u,v)$ for all $u,v \in V$.

Definition 2.3: Let $G = (\sigma,\mu)$ be a fuzzy graph. The degree of a vertex v is $d(v) = \Sigma_{u \neq v}\, \mu(v,u)$

Definition 2.4: Let $G = (\sigma,\mu)$ be a fuzzy graph on $G^* : < V, E >$. If $d_G(v) = k$ for all $v \in V$, i.e., if each vertex has the same degree k, then G is said to be a regular fuzzy graph of degree k or a k-regular fuzzy graph.

Definition 2.5: Let $G = (\sigma,\mu)$ be a fuzzy graph on G*. The total degree of a vertex $u \in V$ is defined by $td_G(u) = \Sigma_{u \neq v}\, \mu(v,u) + \sigma(u)$

Definition 2.6: An **Intuitionistic fuzzy graph** is of the form $G = < V, E >$ where
(i) $V=\{v1,v2,\ldots,vn\}$ such that $\mu_1: V \rightarrow [0,1]$ and $\gamma_1: V \rightarrow [0,1]$ denote the degree of membership and nonmembership of the element $vi \in V$, respectively, and
$0 \leq \mu_1(vi) + \gamma_1(vi) \leq 1$ for every $vi \in V$, $(i = 1,2, \ldots\ldots n)$,
(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \rightarrow [0,1]$ and $\gamma_2: V \times V \rightarrow [0,1]$ are such that
$\mu_2(vi, vj) \leq \min[\mu_1(vi), \mu_1(vj)]$ and $\gamma_2(vi, vj) \leq \max[\gamma_1(vi), \gamma_1(vj)]$
and $0 \leq \mu_2(vi, vj) + \gamma_2(vi,vj) \leq 1$ for every $(vi ,vj) \in E$, $(i, j = 1,2, \ldots\ldots n)$

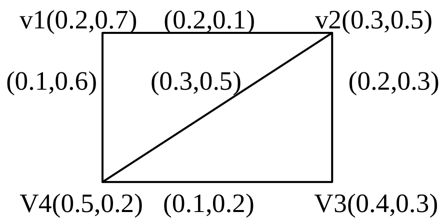

Fig 1: Intuitionistic Fuzzy Graph

Definition 2.7: Let G = < V,E > be an IFG. Then the degree of a vertex v is defined by
$d(v) = (d\mu(v), d\gamma(v))$ where $d\mu(v) = \Sigma_{u\neq v}\, \mu_2(v,u)$ and $d\gamma(v) = \Sigma_{u\neq v}\, \gamma_2(v,u)$.

Definition 2.8: The minimum degree of G is $\delta(G) = (\delta\mu(G), \delta\gamma(G))$ where $\delta\mu(G) = \Lambda\ \{d\mu(v)/v \in V\}$ and $\delta\gamma(G) = \Lambda\ \{d\gamma(v)/v \in V\}$

Definition 2.9: The maximum degree of G is $\Delta(G) = (\Delta\mu(G), \Delta\gamma(G))$ where $\Delta\mu(G) = V\{d\mu(v)/v \in V\}$ and $\Delta\gamma(G) = V\ \{d\gamma(v) / v \in V\}$

Definition 2.10: An edge e = (x, y) of an IFG G = < V, E > is called an effective edge if $\mu_2(x, y) = \mu_1(x)\ \Lambda\ \mu_1(y)$ and $\gamma_2(x, y) = \gamma_1(x)\ V\ \gamma_1(y)$.

Definition 2.11: An Intuitionistic fuzzy graph is complete if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = \max(\gamma_{2i}, \gamma_{2j})$ for all (vi , vj) Є V.

Definition 2.12: The total degree of a vertex 'u' is defined as $td(u) = (td_\mu(u), td_\gamma(u))$, where $td_\mu(u) = \Sigma_{u\neq v}\, \mu_2(u, v) + \mu_1(u)$ and $td_\gamma(u) = \Sigma_{u\neq v}\, \gamma_2(u, v) + \gamma_1(u)$

Definition 2.13: An IFG H = < V', E' > is said to be an Intuitionistic fuzzy subgraph (IFSG) of the IFG, G = < V, E > if V' ⊆V and E' ⊆ E. In other words, if $\mu_{1i}' \leq \mu_{1i}$; $\gamma_{1i}' \geq \gamma_{1i}$ and $\mu_{2ij}' \leq \mu_{2ij}$; $\gamma_{2ij}' \geq \gamma_{2ij}$ for every i, j = 1,2………n.

Definition 2.14: The complement of an IFG G = < V, E > is denoted by $\overline{G}$ = <$\overline{V}$, $\overline{E}$ >and is defined as

i) $\overline{\mu}_1(u) = \mu_1(u)$ and $\overline{\gamma}_1(u) = \gamma_1(u)$ for every uЄ V.

ii) $\overline{\mu}_2(u,v) = \mu_1(u)\ \Lambda\ \mu_1(v) - \mu_2(u, v)$ and $\overline{\gamma}_2(u,v) = \gamma_1(u)\ V\ \gamma_1(v) - \gamma_2(u,v)$ for all (u, v)) Є E.

Definition 2.15: An Intuitionistic fuzzy graph G = <V, E> is said to be regular, if every vertex adjacent to vertices withsame degree.

Example:

Let G= <V,E> be a IFG. The membership and non-membership values of G are defined by,
$(\mu_1(u), \gamma_1(u)) = (0.4, 0.5)$, $(\mu_1(v), \gamma_1(v)) = (0.5, 0.5)$, $(\mu_1(w), \gamma_1(w)) = (0.5,0.3)$, $(\mu_1(x), \gamma_1(x)) = (0.5, 0.4)$, and $(\mu_2(u,v), \gamma_2(u,v)) = (0.3,0.3)$, $(\mu_2(v, w), \gamma_2(v, w)) = (0.4, 0.2)$, $(\mu_2(w, x), \gamma_2(w, x)) = (0.3,0.3)$, $(\mu_2(x, u), \gamma_2(x, u)) = (0.4, 0.2)$. Here G is regular.

III. Irregular Intuitionistic fuzzy graphs

Definition 3.1: Let G = <V,E>be IFG. Then G is irregular, if there is a vertex which is adjacent to vertices with distinct degrees.

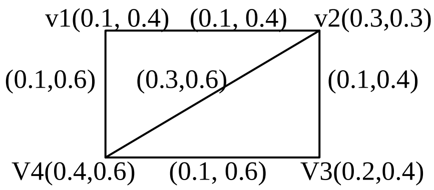

Fig- 2:Intuitionistic fuzzy graph

Here v1 adjacent to v2 and v4 which are having distinct degrees, So G is Irregular IFG.

Definition 3.3: Let G = < V, E >be a connected IFG. G is said to be a neighbourly irregular Intuitionistic fuzzy graph if every two adjacent vertices of G have distinct degree.

Example:

In **Fig -2**, d(v1) = (0.2,1) ; d(v2) = (0.4, 1.4) ; d(v3) = (0.2, 1) ; d(v4) = (0.5, 1.8)
The adjacent vertices v1, v2 and v4 are having distinct degrees, Also the adjacent vertices v2, v3 and v4 are having distinct degrees. (i.e) G is also neighbourly irregular Intuitionistic fuzzy graph.

Definition 3.4: Let G = < V, E > be a connected IFG. Then, G is said to be a highly irregular IFG if every vertex of G is adjacent to vertices with distinct degrees

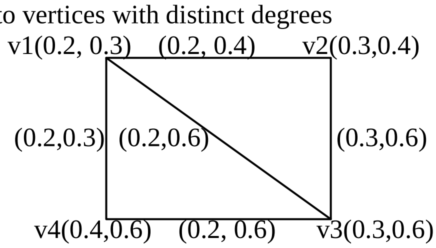

Fig-3: Highly Irregular Intuitionistic fuzzy graph

Here, Every vertex of IFG is adjacent to vertices with distinct degrees.

Proposition 3.5
A highly irregular IFG need not be a neighbourly irregular IFG.

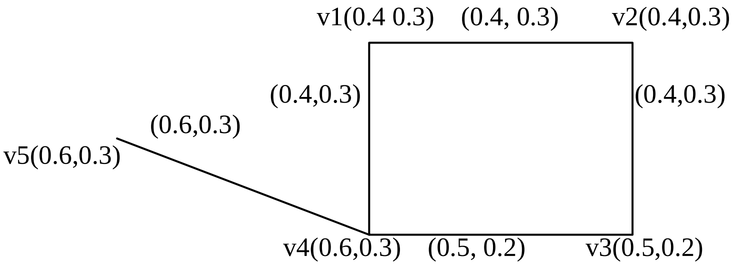

Fig-4: Highly Irregular Intuitionistic fuzzy graph But not neighbourly irregular IFG.

Proposition 3.6:
A neighbourly irregular IFG need not be a highly irregular IFG.
Example:
In **Fig -2**, d(v1) = (0.2,1) ; d(v2) = (0.4, 1.4) ; d(v3) = (0.2, 1) ; d(v4) = (0.5, 1.8)
i.e) neighbourly irregular IFG , But, for the vertex v2, adjacent vertices, d(v1) = d(v3).
So, is not Highly Irregular IFG.

Theorem 3.7
Let G = < V, E > is highly irregular intuitionistic fuzzy graph and neighbourly irregular intuitonistic fuzzy graph if and only if the degrees of all vertices of G are distinct.
Proof:
Let, G be the IFG with n vertices v1, v2,…,vn.
Now, Suppose G is highly irregular and neighbourly irregular IFG. Let the adjacent vertices of v1 be v2, v3,…,vn with degrees (c_2, k_2), (c_3, k_3),…, (c_n, k_n) respectively.
Since G is highly irregular, $c_2 \neq c_3 \neq, \ldots, \neq cn$. Also $k_2 \neq k_3 \neq, \ldots, \neq kn$.
(i.e) d(v1) is not either of (c_2,k_2), (c_3,k_3),…, (c_n, k_n)
Therefore, the degrees of all vertices of G are distinct
Conversely, Assume that the degrees of all vertices of G are distinct.
Which implies that, every two adjacent vertices have distinct degrees and to every vertex, the adjacent vertices have distinct degrees.
That is, G is neighbourly irregular and highly irregular IFG.

Proposition 3.8:
If a IFG G = < V, E > is neighbourly irregular,then G^c is need not be neighbourly irregular
Example 3.9:
Let G= <V,E> be a IFG. Let, $(\mu_1(u), \gamma_1(u)) = (0.3, 0.4)$, $(\mu_1(v), \gamma_1(v)) = (0.4, 0.5)$, $(\mu_1(w), \gamma_1(w)) = (0.5,0.3)$, $(\mu_1(x), \gamma_1(x)) = (0.4, 0.3)$,and $(\mu_2(u, v), \gamma_2(u, v)) = (0.1,0.3)$, $(\mu_2(v, w), \gamma_2(v, w)) = (0.3, 0.4)$, $(\mu_2(w, x), \gamma_2(w, x)) = (0.3,0.2)$, $(\mu_2(x ,u), \gamma_2(x, u)) = (0.2 , 0.3)$.
Here, d(u) = (0.3,0.6) ; d(v) = (0.4,0.7) ; d(w) = (0.6,0.6) ; d(x) = (0.5,0.5)
But in G^c ,
$(\mu_2(u, v), \gamma_2(u, v)) = (0.2,0.2)$, $(\mu_2(v, w), \gamma_2(v, w)) = (0.1, 0.1)$, $(\mu_2(w, x), \gamma_2(w, x)) = (0.1,0.1)$, $(\mu_2(x ,u), \gamma_2(x, u)) = (0.1 , 0.1)$, $(\mu_2(u, w), \gamma_2(u, w)) = (0.3,0.4)$, $(\mu_2(x ,v), \gamma_2(x, v)) = (0.4 , 0.5)$.
Here, d(u) = d(x) = (0.6, 0.7).
That is G^c is not neighbourly irregular IFG.

Proposition 3.10:
The converse of the above result is not true. (i.e) If a IFG G = < V, E > is not neighbourly irregular, then its complement need not neighbourly irregular.

Example 3.11:

Let G= <V,E> be a IFG. We define G as, $(\mu_1(u), \gamma_1(u)) = (0.5, 0.4)$, $(\mu_1(v), \gamma_1(v)) = (0.4, 0.2)$, $(\mu_1(w), \gamma_1(w)) = (0.5,0.3)$, $(\mu_1(x), \gamma_1(x)) = (0.4, 0.3)$, $(\mu_1(y), \gamma_1(y)) = (0.6, 0.3)$ and $(\mu_2(u,v), \gamma_2(u,v)) = (0.4,0.4)$, $(\mu_2(u, w), \gamma_2(u, w)) = (0.1,0.3)$, $(\mu_2(u ,x), \gamma_2(u, x)) = (0.2 , 0.2)$, $(\mu_2(u, y), \gamma_2(u, y)) = (0.5,0.3)$,

$(\mu_2(v, w), \gamma_2(v, w)) = (0.3, 0.2)$, $(\mu_2(w, x), \gamma_2(w, x)) = (0.3,0.1)$, $(\mu_2(x, y), \gamma_2(x, y)) = (0.2,0.3)$.
Clearly G and G^c are not neighbourly irregular IFG, since d(x)=d(y) = d(v) = d(w) =(0.7,0.6) in G and d(w) = d(y) = (1.1,0.7) ; Also d(v) = d(x) =(0.9,0.7) which are adjacent vertices in G^c .

Remark:

(i) A complete IFG need not be neighbourly irregular

(ii) In Fuzzy case, G = < V, E > is neighbourly irregular, then G^c always not neighbourly irregular. But in Intuitionistic Fuzzy case, we can't say always, but may not be neighbourly irregular.

IV. Totally Irregular Intuitionistic Fuzzy Graphs

Definition 4.1: Let G = < V, E > be a IFG. Then G is totally irregular, if there is a vertex which is adjacent to vertices with distinct total degrees.

Example

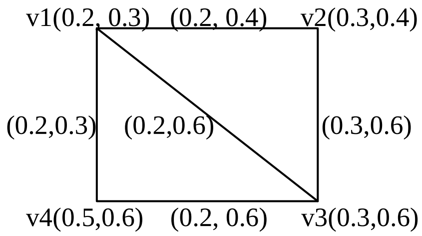

Fig-5: Totally Irregular IFG

Definition 4.2: If every two adjacent vertices of a IFG , G = < V, E > have distinct total degree, then G is said to be a neighbourly total irregular IFG.

Example:

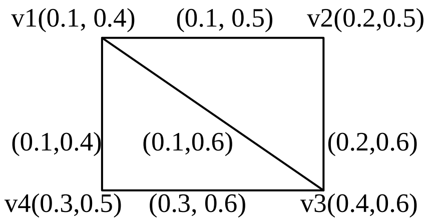

Fig-6: Neibourly totally irregular IFG

.**Proposition 4.3:**

A neighbourly irregular Intuitionistic fuzzy graph need not be a neighbourly total irregular IFG.

Example:

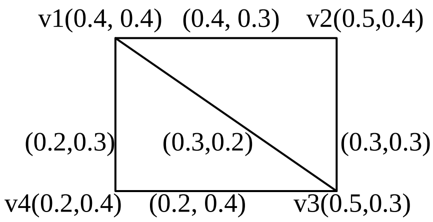

Fig-7: Neibourly irregular IFG but not total Irregular

Proposition 4.5:

A neighbourly total irregular fuzzy graph need not be a neighbourly irregular fuzzy graph

Example:

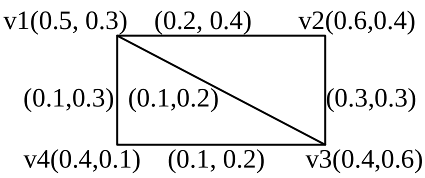

Fig-8: Neighbourly total irregular IFG but not neighbouly Irregular

Theorem 4.6:
Let G = <V, E> be IFG. If G is neighbourly irregular and μ_1 and γ_1 are constant functions, then G is a neighbourly total irregular IFG.

Proof:
Assume G = <V, E> is neighbourly Irregular IFG. i.e) the degrees of every two adjacent vertices are distinct.
Consider two adjacent vertices v1 and v2 with distinct degrees say (c_1, c_2) and (k_1, k_2).
Then $d(v1) = (c_1, c_2)$ and $d(v2) = (k_1, k_2)$ where $c_1 \neq k_1$, $c_2 \neq k_2$.
Also assume $\mu_1(v1) = \mu_1(v2)$ =a and $\gamma_1(v1) = \gamma_1(v2)$ =b ; a, b are constant and in [0,1] .
Therefore, $td_\mu(v1) = d_\mu(v1) + \mu_1(v1) = c_1 + a$; $td_\mu(v2) = d_\mu(v2) + \mu_1(v2) = k_1 + a$
To prove , $td_\mu(v1) \neq td_\mu(v2)$
Suppose, $td_\mu(v1) = td_\mu(v2)$
$c_1 + a = k_1 + a \Rightarrow c_1 - k_1 = a - a = 0 \Rightarrow c_1 = k_1$ which is contradiction to $c_1 \neq k_1$
Therefore the total µ-degrees of vertices are distinct.
Similarly, If γ_1 is constant, the total γ -degrees of vertices are distinct.
That is G is neighbourly total irregular IFG.

Theorem 4.7:
Let G = <V, E> be a neighbourly total irregular IFG and µ1 and γ1 are constant functions, then G is a neighbourly irregular IFG

Proof:
Assume G = <V, E> is neighbourly total Irregular IFG. i.e) the total degrees of every two adjacent vertices are distinct.
Consider two adjacent vertices v1 and v2 with distinct degrees say (c_1, c_2) and (k_1, k_2).Then $d(v1) = (c_1, c_2)$ and $d(v2) = (k_1, k_2)$ where $c_1 \neq k_1$, $c_2 \neq k_2$.
Also, $\mu_1(v1) = \mu_1(v2)$ =a and $\gamma_1(v1) = \gamma_1(v2)$ =b ; a, b are constant and in [0,1]
Assume $td_\mu(v1) \neq td_\mu(v2)$
To prove , $d_\mu(v1) \neq d_\mu(v2)$
As $td_\mu(v1) \neq d_\mu(v2)$
$c_1 + a \neq k_1 + a \Rightarrow c_1 \neq k_1$
Therefore the µ-degrees of adjacent vertices are distinct if µ1 is constant function.
Similarly, we prove If γ_1 is constant, the γ -degrees of adjacent vertices are distinct.
i.e) $c_2 + b \neq k_2 + b \Rightarrow c_2 \neq k_2$
This is true for all the pair of adjacent vertices
Therefore G is neighbourly irregular IFG.

Proposition 4.8:
Let G = <V, E> be neighbourly irregular IFG then Intuitionistic fuzzy subgraph H of G need not be neighbourly irregular IFG.

Example:

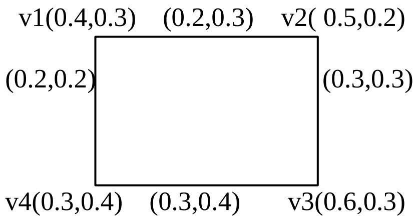

Fig-9: Neighbourly irregular IFG G

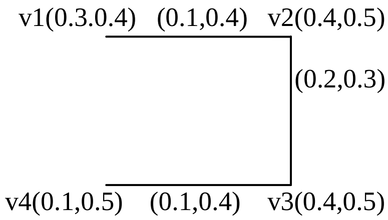

Fig-10: Not neighbourly Irregular Subgraph of G

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