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On Construction of A Control Operator Introduced To ECGM Algorithm for Solving Discrete-Time Linear Quadratic Regulator Control Systems with Delay-I

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Abstract: *In this paper, we constructed a control operator sequel to an earlier constructed control operator in one of our papers which enables an Extended Conjugate Gradient Method (ECGM) to be employed in solving discrete time linear quadratic regulator problems with delay parameter in the state variable. The construction of the control operator places scalar linear delay problems of the type within the class of problems that can be solved with the ECGM and it is aimed at reducing the rigours faced in using the classical methods in solving this class of problem. More so, the authors of this paper desire that, the application of this control operator will further improve the results of the ECGM as well as increasing the variant approaches used in solving the said class of optimal control problem.*
Keywords: *Control Operator, Optimal Control, Discrete-Time Linear Regulator Problem, Extended Conjugate Gradient Method and Differential Delay State.*

I. Introduction

In [1] we have considered the construction of a control operator for continuous-time linear regulator problems with delay parameter. This serves as a spring board and motivated the construction of a similar control operator for discrete-time linear quadratic regulator problems with delay parameter. The continuous-time linear quadratic regulator performance measure to be minimized considered by [6] and the Bolza problem of [4] and [3] as:

Problem (P1):

$$J(x,t_0,t_f,u(.)) = \frac{1}{2}x^T(t_f)Hx(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\{x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)\}dt \qquad 1.1$$

Subject to the differential delay state equation

$$\dot{x}(t) = C_1x(t) + C_2x(t-r) + Du(t), \quad t_0 \le t \le t_f, \qquad 1.2$$

$$x(t) = h(t), -r \le t \le 0 \qquad 1.3$$

where H and $Q(t)$ are real symmetric positive semi-definite $n \times n$ matries. $R(t)$ is a real symmetric positive definite $m \times m$ matrix, the initial time, t_0, and the final time, t_f, are specified. $x(t)$ is an n-dimensional state vector, $u(t)$ is the m-dimensional plant control input vector.$u(t)$ and $x(t)$ are not constrained by any boundaries. C_1, C_2 and D are specified constants which are not necessarily positive, the delay parameter, $r > 0$ and $h(t)$ is a given piecewise continuous function which is of exponential order on $[-r,0]$.

According to [2] and with similar report by [1], the controlled differential–delay constraint (1.2) constitutes an important model which has been used variously. Sequel to this, equation (1.1) can be rewritten as:

$$J = \int_{t_0}^{t_f}\{\frac{d}{dt}(\frac{1}{2}x^T(t_f)Hx(t_f))\}\,dt + \frac{1}{2}\int_{t_0}^{t_f}\{x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)\}dt \qquad 1.4$$

$$J = \int_{t_0}^{t_f}\{x^T(t)H\dot{x}(t)\}\,dt + \frac{1}{2}\int_{t_0}^{t_f}\{x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)\}dt \qquad 1.5$$

$$J = \int_{t_0}^{t_f}\left\{x^T(t)H\dot{x}(t) + \frac{1}{2}x^T(t)Q(t)x(t) + \frac{1}{2}u^T(t)R(t)u(t)\right\}dt. \qquad 1.6$$

As customary with penalty function techniques, constrained problem equations (1.2) and (1.6) may be put into the following equivalent form:

$$\langle z, Gz\rangle_k = min_{(x,u)}\int_{t_0}^{t_f}\{x^T(t)H\dot{x}(t) + \frac{1}{2}x^T(t)Q(t)x(t) + \frac{1}{2}u^T(t)R(t)u(t)$$
$$+ \mu\|C_1x(t) + C_2x(t-r) + Du(t) - \dot{x}(t)\|^2\}dt \qquad 1.7$$

$$x(s) = h(s);\ s \in [-r,0] \qquad 1.8$$

where $\mu > 0$ is the penalty parameter and $\|C_1x(t) + C_2x(t-r) + Du(t) - \dot{x}(t)\|^2$ is the penalty term. Let us denote by $\tilde{k}$ the product space

$$\tilde{k} = \mathcal{H}[t_0,t_f] \times \ell_2[t_0,t_f] \times \ell_2[-r,0] \qquad 1.9$$

of the Sobolev space $\mathcal{H}[t_0,t_f]$ of absolutely continuous function $x(.)$ such that, both $x(.) and\ \dot{x}(.)$ are square integrable over the finite interval $[t_0,t_f]$ and the Hilbert space $\ell_2[\alpha,\beta]$ of equivalence classes of real valued functions on $[\alpha,\beta]$ with norm defined by:

$\|f(.)\|_{\ell_2[\alpha,\beta]} = \left(\int_{\alpha}^{\beta}|f(t)|^2 dt\right)^{\frac{1}{2}}, f(.)\epsilon\ell_2[\alpha,\beta].$ 1.10

Then, the inner product $\langle .,.\rangle_{\tilde{k}}$ on $\tilde{k}$ is given by

$\langle .,.\rangle_{\tilde{k}} = \langle .,.\rangle_{\mathcal{H}[t_0,t_f]} + \langle .,.\rangle_{\ell_2[t_0,t_f]} + \langle .,.\rangle_{\ell_2[-r,0]}.$ 1.11

Suppose $z(t) \in \tilde{k}$ denotes ordered triple pair

$z^T(t) = (x(t), u(t), h(t));\ x(t) \in \mathcal{H}[t_0, t_f], u(t) \in \ell_2[t_0, t_f] and\ h(t) \in \ell_2[-r, 0],$ 1.12

then, we seek to determine the operator **G** on $\tilde{k}$ such that

$$\langle z, \boldsymbol{G}z\rangle_{\tilde{k}} = \int_{t_0}^{t_f}\{x^T(t)H\dot{x}(t) + \tfrac{1}{2}x^T(t)Q(t)x(t) + \tfrac{1}{2}u^T(t)R(t)u(t) + \mu\|C_1x(t) + C_2x(t-r) + Du(t) - \dot{x}(t)\|^2\}dt, \quad 1.13$$

where $\tilde{k}$ is suitably chosen Hilbert space. It follows from (1.9) that $z(t) \in \tilde{k}$ has the norm

$\|z(t)\|_{\tilde{k}}^2 = \|x(t)\|_{\mathcal{H}[t_0,t_f]}^2 + \|u(t)\|_{\ell_2[t_0,t_f]}^2 + \|h(t)\|_{\ell_2[-r,0]}^2.$ 1.14

According to [10], before the numerical procedure of dynamic programming can be applied, the state equation dynamic,(1.2), must be approximated by a difference equation, and the integral in the performance measure must be approximated by a summation. This can be done most conveniently by dividing the time interval $t_0 \le t \le t_f$ into N equal increments, Δt. Then, from (1.2), we have

$\frac{x(t+\Delta t)-x(t)}{\Delta t} \approx C_1x(t) + C_2x(t-r) + Du(t)$ 1.15

$x(t+\Delta t) - x(t) = \Delta tC_1x(t) + \Delta tC_2x(t-r) + \Delta tDu(t)$

$x(t+\Delta t) = [1 + \Delta tC_1]x(t) + \Delta tC_2x(t-r) + \Delta tDu(t).$ 1.16

Here, it will be assumed that Δt is small enough so that the control signal can be approximated by a piecewise constant function that changes only at the instants

$t = 0, \Delta t, 2\Delta t, \dots, (N-3)\Delta t, (N-2)\Delta t, (N-1)\Delta t$ 1.17

Thus,

$t = k\Delta t$ 1.18

putting (1.18) for *t* in (1.16), we obtain

$x((k+1)\Delta t) = [1 + \Delta tC_1]x(k\Delta t) + \Delta tC_2x(k\Delta t - r) + \Delta tDu(k\Delta t),$ 1.19

$\forall\ k = 0,1,\dots,N-1,$ where $x(k\Delta t)$ is referred to as the *kth* value of *x* and is denoted by $x(k)$. With this, the system difference equation (1.19) as can be written as:

$x(k+1) = [1 + \Delta tC_1]x(k) + \Delta tC_2x(k-r) + \Delta tDu(k).$ 1.20

Following the same trend as above, the performance measure, (1.6) becomes

$$\begin{aligned} J = &\int_{t_0}^{\Delta t}\left\{x^T(0)H\dot{x}(0) + \tfrac{1}{2}x^T(0)Qx(0) + \tfrac{1}{2}u^T(0)Ru(0)\right\}dt \\ &+\int_{\Delta t}^{2\Delta t}\left\{x^T(k)H\dot{x}(\Delta t) + \tfrac{1}{2}x^T(\Delta t)Qx(\Delta t) + \tfrac{1}{2}u^T(\Delta t)Ru(\Delta t)\right\}dt \\ &+\int_{2\Delta t}^{3\Delta t}\left\{x^T(2\Delta t)H\dot{x}(2\Delta t) + \tfrac{1}{2}x^T(2\Delta t)Qx(2\Delta t) + \tfrac{1}{2}u^T(2\Delta t)Ru(2\Delta t)\right\}dt + \cdots \\ &+\int_{(N-1)\Delta t}^{N\Delta t}\left\{\begin{matrix} x^T(N-1)\Delta tH\dot{x}(N-1)\Delta t + \frac{1}{2}x^T(N-1)\Delta tQx(N-1)\Delta t \\ +\frac{1}{2}u^T(N-1)\Delta tRu(N-1)\Delta t \end{matrix}\right\}dt \end{aligned} \quad 1.21$$

$$\begin{aligned} J = &\ t\left(x^T(0)H\dot{x}(0) + \tfrac{1}{2}x^T(0)Qx(0) + \tfrac{1}{2}u^T(0)Ru(0)\right)\Big|_0^{\Delta t} \\ &+t\left(x^T(\Delta t)H\dot{x}(\Delta t) + \tfrac{1}{2}x^T(\Delta t)Qx(\Delta t) + \tfrac{1}{2}u^T(\Delta t)Ru(\Delta t)\right)\Big|_{\Delta t}^{2\Delta t} \\ &+t\left(x^T(2\Delta t)H\dot{x}(2\Delta t) + \tfrac{1}{2}x^T(2\Delta t)Qx(2\Delta t) + \tfrac{1}{2}u^T(2\Delta t)Ru(2\Delta t)\right)\Big|_{2\Delta t}^{3\Delta t} + \cdots \\ &+t\left(\begin{matrix} x^T(N-1)\Delta tH\dot{x}(N-1)\Delta t + \frac{1}{2}x^T(N-1)\Delta tQx(N-1)\Delta t \\ +\frac{1}{2}u^T(N-1)\Delta tRu(N-1)\Delta t \end{matrix}\right)\Big|_{(N-1)\Delta t}^{N\Delta t} \end{aligned} \quad 1.22$$

$$\begin{aligned} J = &\ \Delta t\left(x^T(0)H\dot{x}(0) + \tfrac{1}{2}x^T(0)Qx(0) + \tfrac{1}{2}u^T(0)Ru(0)\right) \\ &+\Delta t\left(x^T(\Delta t)H\dot{x}(\Delta t) + \tfrac{1}{2}x^T(\Delta t)Qx(\Delta t) + \tfrac{1}{2}u^T(\Delta t)Ru(\Delta t)\right) \\ &+\Delta t\left(x^T(2\Delta t)H\dot{x}(2\Delta t) + \tfrac{1}{2}x^T(2\Delta t)Qx(2\Delta t) + \tfrac{1}{2}u^T(2\Delta t)Ru(2\Delta t)\right) \\ &+ \dots + \Delta t\left(\begin{matrix} x^T(N-1)\Delta tH\dot{x}(N-1)\Delta t + \frac{1}{2}x^T(N-1)\Delta tQx(N-1)\Delta t \\ +\frac{1}{2}u^T(N-1)\Delta tRu(N-1)\Delta t \end{matrix}\right). \end{aligned} \quad 1.23$$

On applying the same condition as in (1.20) to (1.23), we obtain

$$J = \Delta t\left(x^T(0)H\dot{x}(0) + \tfrac{1}{2}x^T(0)Qx(0) + \tfrac{1}{2}u^T(0)Ru(0)\right)$$
$$+\Delta t\left(x^T(1)H\dot{x}(1) + \tfrac{1}{2}x^T(1)Qx(1) + \tfrac{1}{2}u^T(1)Ru(1)\right)$$
$$+ \Delta t\left(x^T(2)H\dot{x}(2) + \tfrac{1}{2}x^T(2)Qx(2) + \tfrac{1}{2}u^T(2)Ru(2)\right)$$
$$+ \dots + \Delta t\begin{pmatrix} x^T(N-1)H\dot{x}(N-1) + \tfrac{1}{2}x^T(N-1)Qx(N-1) \\ + \tfrac{1}{2}u^T(N-1)Ru(N-1)\end{pmatrix}. \qquad 1.24$$

Then, (1.24) can now be written using summative convention as:

$$J = \Delta t\sum_{k=0}^{N-1}[x^T(k)H\dot{x}(k) + \tfrac{1}{2}x^T(k)Qx(k) + \tfrac{1}{2}u^T(k)Ru(k)]. \qquad 1.25$$

From the above, the performance measure of the discrete-time linear regulator problem subject to the difference delay state equation of the form (1.20) and (1.25) can now be written in the constrained form:

$$\langle t_f, Gt_f\rangle = \underset{(x,u)}{Min} \sum_{k=0}^{N-1}\begin{Bmatrix} \Delta t[x^T(k)H\dot{x}(k) + \tfrac{1}{2}x^T(k)Qx(k) + \tfrac{1}{2}u^T(k)Ru(k)] \\ +\mu\|x(k+1) - [1+\Delta tC_1]x(k) - \Delta tC_2x(k-r) - \Delta tDu(k)\|^2\end{Bmatrix}. \qquad 1.26$$

Using [5] and [8], the discretized unconstrained problem is as:

$$Min\, J(x,u,\mu) = \sum_{k=0}^{N-1}\begin{Bmatrix} \Delta t[x^T(k)H\dot{x}(k) + \tfrac{1}{2}x^T(k)Qx(k) + \tfrac{1}{2}u^T(k)Ru(k)] + \mu(x(k+1) \\ -[1+\Delta tC_1]x(k) - \Delta tC_2x(k-r) - \Delta tDu(k))^2\end{Bmatrix} \qquad 1.27$$

On substituting the RSH of (1.20) into (1.27), we obtain

$$Min\, J(x,u,\mu) = \sum_{k=0}^{N-1}\begin{Bmatrix} \Delta t[x^T(k)H[1+\Delta tC_1]x(k) - \Delta tC_2x(k-r) - \Delta tDu(k)] \\ + \tfrac{1}{2}x^T(k)Qx(k)] + \tfrac{1}{2}u^T(k)Ru(k)] + \mu\,(x(k+1) \\ -[1+\Delta tC_1]x(k) - \Delta tC_2x(k-r) - \Delta tDu(k))^2\end{Bmatrix} \qquad 1.28$$

Expanding and simplifying (1.28) at $\Delta t = 1$,we obtain

$$Min\, J(x,u,\mu) = \sum_{k=0}^{N-1}\begin{Bmatrix} (1+C_1)x^T(k)Hx(k) - C_2x^T(k)Hx(k-r) - x^T(k)Du(k) \\ + \tfrac{1}{2}x^T(k)Qx(k) + \tfrac{1}{2}u^T(k)Ru(k) \\ +\mu[x^2(k+1) - 2(1+C_1)x(k+1)x(k) - 2C_2x(k+1)x(k-r) \\ -2x(k+1)Du(k) + 2C_2x(k-r)Du(k) + 2C_2(1+C_1)x(k)x(k-r) \\ 2(1+C_1)x(k)Du(k) + (1+C_1)^2x^2(k) + C_2^2x^2(k-r) + D^2u^2(k)]\end{Bmatrix} \qquad 1.29$$

The next section shall focus on discussing the CGM algorithm for completeness sake, since the constructed operator is to be applied in the ECGM algorithm which is an application of the CGM to optimal control problems.

II. Conjugate Gradient Method Algorithm

The conjugate Gradient Method (CGM) is a variant of the gradient method. In its simplest form, the gradient method uses the iterative scheme

$$x_{i+1=}\ x_i - \alpha\nabla F(x) \qquad 2.1$$

to generate a sequence $\{x_i\}_{i=1}^{n}$ of vectors which converge to the minimum of F(x). The parameter α appearing in (2.1) denotes the step length of the descent direction sequence. In particular, if F is a functional on a Hilbert space $\mathcal{H}$ such that in $\mathcal{H}$, F admits a Taylor series expansion

$$F(x) = F_0 + \langle a, x\rangle_H + \tfrac{1}{2}\langle x, Gx\rangle_H \qquad 2.2$$

where a, $x \in \mathcal{H}$ and is a positive definite, symmetric, linear operator, then it can be shown by [7] that F possesses a unique minimum x^* say in $\mathcal{H}$, and that $\nabla F(x^*) = 0$. The CGM algorithm for iteratively locating the minimum x^* of $F(x)$ in $\mathcal{H}$ as described by [7] is as follows:

Step 1: Guess the first element $x_0 \in \mathcal{H}$ and compute the remaining members of the sequence with the aid of the formulae in the steps 2 through 6.

Step 2: Compute the descent direction $p_0 = -g_0$ 2.3a

Step 3: Set $x_{i+1=}\ x_i + \alpha_i p_i$; where $\alpha_i = \frac{\langle g_i, g_i\rangle_{\mathcal{H}}}{\langle p_i, Gp_i\rangle_{\mathcal{H}}}$ 2.3b

Step 4: Compute $g_{i+1} = g_i + \alpha_i Gp_i$ 2.3c

Step 5: Set $p_{i+1} = -g_{i+1} + \beta_i p_i$; $\beta_i = \frac{\langle g_{i+1}, g_{i+1}\rangle_{\mathcal{H}}}{\langle g_i, g_i\rangle_{\mathcal{H}}}$ 2.3d

Step 6: If $g_i = 0$ for some *i*, then, terminate the sequence; else set *i* = *i* + 1 and go to step 3.

In the iterative steps 2 through 6 above, p_i denotes the descent direction at *i*-th step of the algorithm,α_i, is the step length of the descent sequence $\{x_i\}$ and g_i denotes the gradient of F at x_i. Steps 3, 4 and 5 of the algorithm reveal the crucial role of the linear operator G in determining the step length of the descent sequence and also in generating a conjugate direction of search.

III. Necessary Ingredients for Conjugate Gradient Method

In a view to constructing the control operator, there is need to determine the ingredients of the operator in what follows as:

3.1 The Gradient of the Algorithm

From (1.29), the gradient is given as:

$$\begin{pmatrix} \nabla J_{x_k} \\ \nabla J_{u_k} \\ \nabla J_{x_{k+1}} \end{pmatrix} = \begin{pmatrix} \sum_{k=0}^{N-1} \begin{Bmatrix} 2(1+C_1)x(k)H - C_2Hx(k-r) - HDu(k) + x(k)Q \\ +\mu[-2(1+C_1)x(k+1) + 2C_2(1+C_1)x(k-r) \\ +2(1+C_1)Du(k) + 2(1+C_1)^2x(k)] \end{Bmatrix} \\ \sum_{k=0}^{N-1} \begin{Bmatrix} -x^T(k)HD + u(k)R + \mu[-2x(k+1)D + 2C_2x(k-r)D \\ +2(1+C_1)x(k)D + 2D^2u(k)] \end{Bmatrix} \\ \sum_{k=0}^{N-1}\{\mu[2x(k+1) - 2(1+C_1)x(k) - 2C_2x(k-r) - 2Du(k)\} \end{pmatrix} \quad 3.1$$

3.2 Descent Direction of the Algorithm

Also, from (1.29) and (3.1), the descent direction to be used in the CGM algorithm is given as:

$$P = -\begin{pmatrix} \nabla J_{x_k} \\ \nabla J_{u_k} \\ \nabla J_{x_{k+1}} \end{pmatrix} = \begin{pmatrix} \sum_{k=0}^{N-1} \begin{Bmatrix} -2(1+C_1)x(k)H + C_2Hx(k-r) + HDu(k) - x(k)Q \\ -\mu[-2(1+C_1)x(k+1) + 2C_2(1+C_1)x(k-r) \\ +2(1+C_1)Du(k) + 2(1+C_1)^2x(k)] \end{Bmatrix} \\ \sum_{k=0}^{N-1} \begin{Bmatrix} x^T(k)HD - u(k)R - \mu[-2x(k+1)D + 2C_2x(k-r)D \\ +2(1+C_1)x(k)D + 2D^2u(k)] \end{Bmatrix} \\ \sum_{k=0}^{N-1}\{-\mu[2x(k+1) - 2(1+C_1)x(k) - 2C_2x(k-r) - 2Du(k)\} \end{pmatrix} \quad 3.2$$

3.3 The Control Operator

As shown by [7] in (2.2), F admits a Taylor series expansion and the linear operator, G, is a Hessian matrix which we shall go on to determine in what follows from (1.29) as:

$\nabla^2 J_{x_k} = \sum_{k=0}^{N-1}\{2(1+C_1)H + Q + 2\mu(1+C_1)^2\}$, 3.3a

$\nabla^2 J_{x_k u_k} = \sum_{k=0}^{N-1}\{-HD + 2\mu(1+C_1)D\}$, 3.3b

$\nabla^2 J_{x_k x_{k+1}} = \sum_{k=0}^{N-1}\{-2\mu(1+C_1)\}$, 3.3c

$\nabla^2 J_{u_k x_k} = \sum_{k=0}^{N-1}\{-HD + 2\mu(1+C_1)D\}$, 3.3d

$\nabla^2 J_{u_k u_k} = \sum_{k=0}^{N-1}\{R + 2\mu D^2\}$, 3.3e

$\nabla^2 J_{u_k x_{k+1}} = \sum_{k=0}^{N-1}\{-2\mu D\}$, 3.3f

$\nabla^2 J_{x_{k+1} x_k} = \sum_{k=0}^{N-1}\{-2(1+C_1)\}$, 3.3g

$\nabla^2 J_{x_{k+1} u_k} = \sum_{k=0}^{N-1}\{-2D\}$ and 3.3h

$\nabla^2 J_{x_{k+1} x_{k+1}} = \sum_{k=0}^{N-1}\{2\mu\}$. 3.3i

$$G = \sum_{k=0}^{N-1} \begin{pmatrix} 2(1+C_1)H + Q + 2\mu(1+C_1)^2 & -HD + 2\mu(1+C_1)D & -2\mu(1+C_1) \\ -HD + 2\mu(1+C_1)D & R + 2\mu D^2 & -2\mu D \\ -2(1+C_1) & -2\mu D & 2\mu \end{pmatrix} \quad 3.3$$

The required control operator is the product of (3.2) and (3.3), given by our next equation as:

$$GP = -\sum_{k=0}^{N-1} \begin{pmatrix} 2(1+C_1)H + Q + 2\mu(1+C_1)^2 & -HD + 2\mu(1+C_1)D & -2\mu(1+C_1) \\ -HD + 2\mu(1+C_1)D & R + 2\mu D^2 & -2\mu D \\ -2(1+C_1) & -2\mu D & 2\mu \end{pmatrix} \begin{pmatrix} \nabla J_{x_k} \\ \nabla J_{u_k} \\ \nabla J_{x_{k+1}} \end{pmatrix} \quad 3.4$$

From (3.4), we obtain the entries of the control operator as:

$$GP_{11} = \sum_{k=0}^{N-1} \begin{Bmatrix} [2(1+C_1)H + Q + 2\mu(1+C_1)^2]\{-2(1+C_1)x(k)H + C_2Hx(k-r) \\ +HDu(k) - x(k)Q + \mu[-2(1+C_1)x(k+1) + 2C_2(1+C_1)x(k-r) \\ +2(1+C_1)Du(k) + 2(1+C_1)^2x(k)]\} + [-HD + 2\mu(1+C_1)D] \\ \times\{x^T(k)HD - u(k)R - \mu[-2x(k+1)D + 2C_2x(k-r)D \\ +2(1+C_1)x(k)D + 2D^2u(k)]\} + [-2\mu(1+C_1)]\{\mu[2x(k+1) \\ -2(1+C_1)x(k) - 2C_2x(k-r) - 2Du(k)\} \end{Bmatrix} \quad 3.5$$

$$GP_{21} = \sum_{k=0}^{N-1} \left\{ \begin{array}{c} [-HD + 2\mu(1 + C_1)D]\{-2(1 + C_1)x(k)H + C_2Hx(k-r) \\ +HDu(k) - x(k)Q + \mu[-2(1 + C_1)x(k+1) + 2C_2(1 + C_1)x(k-r) \\ +2(1 + C_1)Du(k) + 2(1 + C_1)^2x(k)]\} + [R + 2\mu D^2] \\ \times \{x^T(k)HD - u(k)R - \mu[-2x(k+1)D + 2C_2x(k-r)D \\ +2(1 + C_1)x(k)D + 2D^2u(k)]\} + [-2\mu D]\{\mu[2x(k+1) \\ -2(1 + C_1)x(k) - 2C_2x(k-r) - 2Du(k)\} \end{array} \right\} \quad 3.6$$

$$GP_{31} = \sum_{k=0}^{N-1} \left\{ \begin{array}{c} [-2\mu(1 + C_1)]\{-2(1 + C_1)x(k)H + C_2Hx(k-r) + HDu(k) \\ - x(k)Q + \mu[-2(1 + C_1)x(k+1) + 2C_2(1 + C_1)x(k-r) \\ +2(1 + C_1)Du(k) + 2(1 + C_1)^2x(k)]\} + [-2\mu D]\{x^T(k)HD \\ -u(k)R - \mu[-2x(k+1)D + 2C_2x(k-r)D + 2(1 + C_1)x(k)D \\ +2D^2u(k)]\} + [2\mu]\{\mu[2x(k+1) - 2(1 + C_1)x(k) - 2C_2x(k-r) \\ - 2Du(k)\} \end{array} \right\}. \quad 3.7$$

Then control operator, $GP = (GP_{11} \quad GP_{21} \quad GP_{31})^T$, can now be used in steps 3, 4 and 5 of the CGM algorithm in determining the step length, α, of the descent sequence and also in generating a conjugate descent direction of search.

IV. Conclusion:

It follows from here that, while [9] constructed an operator for CLRP, [2] focuses on same class of optimal control problem but with delay parameter in the state variable. The construction of this control operator, G, helps to bridge the gap between Bolza problems and CLRP with delay parameter via discretization of the continuous linear regulator problem. This makes the construction of the operator very important and relevant in that, it takes cares of the CLRP without delay parameter in the state variable.

Based on this, our next paper shall be devoted to a detailed exposition of the construction and computational application of the control operator to CGM algorithm in solving Discrete-Time Linear Regulator Problems with delay parameter in the state variable with numerical comparison of the operator with [1] shall be exhibited.

References

[1] ADEBAYO,K. J. and ADERIBIGBE, F. M., (2014), On Construction of A Control Operator Applied In Conjugate Gradient Method Algorithm For Solving Continuous Time Linear Regulator Problems With Delay – I

[2] Aderibigbe, F. M., (1993), "An Extended Conjugate Gradient Method Algorithm For Control Systems with Delay-I, Advances in Modeling & Analysis, C, AMSE Press, Vol. 36, No. 3, pp 51-64.

[3] Athans, M. and Falb, P. L., (1966), Optimal Control: An Introduction to the Theory and Its Applications, McGraw-Hill, New York.

[4] David, G. Hull, (2003), Optimal Control Theory for Applications, Mechanical Engineering Series, Springer-Verlag, New York, Inc., 175 Fifth Avenue, New York, NY 10010.

[5] Gelfand, I. M. and Fomin, S. V., (1963), Calculus of Variations, Prentice Hall, Inc., New Jersey.

[6] George M. Siouris, (1996), An Engineering Approach To Optimal Control And Estimation Theory, John Wiley & Sons, Inc., 605 Third Avenue, New York, 10158- 00 12.

[7]. Hasdorff, L. (1976), Gradient optimization and Nonlinear Control. J. Wiley and Sons, NewYork.

[8] Hestenes, M. R. and Stiefel, E.,(1952), Method of Conjugate Gradients for Solving Linear Systems, J. Res. Nat. Bur. Standards, Vol. 49, pp 409-436.

[9] Ibiejugba, M. R. and Onumanyi, P., (1984), "A Control Operator and some of its Applications, J. Math. Anal. Appl. Vol. 103, No. 1, Pp. 31-47.

[10] Kirk, E. Donald, (2004), Optimal control theory: An introduction, Prentice-Hall, Inc., Englewood Cliff, New Jersey.