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Comparison of Service Delivery by Atm in Two Banks: Application of Queuing Theory.

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Abstract: *Queuing Theory probabilistic methods as well as Birth- Death processing were discussed in this paper. The above method was applied to Access Bank and United Bank of Africa, Ado Ekiti branch. The distribution of arrival rate was Poisson distribution while queuing discipline in the branch was first come first served. This paper also emphasized on Markovian in Queues. From analysis, we were able to obtain the average arrival rate, average service rate, average time spent in the queue for Access bank as 2.01, 1.65, 0.5 respectively and UBA as 3.28, 1.75, 1.67minutes, respectively. The average number of waiting in the system and idle time were obtained for Access as 3minutes, 0.61(61%) and UBA as 7minutes, 0.86(86%) respectively. The utilisation factor for Access bank is 0.58 and that of UBA is 0.38.*
Keywords: *Queue, Balking, Reneging, Traffic Intensity, Customers, Arrival Rate, M/M/1, Server.*

I. Introduction

Waiting for service is part of our daily life. We wait to eat in restaurants, 'we queue up' at the check-out counters in grocery stores and we 'line up' for service in post offices. And the waiting phenomenon is not an experience limited to human beings only. Jobs wait to be processed on a machine, planes circle in stack before given permission to land at an airport and cars stop at traffic lights. Unfortunately, we cannot eliminate waiting without incurring inordinate expenses. In fact, all we can hope to achieve is to reduce its adverse impact to tolerable levels. From analysis, we were able to obtain the average arrival rate, average service rate, average time spent in the queue, among other things, for the two banks, from which necessary comparisons were made. J. Jackson (1957, 1963) made notable contributions to the development of queuing network models. A Jackson model is probably the most researched and widely applied network model in various fields, including the healthcare field and Banking sector. Jackson's major contribution was to find a "product-form" steady-state solution for in open and closed models with a tandem or a feed-forward flow configuration. In a network model, various numbers of entities can exist at multiple stations, and the state of the system is described by the joint probability distribution for the number of entities at each station.

Numerous theoretical works were published which expanded the Jackson model, and many of those examined or modified the Jackson properties (i) and (ii), in particular, in an open model. Among those, Disney (1981) and Melamed (1979) are widely known. Disney (1981) examined the internal arrival rate distribution with feedback flow as a generalization of Jackson model. His research showed that when a system has any kind of feedback flow, the internal flows in the system do not follow the Poisson distribution. Thus, the assumption of Poisson arrival is justified only when the system under consideration has either a tandem or arbitrarily linked network configuration with feed-forward flows. It is, however, known that the Jackson's product-form solution holds regardless of whether or not internal flows are Poisson. 14 Melamed (1979) extended Burke's finding in an open Jackson system. He showed that departure rates from internal stations to outside the system are mutually independent if arriving rates to all internal stations follow the Poisson distribution. The finding, in turn, means that the sum of all departure rates from the network must also be Poisson.

In general, a queue is formed at a queuing system when either customers (human beings or physical entities) requiring service wait due to number of customers exceeds the number of service facilities or service facilities do not work efficiently and take more time than prescribed to serve a customer. Therefore, a queue is situation where people or items are on a line waiting to be served.

Queuing theory can be applied to a variety of operational situation where it is not possible to predict accurately the arrival rate or time of customers and service rate or time of service facilities. In particular, it can be used to determine the level of service either the service rate or the number of service facilities that balances the following two conflicting cost;

- Cost of offering the service
- Cost incurred due to delay in offering service.

This paper analyses the queuing situation at Access Bank and UBA plc Ado Ekiti in order to secure some characteristics that measure the performances of the bank's operation.

Definition of Terms
Queue: A group of items waiting for service in a service station is known as Queue. It may be finite or infinite.
Server: A server is a person by whom service is rendered.
Queuing System: System considering of arrival of customers, waiting in queue, picked up for service, being served and departure of customers.
Waiting Time: The time to which a customer has to wait in queue before taken to service.
Traffic Intensity: the ratio between mean arrival time rate and mean service rate.
λ stands for mean arrival rate of customers.
μ stands for mean service rate of customers.

II. Matrials And Methods

The data used in this paper were collected in respect of the arriving customers and served customers in interval of time. The data collected were analysed using the following techniques:

ARRIVAL RATE (λ) : ACCESS BANK

Since arrival is Poisson with probability density function defined as

$$f(t) = \frac{\lambda e^{-t}}{t!}, \quad t = 0,1......$$

with mean X.

The mean/average number of arrival(X) is given as

$$\lambda = \frac{\sum XiFi}{\sum fi}$$

- **SERVICE RATE (μ)**

$$\mu = \frac{Total\ number\ of\ customers\ served}{Total\ service\ time}$$

Recall that the service rate is exponential.
The exponential probability density function is defined as:

$$f(t) = \sum_{0}^{\beta} e^{-\beta t} \qquad \begin{matrix} t>0 \\ t\leq 0 \end{matrix}$$

With mean $1/\beta$

$\therefore$the mean service rate $(\mu) = 1/\beta$

- **Traffic Intensity (ρ)**

$$\rho = \frac{1/\mu}{1/\lambda}$$

PERFORMANCE MEASURE

PROBABILITY THAT THE SYSTEM IS IDLE (P_O), NO CUSTOMER

$$P_O = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{S!} \left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{S\mu - \lambda}\right]^{-1}$$

- **EXPECTED WAITING TIME OF A CUSTOMER IN THE QUEUE (W_q)**

$$W_q = \frac{L_q}{\lambda}$$

- **EXPECTED WAITING TIME THAT A CUSTOMER SPENDS IN THE SYSTEM (W_s)**

$$W_s = W_q + \frac{1}{\mu}$$

- **PROBABILITY THAT ALL SERVERS ARE SIMULTANEOUSLY BUSY (UTILIZATION FACTOR)**

$$= \frac{1}{S!}\left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu - \lambda} P_O$$

Transient-State And Steady –State

When a service system is started, it progresses through a number of changes. However, it attains stability after some time. Before the start of the service operations, it is very much influenced by the initial conditions (number of customers in the influenced system) and the elapsed time. This period of transition is termed as transient-state. However, after sufficient time has passed, the system becomes independent of the initial condition and of the elapsed time (except under very special conditions) and enters a steady-state condition.

In the development of queuing theory models, it is assumed that the system has entered a steady state primarily for two reasons:

- System most of the time operates in the steady state condition
- Transient case is much more complex.

Let P_n (t) dente the probability that there are customers in the system at time t. the rate of change in the value P_n (t) with respect to time t is denoted by the derivative of P_n with respect to t that is P_n (t). in the case of the steady-state we have:

If in some cases, arrival rate of customers t the system is more than the service rate, then a steady-state cannot be reached regardless of the length of the elapsed time.

Relationship Among Perfomance Measures

By definition of various measures of performance, we have:

$L_s = \sum nP_n$ and n-o

$L_a = \sum(n\text{-}s)pn$ n-s

General relationships among various performance measures is as follows

- Expected number of customers in the system is equal to the expected number of customer in queue plus in service.

  $L_s = L_a$ + expected number customers in service

  $L_s = L_a + \lambda/\mu$
- Expected waiting time of customers in the system is equal to the average waiting time on queue plus the expected service time:

  $W_s = W_a + 1/\mu$
- Probability of being in the system (waiting and being served) longer than time t is given by

  $P(T\text{-}t) = e^{-(\mu-\lambda)t}$; $P(T\text{-}t) = 1\text{-}P(T\text{-}t)$

  Where T = time spent in the system

  t = specified time period

  e = 2.718
- Probability of only waiting for service longer than the time t is given by:

  $P(T\text{-}t) = \lambda/\mu(e^{-(\mu-\lambda)t})$
- Probability of exactly n customers in the system is given by:

  $P_n\ (1 \text{ x } \lambda/\mu)(\ \lambda/\mu \text{ x } n)$
- Probability that the number of customers in the system n exceeds a given number is given by:

  $P(n>r) = (\lambda/\mu)r+1$
- Expected number of customers in the system is equal to the average number of arrivals per unit of time multiplied by the average time spent by the customer in the system:

  $L_s = \lambda W_s$ or $W_s = W_a + 1/\mu = 1/\mu\ L_s$:

  $L_a = L_s + \lambda/\mu = \lambda W_a$ or $W_a = W_s + 1/\mu = 1/\lambda(L_a)$

For applying formula (vii) and (viii) for system of finite queue, instead of using λ its effective value $\lambda(1\text{-}P_N)$ must be used.

The probability, P_n of n customers in the queuing system at any time can be used to determine all the basic measures of performance on the following order:

$L_s = \sum/n{=}o$ x npn --------$W_s = L_s/\lambda$ x $W_a = W_s + 1/\mu$ ------- $L_a = \lambda W_a$

Probability Distributions In Queuing System

It is assumed that customers joining a queuing system arrive in random manner and follow a poisson distribution or equivalently the inter-arrival times follow exponential assumed to be exponentially distributed. It implies that the probability of service completion in any short-time interval is constant and independent of the length of time that the service has been in progress. The basic reasons for assuming exponential service is that it helps in formulating simple mathematical models which ultimately help in analyzing a number of aspects of queuing problems.

The number of arrivals and departures during an interval of time in a queuing system is controlled by the following assumptions also axioms:

- The probability of an event (arrival or departure) occurring during the time interval $(t, t + \Delta t)$ depends on the length of time interval Δt that is, probability of the event not depending either on number of events that occur up to time t or the specific value of t, meaning that the events that occur in non-over lapping time are statistically independent. The probability of more than one event occurring during the time interval, $(t, t + \Delta t)$ is negligible. It is denoted by $0(\Delta t)$.
- Almost one event (arrival or departure) can occur during a small time interval Δ t. The probability of an arrival during the time interval $(t, t + \Delta t)$ is given by; $P_t(\Delta t) = (\Delta t + 0(\Delta t))$ where λ is a constant and independent of the total number of arrivals up to t; Δt is a small time interval and $0(\Delta t)$ represents the quantity that becomes negligible when compared to Δt as $\Delta t \to 0$ i.e $\lim_{\Delta t \to 0} \left(\frac{0(\Delta t)}{\Delta t}\right) = 0$

Distribution Of Arrival (Birth Process)

The arrival process assumes that the customers arrive at the queuing system and never leave it. Such a process is called pure birth process. The aim is to derive an expression for the probability $P_n(t)$ of arrivals during time interval $(t, t + \Delta t)$: the terms commonly used in the development of various queuing models are the following.

Δt = a time interval so small that the probability of more than one customer's arrival is negligible.

Δt = probability that a customer will arrive in the system during time Δt.

$1-\lambda\ \Delta t$ = probability that no customer will arrive in the system during the time Δt.

If the arrivals are completely random, then the probability distribution of a number of arrivals in a fixed time interval follows a Poisson distribution.

Distribution Of Inter Arrival Times

(Exponential Process): if the number of arrival, n in time t follows the Poisson distribution then:

$$P_n(t) = (\lambda t)/n! \quad e - \lambda t,\ n = 0,1,2, \ldots$$

Is an associated random variable defined as the inter-arrival time T follow the exponential distribution $F(t) = \lambda e^{-\lambda t}$ and vice-versa.

The markovian property of inter-arrival time states that the probability that a customer currently in service is completed at some time t is dependent on how long it has already been in service, that is;

$$P(T \geq t_1 / T \geq t_0) = P(0 \leq T \leq t_1 - t_0)$$

Where T = the time between successive arrivals

Distribution Of Departure (Death Process)

The departure process assumes that no customer joins the system while service is continued for those who are steady in the system. Let, at time t = 0 (starting time) there be $\mu \geq 1$ customers in the system. Since service is being provided at the rate of μ, therefore customers leave the system at the rate of μ after being serviced. Such a process is called pure death process.

AXIOMS:

- Probability of the departure during time Δt is $\mu\ \Delta t$.
- Probability of more than one departure between time t and $t + \Delta t$ is negligible.
- The number of departures on non-overlapping intervals is statistically independent.

The following terms are used in the development of various queuing models.

$\mu\ \Delta t$ = probability that a customer in service at time t will complete service during time Δt.

$1-\mu\ \Delta t$ = probability that the customer in service at time t will not complete service during time Δt

Distribution Of Service Terms

The probability density functions (t) of service time is given by:

$$S(t) = \begin{matrix} \mu e - \ \mu t & 0 \leq t \leq & \infty \\ 0 & ; t < & 0 \end{matrix}$$

This shows that service times follow negative experimental distribution with mean 1/μ and variance $1/\mu^2$. As the service times increase, the probability of occurrence tails off (experimentally) towards zero probability. The area under the negative exponential distribution curve is determined as $F(T) = \mu e^{-\mu tdt}$ = -μe - μt 0 t

$$=-e^{-\mu t} + e^0 = 1 - e^{-\mu t}$$

III. Results And Discussion

A total number of three hundred and sixty customers(360) were examined between the hours of 8.00a.m and 2.00p.m at the interval of five minutes for arrival. It was found out that the mean arrival rate and mean service rate for both banks are:

- ACCESS BANK: 2.01 and 1.65

This showed that the arrival rate is greater than the service rate. This shows that there is queue. It can be concluded that 2 customers entered the system in a 5 minutes interval and the service rate for each customer is1.65mins

- UBA : 3.275 and 1.75

This showed that the arrival rate is greater than the service rate. This shows that there is queue. It can be concluded that 3 customers entered the system in a 5 minutes interval and the service rate for each customer is1.75mins

It can be concluded that the queue in UBA is more than that of Access bank.

While 2 ATM machines were observed.

- The potential utilisation was calculated to be 0.58% for Access Bank and 0.38% for UBA. This shows that UBA was far below efficiency compared to Access Bank.
- The traffic intensity for Access Bank is 0.61 i.e. 61% and that of UBA is 0.86 which amount to 86%.

BANK	P_o	L_q	L_s	W_q	W_s	$W_{q+} W_s$
ACCESS	0.25	1	3	0.5	1.11	1.61
UBA	0.08	5	7	1.67	2.24	3.91

Looking at the waiting time of customers in line and the time spent on the ATM machine that is ($Wq_+ W_s$), we discovered that customers in UBA spent more time before being served both on queue and on the ATM machine than that of Access bank. (1.61minutes compared to 3.91minutes)

IV. Conclusion

The queuing theory and models are among the quantitative technique widely used today. Their application in organisation, where people would have to queue like banks would serve as guides to decision making based on the results of the analysis performed. The situation or conditions experienced in this branch during the period of study are similar to what operate in other branches and other banks in the country. Excessive waste of time in the banking hall would have a negative impact on the economy of the country in terms of the opportunity cost, i.e. excess time would have been used elsewhere.

Recommendation

The following recommendations are made based on the outcome of the data analysis and the experience gathered during the period.

- We recommended that the management of Access Bank should ensure that their other two ATM machines are working since they have four ATM machines to ease the problem their customers face on queuing.
- To the management of UBA, we recommended that they should get more ATM machine in order to improve on their service delivery.
- Finally, in a nut shell, more ATM machines should be added to the present ones in both banks since queuing is evident in both banks.

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