Modification of Generalization of Equivalence Relation

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Abstract :*In this paper contains description of certain relations called partial equivalence relations .Moreover ,it includes the formal algorithms for determining the order of relations like partial reflexive, partial symmetric and partial transitive relations .Some examples are also presented for the clarification of these concept.*
Key Words*:-Partial reflexive, Partial symmetric and Partial transitive .*

I.   Introduction:

Mathematical meaning of the word "Relation" can be interpreted as the association or dissociation ,interaction or interconnectedness of elements of two sets .The word "Relation" may be interpreted as the quantification of propositions .In mathematics,various form of relations available. Most commonly used relations are "Preorder","Order relation", "Partial order relation" , "Strict ordering relation". "Binary relation", "n-arrey Relation", "Equivalence relation",Fuzzy set and Rough set relation etc. Whatever may be the forms ,any relation is a special type of set that connects some other sets in a proper way.

In this paper I want to discuss certain type of modification of generalization of equivalence relations. The Fuzzy relation describes some of the incomplete equivalence relations ,but this description is insufficient .There exists certain relations ,which often used in the day-to-day linguistic expressions and need more attention for the application point of view.

II.   Comparison Of Partial Equivalence Relation With Fuzzy Relations :-

The readers who are not familiar with the partial relation may think why one should define partial relation although the definition of fuzzy relation already exists ? To answer this query , in fuzzy relation the membership function itself is a relationship between the elements of the set on which the fuzzy relation $\bar{R}$ is defined that's why the relation $\bar{R}$ is associated with another relation. Fuzzy relation and partial equivalence relation are different concepts .Those are summarized below.

2.1Fuzzy Relation
a. The knowledge of the memberships of Individual elements within the domain of fuzzy relation is essential.
.
b. Only one membership relation is required to represent a fuzzy relation.
c. Membership function differs as per the variation of the rùles.

2.2Partial Equivalence relation a .Partial equivalence relation does not require it essentially.
b . Among them two relations are the subset of the Cartesian product of a set is required to define a partial relation.   1
c. Partial relation neither contain a membership function nor it changes as the variation of the rules.

III.   Implementation:-

Example:-3.1
Let us consider the grid as shown in the figure. The English alphabet B and P can traced by using this grid .The trace of B and P are
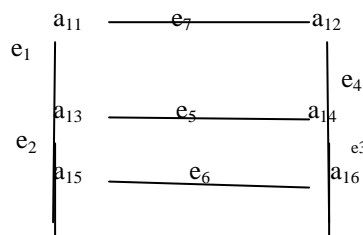
*(Fig.3.1)
Trace of B ={$(a_{15},a_{13}),(a_{13},a_{11}),(a_{11},a_{12}),(a_{12},a_{14}),(a_{14},a_{13}),(a_{14},a_{16}),(a_{15},a_{16})$}
Trace of P={$(a_{15},a_{13}),(a_{13},a_{11}),(a_{11},a_{12}),(a_{12},a_{14}),(a_{14},a_{13})$}

Let the relationship between B and P which can be modeled as the traverse of maximal similar path in the grid.
Let us consider the trace of B and P are the relationships between R_1 and R_2 is R_{12} respectively then
$R_{12}=\{(a_{15},a_{13}),(a_{13},a_{11}),(a_{11},a_{12}),(a_{12},a_{14}),(a_{14},a_{13})\}$
The order of similar appearance of B with P is I Trace of B∩ Trace of PI/ I Trace of PI =5/5=1
The order of partial appearance of B with P is I Trace of B∩ Trace of PI/ I Trace of BI =5/7=0.7142857.

The order of partial reflexive relation is min{1,5/7}=5/7=0.7142857

Remark:-1
The order of partial reflexivity is greater than or equal to zero or lesser than or equal to one .
i={1,2,3,4,5} the index used for the edges i on the grid for tracing P.
j= {1,2,3,4,5,6,7} the index used for the edges j on the grid for tracing of B.   2.
Formal Algorithm:-1 The algorithm for forming matrix as follows e_{ij}
=1 if both the edges e_{ij} are used for i=j
    =0 if at least e_i and e_j are not used if i≠j

Trace of B →

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	1	0	0	0	0	0	0
e_2	0	1	0	0	0	0	0
e_3	0	0	1	0	0	0	0
e_4	0	0	0	1	0	0	0
e_5	0	0	0	0	1	0	0

Trace of P→   (fig:3.2)

Proposition:-3.1.1
Every partial reflexive relation is reflexive but converse is not true.
Proof is not needed and left it.

Example :-3.2
Let us consider the grid as shown in the figure 3.1
The English alphabets B and H can be traced by using this grid .
The trace of B and trace of H are
Trace of B =$\{(a_{15},a_{13}),(a_{13},a_{11}),(a_{11},a_{12}),(a_{12},a_{14}),(a_{14},a_{13}),(a_{14},a_{16}),(a_{15},a_{16})\}$
Trace of H=$\{(a_{11},a_{13}),(a_{13},a_{15}),(a_{12},a_{14}),(a_{16},a_{14}),(a_{13},a_{14})\}$
Let the relationship between B and H be R_{12} , i.e ,the similarity in the appearance of B with H . The relationship can be measured by enlisting all the common edges those appears in the trace of B and H in the grid .If we consider the traces of B and H in the grid . if we consider the traces of B and H as the relationships R_1 and R_2 is R_{12} then
$R_{12} =\{(a_{11},a_{13}),(a_{13},a_{15}),(a_{12},a_{14}),(a_{16},a_{14}),(a_{13},a_{14})\}$
The order of similar appearance of B with H = I Trace of B∩ Trace of HI/ I Trace of HI =5/5=1
The of partial appearance of H with B is I Trace of B∩ Trace of HI/ I Trace of BI =5/7=0.7142857
Order of partial symmetric relation is min{1 ,5/7} =5/7=0.7142857
                                                                                    3.
Remark-2
The order of partial symmetric is always greater then equal to zero and lesser then equal to one.

The partial symmetric relationship can be expressed in the matrix form .Here we express such algorithm by illustrating the following partial symmetric relationship of the trace of H with the trace B .The edges consisting the trace of H are taken in the row and the edges consists of trace of B are taken in columns .The algorithm for forming matrix as follows.

Formal Algorithm-2
$e_{ij} = e_{ji} =1$, if both e_i and e_j Є Trace of H ∩Trace of B and e_i ,$e_j$are connected to at least a node .
    =0,if either any of e_i or e_j not Є Trace of H ∩ Trace of B.

Trace of B→

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	1	1	0	0	1	0	0
e_2	1	1	0	0	1	0	0
e_3	0	0	1	1	1	0	0
e_4	0	0	1	1	1	0	0
e_5	1	1	1	1	1	0	0

Trace of H→ (rows)   (Fig3.3)

Proposition:-3.1.2

Every partial symmetric relation is symmetric relation but converse is not true.

Proof:- In this matrix fig 3.3

Let x Є Trace of H and y Є Trace of B.

x R y is called a trace of H R trace of B.

x R y C (Trace of H) R (Trace of B)

Similarly transpose of fig 3.3 is written as follows y R x => (Trace of B)R(Trace of H) is called the transpose of the matrix.This is the complete proof .

Theorem:-3.1

The matrix associated with the partial symmetric relation R_{12} is equal to the similar matrix associated with the partial symmetric relation R_{21} Provided $I R_1 I = IR_2 I$.

Proof.:-In order to prove this theorem two cases arises   4.

Case:-1 Suppose $I R_1 I = IR_2 I$

Then the proof is obvious as for Formal Algorithm-2

Case:-2 $I R_1 I \neq IR_2 I$

Let us consider the Sub matrix of order min$\{ I R_1 I , IR_2 I \} X$ min$\{ I R_1 I , IR_2 I \}$ and this sub matrix must be associated with the elements those are included from the first row and first column to min $\{ I R_1 I , IR_2 I \}$ row and column respectively. The elements those are below of the main diagonal of this sub matrix are equal. That neighborhood $a_{ij}=a_{ji}$ for$\{1,2,3,\dots$ min$\{ I R_1 I , IR_2 I \}$.

Example -3.3

Let us consider the grid as shown in the figure-3.1.The English alphabets B,H and I can be traced by using the grid.

The trace of B,H and I with the relations R_1, R_2 and R_3 are

Trace of B =$\{(a_{15},a_{13}),(a_{13},a_{11}),(a_{11},a_{12}),(a_{12},a_{14}),(a_{14},a_{13}),(a_{14},a_{16}),(a_{15},a_{16})\}$

Trace of H=$\{(a_{11},a_{13}),(a_{13},a_{15}),(a_{12},a_{14}),(a_{16},a_{14}),(a_{13},a_{14})\}$

Trace of I =$\{(a_{11},a_{31}),(a_{15},a_{13})\}$

Let the trace of B and trace of H is R_{12} and the trace between H and I is R_{23} .Then the trace of B and trace of I is R_{13} .

The order of similar appearance of B with H is I Trace of B ∩ Trace of HI / I Trace of H I =5/5=1

The order of similar appearance of B with I is I Trace of B ∩ Trace of I I / I Trace of I I =2/2=1

The order of partial appearance of I with B is I Trace of I ∩ Trace of B I / I Trace of B I =2/7=0.2857143

The order of partial transitive relation is min$\{1,1,2/7\}$ =2/7=0.2857143

Formal Algorithm:-3 $e_{ij}=1$,if both the edges e_i and e_j has a link for i=j

=0,if both the edges of e_i and e_j has no link.

Trace of B→

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	1	0	0	0	0	0	0
e_2	0	1	0	0	0	0	0
e_3	0	0	1	0	0	0	0
e_4	0	0	0	1	0	0	0
e_5	0	0	0	0	1	0	0

Trace of H→

(Fig:-3.4)   5.

Trace of H→

	e_1	e_2	e_3	e_4	e_5
e_1	1	0	0	0	0
e_2	0	1	0	0	0

Trace of I→

(Fig:-3.5)

Trace of I→

	e_1	e_2
e_1	1	0
e_2	0	1
e_3	0	0
e_4	0	0
e_5	0	0
e_6	0	0
e_7	0	0

Trace of B→

(Fig -3.6)

Proposition:-3.3.3 Every partial transitive relation is transitive but converse is not true .
Proof:-In the matrix of the fig-3.4,3.5 &3.5.Let x Є Trace of B, y Є Trace of H and z Є Trace of I.
x R y is called a Trace of B R Trace of H ,x R y C (Trace of B) R (Trace of H)
y R z is called a Trace of H R Trace of I , y R z C (Trace of B) R (Trace of I)
Then x R z => (Trace of B) R (Trace of I).Thus this establishes the proof .

IV.   Conclusion :-

This paper defines theorem and propositions to help the many development of formulae. Using *formal algorithm* to studied many theoretical and implicational problem in near future . The order of partial reflexive ,partial symmetric and partial transitive to give better clarification of the concept.

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