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# Thermal stresses of semi infinite Rectangular slab with internal heat source

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**Abstract:** *The main purpose of this paper to solve three dimensional thermoelastic problem of thermal stresses of semi infinite rectangular slab with internal heat source to determine temperature distribution, thermal displacements stress functions, at any point of the rectangular slab with given boundary conditions by applying the finite Marchi-Fasulo integral transform and infinite Fourier cosine transform technique.*
**Keywords:** *Rectangular slab, three dimensional thermoelastic problem, temperature distribution, thermal displacements, stress functions, integral transform.*

## I. Introduction

The primary objective of the present paper is to gain an effective solution and a better understanding of thermal stresses in thin rectangular slab with internal heat supply. Recently, Khobragade,N.W., H. S. Roy and S. H. Bagade [1] investigate an thermal stresses of semi infinite rectangular beam, (2013) and khogragade N.W. and Parveen H.[2] discussed thermoelastic solution of a semi infinite rectangular beam due to heat generation,(2012). Tanigawa and Komatsubara [3], Vihak et al [5] and Adams and Bert [4] have studied the direct problem of thermoelasticity in a rectangular plate under thermal shock. Khobragade and Wankhede [6] have studied the inverse unsteady-state thermoelastic problem to determine the temperature displacement function and thermal stresses at the boundary of a thin rectangular plate. They have used the finite Fourier sine transform technique.

In this paper we consider the three dimensional heat conduction problem with internal heat source and determine the temperature distribution, thermal displacement, stress function of the rectangular slab defined in the space D:$0 \le x \le \infty$, $0 \le y \le \infty$, $-h \le z \le h$ with the given boundary condition. The heat conduction equation is solved by using Marchi-Fasulo integral transform and Fourier cosine transform. The result is obtained in the form of infinite series.

## II. Statement Of The Problem

Consider a thin rectangular plate occupying the space D:$0 \le x \le \infty, 0 \le y \le \infty, -h \le z \le h$.The displacement components $u_x$, $u_y$, $u_z$ in the x, y, z direction respectively are in the integral form as

$$u_x = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu\frac{\partial^2 U}{\partial x^2}\right) + \lambda T\right] dx \qquad (1)$$

$$u_y = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} - \nu\frac{\partial^2 U}{\partial y^2}\right) + \lambda T\right] dy \qquad (2)$$

$$u_z = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \nu\frac{\partial^2 U}{\partial z^2}\right) + \lambda T\right] dz \qquad (3)$$

Where E, $\nu$ and $\lambda$ are the Young modulus, the poisson ratio and the linear coefficient of thermal expansion of the material of the plate respectively, U(x, y ,z ,t) is the Airy stress function which satisfies the differential equation.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z,t) = -\lambda E\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)^2 T(x,y,z,t) \qquad (4)$$

Here $T(x,y,z,t)$ denotes the temperature of the thin rectangular plate satisfying the following differential equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g(x,y,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (5)$$

Where k is thermal conductivity and α is the thermal diffusivity of the material of the plate subject to initial conditions

$$T(x,y,z,t) = T_0 \qquad (6)$$

The boundary conditions and interior condition are

$$\left[\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=\infty} = F_1(y,z,t) \qquad (7)$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=0} = F_2\,(y,z,t) \tag{8}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=\infty} = F_3\,(x,z,t) \tag{9}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=0} = F_4\,(x,z,t) \tag{10}$$

$$\left[\mathrm{T(x,y,z,t)} + \mathrm{k}_1 \frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=-h} = F_5\,(x,y,t) \tag{11}$$

$$\left[\mathrm{T(x,y,z,t)} + \mathrm{k}_2 \frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=h} = F_6(x,y,t) \tag{12}$$

The stress functions in term of U(x, y, z, t) are given by

$$\sigma_{xx} = \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right) \tag{13}$$

$$\sigma_{yy} = \left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2}\right) \tag{14}$$

$$\sigma_{zz} = \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) \tag{15}$$

The equations(1) to (15) constitute the mathematical formulation of the problem under consideration.

## III. Solution of the problem

The finite Marchi-Fasulo integral transform of f(x), -a < z <a is defined to be

$$\bar{F} = \int_{-a}^{a} f(x)\, P_n(x) dx \tag{16}$$

then at each point of (-a, a) at which f(x) is continuous. Also the inverse finite Marchi-Fasulo transform is defined as

$$f(x) = \sum_{n=1}^{\infty} \frac{\bar{F}(n)}{\lambda_n} P_n(x) \tag{17}$$

Where,

$$P_n(x) = Q_n \cos(c_n x) - W_n \sin(c_n x)$$

$$Q_n = c_n(\alpha_1 + \alpha_2)\cos(c_n x) + (\beta_{1-}\beta_2)\sin(c_n x)$$

$$W_n = (\beta_{1+}\beta_2)\cos(c_n x) + (\alpha_1 - \alpha_2)c_n \sin(c_n x)$$

$$\lambda_n = \int_{-a}^{a} {P_n}^2\,(x) dx$$

$$= a\left[{Q_n}^2 + {W_n}^2\right] + \frac{\sin(2c_n x)}{2c_n}\left[{Q_n}^2 - {W_n}^2\right] \tag{18}$$

The eigen values $c_n$ are the solutions of the equation

$$[\alpha_1\, c_n \cos(a_n a) + \beta_1 \sin(c_n a)] \times [\beta_2 \cos(c_n a) + \alpha_2\, c_n \sin(c_n a)]$$
$$= [\alpha_2\, c_n \cos(c_n a) - \beta_2 \sin(c_n a)] \times \quad [\beta_1 \cos(c_n a) - \alpha_1\, c_n \sin(c_n a)] \tag{19}$$

Where $\alpha_{1,}\alpha_2, \beta_1, \beta_2$ are constants.

Fourier Cosine integral formula for f(y) is (y>0)

$$f(y) = \frac{2}{\pi}\int_0^{\infty}\left(\int_0^{\infty} f(t) cosstdt\right) cossy\, ds \tag{20}$$

Then,

$$F_c(s) = \int_0^{\infty} f(t) cosstdt$$

is called Fourier cosine transform of f(t)

$$f(t) = \frac{2}{\pi}\int_0^{\infty} F_c\,(s) cosstds$$

is the inverse Fourier cosine transform of $F_c(s)$.

and using the following property of Fourier cosine transform with usual notation and assumption $\lim_{!t!\to 0} f(t) = 0 = \lim_{!t!\to 0} f'(t)$ then

$$f_c\{f''(t); s\} = -s^2 f_c(s) - \sqrt{\frac{2}{\pi}}\, f'(0) \tag{21}$$

Applying Fourier cosine transform two times stated in (20) to (21) and then finite Marchi-Fasulo integral transform stated in (16) to (19) , using conditions (6) to (12) it gives

$T = \frac{4}{\pi^2}\int_0^\infty \int_0^\infty \sum_{n=1}^\infty [e^{-\lambda t(l^2+m^2+a_n{}^2)}\left(\int_0^\infty e^{\lambda t(l^2+m^2+a_n{}^2)} + \lambda\phi\right)dt + \bar{\bar{T}}_0{}^* - \int \lambda\phi\, dt]\ \frac{P_n(z)}{\lambda_n}\ cosmycoslxdldm$
(22)

Where,

$$\emptyset = \frac{P_n(h)}{\alpha_1}\ F_5 - \frac{P_n(-h)}{\alpha_2}\ F_6\ - \overline{F_2}^* - F_4{}^* + \frac{\bar{\bar{g}}^*}{k}$$

Now, substituting value from (22) in (4) it gets Airy's function $U(x,y,z,t)$ as,

$U = -\frac{4\lambda E}{\pi^2}\int_0^\infty \int_0^\infty \sum_{n=1}^\infty [e^{-\lambda t(l^2+m^2+a_n{}^2)}\left(\int_0^\infty e^{\lambda t(l^2+m^2+a_n{}^2)} + \lambda\phi\right)dt + \bar{\bar{T}}_0 - \int \lambda\phi\, dt]\ \frac{P_n(z)}{\lambda_n}\ cosmycoslxdldm$

(23)

## IV. Determination of thermoelastic displacement

Substituting the value of $U(x,y,z,t)$ from (23) in (1),(2) and (3) it gets,

$$u_x = \frac{4\lambda}{\pi^2}\int_0^\infty [\sum_{n=1}^\infty \{\frac{P_n(z)}{\lambda_n}(F_6 - 2F"_6 + \nu F"_6) - F_6\frac{P"n(z)}{\lambda_n} - 2F'_6\frac{P'n(z)}{\lambda_n}\}]\ dx \quad (24)$$

$$u_y = \frac{4\lambda}{\pi^2}\int_0^\infty [\sum_{n=1}^\infty \{\frac{P_n(z)}{\lambda_n}(F_6 + (\nu-2)F"_6) - 2F'_6\frac{P'n(z)}{\lambda_n} - F_6\frac{P"n}{\lambda_n}\}]dy \quad (25)$$

$$u_z = \frac{4\lambda}{\pi^2}\int_{-h}^h [\sum_{n=1}^\infty \{\frac{P_n(z)}{\lambda_n}(F_6 + (\nu-2)F"_6) + 2F'_6\frac{P'n(z)}{\lambda_n} + F_6\frac{P"n(z)}{\lambda_n}\}]dz \quad (26)$$

Where
$F_6(x,y,z,t) = \iint_0^\infty [e^{-\lambda t(l^2+m^2+a_n{}^2)}\int\left(e^{\lambda t(l^2+m^2+a_n{}^2)} + \lambda\phi\right)dt + \bar{\bar{T}}_0{}^* - \int \lambda\phi dt]coslπxcosmπydldm$

And $\quad \emptyset = \frac{P_n(h)}{\alpha_1}\ F_5 - \frac{P_n(-h)}{\alpha_2}\ F_6\ - \overline{F_2}^* - F_4{}^* + \frac{\bar{\bar{g}}^*}{k}$

## V. Determination of stress function

Using (23) in(13),(14),(15) it gets stress function as

$$\sigma_{xx} = \frac{-4\lambda E}{\pi^2}\sum_{n=1}^\infty [2F"_6\frac{P_n(z)}{\lambda_n} + 2F'_6\frac{P'n(z)}{\lambda_n} + F_6\frac{P"n(z)}{\lambda_n}] \quad (27)$$

$$\sigma_{yy} = \frac{-4\lambda E}{\pi^2}\sum_{n=1}^\infty [2F"_6\frac{P_n(z)}{\lambda_n} + 2F'_6\frac{P'n}{\lambda_n} + F_6\frac{P'n(z)}{\lambda_n}] \quad (28)$$

$$\sigma_{zz} = -\frac{8\lambda E}{\pi^2}\sum_{n=1}^\infty F"_6\frac{P_n(z)}{\lambda_n} \quad (29)$$

Where,

$$F_6(x,y,z,t) = \iint_0^\infty [e^{-\lambda t(l^2+m^2+a_n{}^2)}\int\left(e^{\lambda t(l^2+m^2+a_n{}^2)} + \lambda\phi\right)dt + \bar{\bar{T}}_0{}^* - \int \lambda\phi dt]coslπxcosmπydldm$$

And $\quad \emptyset = \frac{P_n(h)}{\alpha_1}\ F_5 - \frac{P_n(-h)}{\alpha_2}\ F_6\ - \overline{F_2}^* - F_4{}^* + \frac{\bar{\bar{g}}^*}{k}$

## VI. Special case

Set, $\quad T(x,y,z,t) = t(z+h)(z-h)e^{-(x+y)} + T_0e^{-txyz} + xyzt$

And $\quad g(x,y,z,t) = \delta(x-x_1)\delta(y-y_1)\ \delta(z-z_1)\delta(t-t_1)$
Where $\delta$ is dirac delta function. Substituting value of T in equations (7) to (12) and then,

Applying Fourier cosine transform two times to $g$ as stated in (20) to (21) and again finite Marchi-Fasulo transform to $'g'$ only stated in (17)to (19) then it gets

$$F_1(y,z,t) = yzt \tag{30}$$

$$F_2(y,z,t) = -t(z+h)(z-h)e^{-y} - T_0tyz + yzt \tag{31}$$

$$F_3(x,z,t) = xzt \tag{32}$$

$$F_4(x,z,t) = -t(z+h)(z-h)e^{-x} + T_0e^{-txyz} + xyzt \tag{33}$$

$$F_5(x,y,t) = T_0e^{-txyh} + xyht - 2k_1\mathrm{h}\, te^{-(x+y)} - k_1txyT_0e^{-txyh} + k_1xyt \tag{34}$$

$$F_6(x,y,t) = T_0e^{txyh} - xyht - 2k_2\mathrm{h}\, te^{-(x+y)} - k_2txyT_0e^{txyh} + k_2xyt \tag{35}$$

$$\overline{F_2}^* = \frac{-4kmt(k_1+k_2)}{(1+m^2)a_n{}^2}[a_nhcos^2(a_nh) + \cos(a_nh).\sin(a_nh)] \tag{36}$$

$$F_4{}^* = \frac{-4te^{-t}(k_1+k_2)}{a_n{}^2}[a_nhcos^2(a_nh)\cos(a_nh)\sin(a_nh)] + xt(1-T_0)[2\cos(a_nh) + (k_2 - k_1)a_n\sin(a_nh)]\ [\frac{2ha_n\cos(a_nh)+\sin(a_nh)}{a_n{}^2}] \tag{37}$$

$$\bar{\bar{g}}^* = A \tag{38}$$

Substituting all the values from (30) to (38) in (22) it gets,

$$T = \frac{4}{\pi^2}\int_0^\infty\int_0^\infty\sum_{n=1}^\infty[e^{-\lambda t(l^2+m^2+a_n{}^2)}\left(\int_0^\infty e^{\lambda t(l^2+m^2+a_n{}^2)} + \lambda\phi\right)dt + \bar{\bar{T_0}}^* - \int\lambda\phi\ dt]\ \frac{P_n(z)}{\lambda_n}\ cosmycoslxdldm$$

Where,

$$\begin{aligned}\emptyset = \frac{P_n(h)}{\propto_1}&\left[T_0e^{-txyh} + xyht - 2k_1\mathrm{h}\, te^{-(x+y)} - k_1txyT_0e^{-txyh} + k_1xyt\right]\\ &- \frac{P_n(-h)}{\propto_2}\left[T_0e^{txyh} - xyht - 2k_2\mathrm{h}\, te^{-(x+y)} - k_2txyT_0e^{txyh} + k_2xyt\right]\\ &+\left[\frac{4kmt(k_1+k_2)}{(1+m^2)a_n{}^2}[a_nhcos^2(a_nh) + \cos(a_nh).\sin(a_nh)]\right]\\ &+[\frac{4te^{-t}(k_1+k_2)}{a_n{}^2}[a_nhcos^2(a_nh)\cos(a_nh)\sin(a_nh)] + xt(1-T_0)[2\cos(a_nh) + (k_2\\ &- k_1)a_n\sin(a_nh)]\ [\frac{2ha_n\cos(a_nh)+\sin(a_nh)}{a_n{}^2}]] + \frac{\bar{\bar{g}}^*}{k}\end{aligned}$$

## VII. Numerical Results

Set, h=2cm, α=0.85, t=1sec, k=1.5, A=2.5,
λ=16.5μm.m$^{-1}$.k$^{-1}$, $\alpha_1$=2, $\alpha_2$=3 $T_0$=4 then

$$T = \frac{4}{\pi^2}\int_0^\infty\int_0^\infty\sum_{n=1}^\infty[e^{-16.5(l^2+m^2+a_n{}^2)}\left(\int_0^\infty e^{16.5(l^2+m^2+a_n{}^2)} + 16.5\phi\right)dt + \bar{\bar{T_0}}^* - \int 16.5\phi\ dt]\ \frac{P_n(z)}{\lambda_n}\ cosmycoslxdldm$$

Where, $\emptyset = \frac{P_n(2)}{2}\ F_5 - \frac{P_n(-2)}{3}\ F_6\ - \overline{F_2}^* - F_4{}^* + \frac{\bar{\bar{g}}^*}{1.5}$

## VIII. Conclusion

In this paper, we discussed the thermal stresses of semi infinite rectangular slab with internal heat source where boundary condition of second kind as well as third kind are taken. The marchi-Fasulo integral transform and fourier cosine transform is used to obtained the numerical results. The temperature obtained can be applied to design the structures in industrial applications. Also stress function, displacements are useful in various industry applications.

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