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Even-even gracefulness of some families of graphs

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Abstract: *In this paper, we prove that the Dumbbell graph, Star graph, Cartesian product* $P_2 \times C_n$ *and* $K_1 + C_n$ *are even-even graceful. The even-even graceful labeling of a graph G with q edges means that there is an injection f: E (G) → {2, 4,…, 2q} so that induced map* f^**: V (G) →{0, 2,…,(2k-2)} defined by* $f^*(x) \equiv \Sigma f(x, y) \pmod{2k}$ *where k = max {p, q} makes all distinct and even.*
Keywords: *Even-even graceful labeling, Dumbbell graph, Star graph and wheel graph.*

I. Introduction

Most graph labeling methods trace their origin to one introduced by Rosa [1] in 1967, or the one given by Graham and Sloane [2] in 1980. Rosa [1] called a function f a β-valuation of a graph G with q edges if f is an injection from the vertices of G to the set {0, 1, . . . , q} such that, when each edge xy is assigned the label |f(x) − f(y)|, the resulting edge labels are distinct. Golomb subsequently called such labeling graceful and this is now the popular term. For all terminology and notation Bondy[3] has been followed. Solairaju and Chithra [4] have introduced the concept of edge-odd gracefulness. Gayathri and Duraisamy have introduced the concept of even edge-graceful labeling. A graph is even vertex-graceful if there exists an injective map f : E (G) → {1,2,…,2q} so that the induced map f^+ : V(G) →{0,2,4,…,2k-2} defined by $f^+(x) = \Sigma f(xy) \pmod{2k}$ where k = max { p, q } makes all distinct. R.Sridevi, S. Navaneethakrishnan, A.Nagarajan and K. Nagarajan [5] have introduced the concept of odd-even gracefulness. They proved that some well known graphs namely P_n, P^+_n, $K_{1,n}$, $K_{1,2,n}$, $K_{m,n}$, $B_{m,n}$ are odd-even graceful. In this paper we introduce the definition even-even gracefulness and also prove that some well known graphs namely S_n, *D (m,n) and* $P_2 \times C_n$ etc are even-even graceful.

Definition1.1

The odd-even graceful labeling of a graph G with q edges is an injection f : V(G) →{1, 3, 5,…,2q + 1} such that, when each edge uv is assigned the label |f(u)−f(v)|, the resulting edge labels are {2,4, 6,…,2q}. A graph which admits an odd-even graceful labeling is called an odd-even graceful graph.

Definition1.2

A graph is even vertex graceful if there exists an injective map f : E (G)→{1,2,…,2q} so that the induced map f^+:V(G) →{0,2,4,…,2k-2} defined by $f^+(x) = f(xy) \pmod{2k}$ where k = max {p, q } makes all distinct.

Definition1.3

A graph is even-even graceful if there exists an injective map f: E (G) → {2, 4,…, 2q} so that the induced map f^*: V (G) →{0, 2,…,(2k-2)} defined by $f^*(x) \equiv \Sigma f(x, y) \pmod{2k}$ where k = max {p, q} makes all distinct and even.

II. Main Results

Definition 2.1 A star S_n is the complete bipartite graph $K_{1,n}$. It is a tree with one internal node and n leaves.
Theorem 2.1 A star graph S_n is even-even graceful when n is even.
Proof: Let G be a star graph with n+ 1 vertices and n edges.
Let $\{e_1,e_2,\dots,e_n\}$be the edge set of S_n.
Define f: E (G)$\rightarrow \{2,4,\dots,2q\}$ such that (here q = n) f (e_i) = 2i; i = 1, 2,…, n.
The internal vertex of S_n has induced label

$$2+4+6+\dots+2n = 2(1+2+3+\dots+n)$$
$$= \frac{2n(n+1)}{2}$$
$$= n(n+1)$$
$$= n.k \text{ where } k = p = n+1$$

$2+4+6+\ldots+2n \equiv 0 \pmod{2k}$ when n is even

Hence, the induced label of internal vertex is '0' and other vertices have induced label from 2 to 2n.

Example 2.1 The star graph S_4 is even-even graceful.

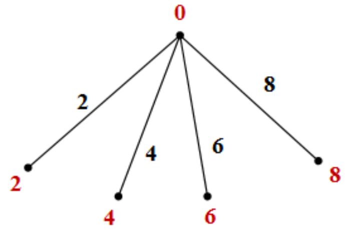

Figure: 1

Definition 2.2 The Dumbbell graph D (m,n) is formed by two disconnected cycles C_m and C_n joined by an edge.

Theorem 2.2 Dumbbell graph D (m,n) is even-even graceful for m = n.

Proof: For any $n \geq 3$, the Dumbbell graph D(m,n) has 2n vertices and 2n+1 edges. Let $\{e_1,e_2,\ldots,e_n\}$be the edge set of the first cycle C_n . 'e_{n+1} ' be a connecting edge. $\{e_{n+2},e_{n+3},\ldots,e_{2n+1}\}$ be the edge set of the second cycle C_n .

We begin with the first cycle C_n by labeling 2 to 2n to each edge anticlockwise consecutively from one side of the connected vertex. Then we label 2n+2 to the connected edge .Finally we label 2n+4 to 4n+2 to each edge of the second cycle C_n clockwise from one side of the connecting vertex.

Hence, the vertices of first cycle C_n has induced labels f (v_i) = 4i-2 ; i = 1,2,…,n and the vertices of second cycle C_n has induced labels f (v_i^1) = 4i ; i = 1,2,3,…,n.

Example: 2.2 The Dumbbell graph with even-even graceful labeling.

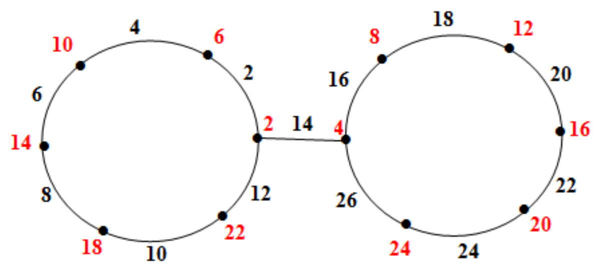

Figure: 2

Theorem 2.3 The ladder graph $P_2 \times C_n$ is even-even graceful.

Proof:

The graph $P_2 \times C_n$ has 2n vertices and 3n edges. First we consider e_1 and e_{3n}, the two outer edges of $P_2 \times C_n$.
Let $\{e_2,e_3,\ldots,e_n\}$be the edge set of one of the long sides of the ladder and $\{e_{2n+1},e_{2n+2},\ldots,e_{3n-1}\}$be the edge set of the other long side of ladder. Finally let $\{e_{n+1},e_{n+2},\ldots,e_{2n}\}$be the edge set of rungs of ladder.

Define f: E(G)$\rightarrow \{2,4,\ldots,2q\}$ such that

f (e_1) = 2; f (e_{3n}) = 6n and f (e_i) = 2i; i = 2,3,…,3n-1.

From the above labeling, the induced vertex labels of the two paths P_n are

$f^+(v_i)$ = 4(n+1)+2i for i = 1,2,…,n-3;

$f^+(v_{n-2})$ = 0 ; $f^+(v_{n-1})$ = 2 & $f^+(v_n)$ = 4(n+1);

$f^+(v_1^1)$ = 2(n+1);

$f^+(v_i^1)$ = 2i for i = 2,3,…,n.

Hence the graph $P_2 \times C_n$ is an even-even graceful.

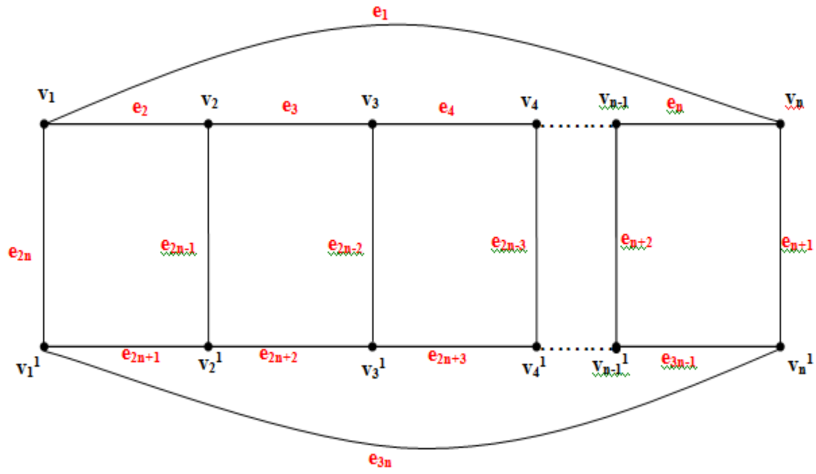

Figure: 3

Example: 2.3 The following figure shows that the graph $P_2 \times C_5$ is even-even graceful.

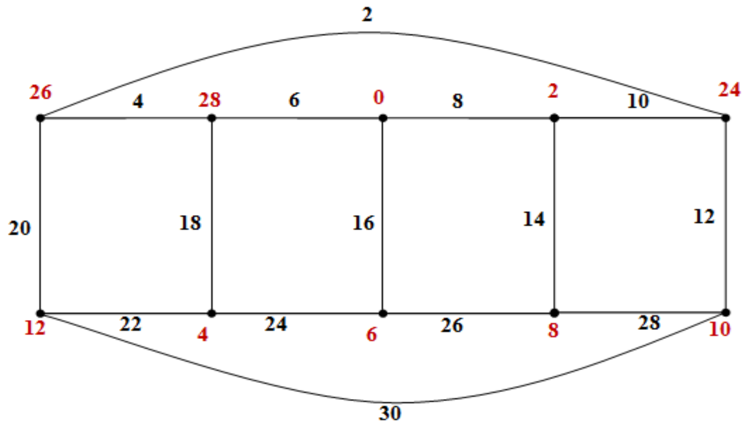

Figure: 4

Definition 2.3

The wheel, W_n, is the graph obtained by joining every vertex of the cycle C_n to exactly one isolated vertex called the center. The edges incident to the center are called spokes.

Theorem 2.4 The wheel W_n is even-even graceful when $n \equiv 0 (\text{mod} 4)$

Proof:

The graph W_n has n+1 vertices and 2n edges. Let $\{e_1, e_2, e_3, \ldots, e_n\}$ be the edge set of the spokes and $\{e_{n+1}, e_{n+2}, \ldots, e_{2n}\}$ be the edge set of consecutive cycle. Let 'v_0'be a center vertex and $v_1, v_2, \ldots, v_n$ be the consecutive cycle vertices.

Define f: E(G)$\rightarrow \{2,4, \ldots, 2q\}$ such that

$f(v_0v_i) = 2i$ for i = 1,2,….n

$f(v_iv_n) = 4n-2(i-1)$ for i = 1,2,…,n

Hence the induced mapping are $f^*(v_0) = n$;

$f^*(v_1) = 2n+4$;

$f^*(v_2) = 2$;

$f^*(v_3) = 0$ and

$f^*(v_i) = 4n-2i+6$ for i = 4,5,…,n.

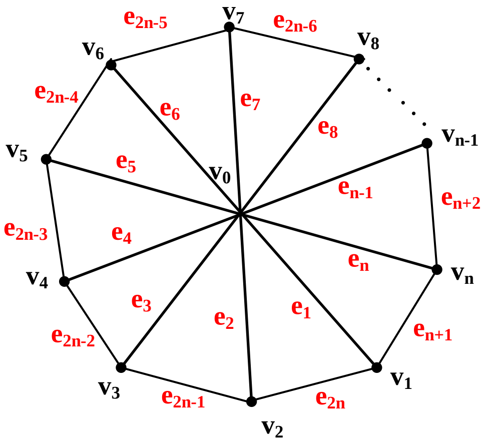

Figure: 5

Example: 2.4 The following figure shows that the graph W_8 is an even-even graceful.

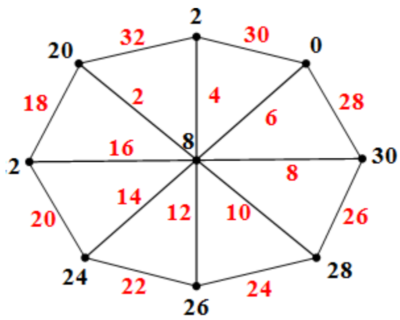

Figure: 6

Definition 2.4 The join of graphs K_1 and C_n, $K_1 + C_n$, is obtained by joining every vertex of K_1 with every vertex of C_n with an edge.

Theorem 2.5 The graph $K_1 + C_n$ is eve-even graceful if n is a multiple of 4.

Proof: The graph $K_1 + C_n$ has n+1 vertices and 2n edges. Let 'v' be vertex of K_1 and $v_1, v_2, \ldots, v_n$ be a vertices of the cycle. Start at the first edge which are incident to the K_1 with 2 and continue in strictly increasing order by 2. $\therefore$The smallest edge label is 2 and largest edge label is 2n.

Similarly, label the edges of C_n, start from right hand side with 2n+2 and continue in strictly increasing order by 2. So the smallest edge label of C_n is 2n+2 and largest edge label is 4n.

Hence the induced labels of vertices are,

$f^*(v) = n$; $f^*(v_1) = 0$; $f^*(v_n) = 2$ and $f^*(v_i) = 4n-2i+2$ if $n = 2,4,\ldots,n-1$

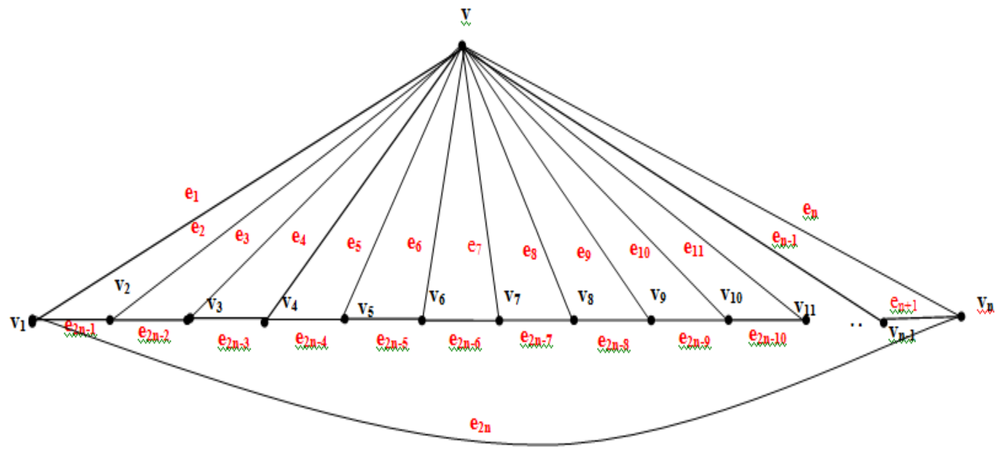

Figure: 7

Example: 2.5 The graph $K_1 + C_{12}$ and its even-even graceful labeling are shown in the following Figure.

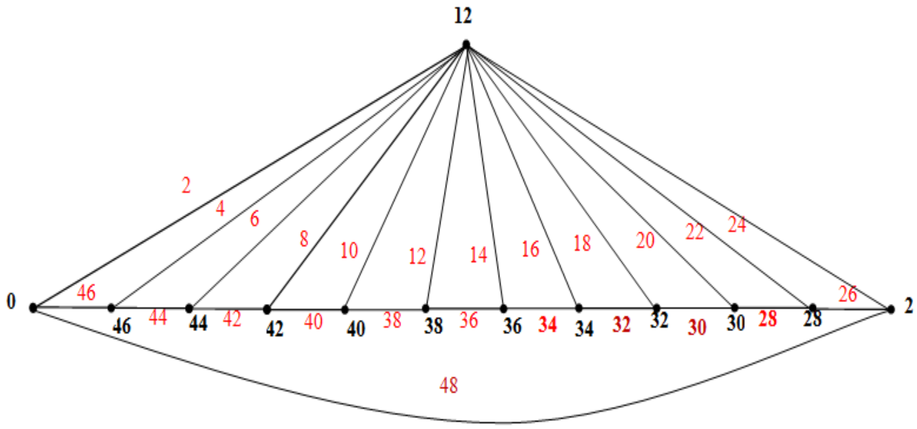

Figure: 8

III. Conclusion

In this paper we have introduced the definition for 'even-even graceful labeling'. We have proved that the Dumbbell graph, Star graph, Cartesian product $P_2 \times C_n$ and $K_1 + C_n$ are all even-even graceful. We have also proved that the wheel W_n is even-even graceful when $n \equiv 0 (mod 4)$.

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