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Interval valued fuzzy generalized semi-preclosed mappings

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Abstract: *In this paper, we introduce interval valued fuzzy generalized semi-preclosed mappings and interval valued fuzzy generalized semi-preopen mappings. Also we investigate some of their properties.*

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I. Introduction

The concept of fuzzy subset was introduced and studied by L. A. Zadeh [15] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper: C. L. Chang [3] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like this concept and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] introduced semi-preclosed sets and Dontchev [4] introduced generalized semi-preclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by Saraf and Khanna [14]. Tapas Kumar Mondal and S. K. Samantha [9] introduced the topology of interval valued fuzzy sets and Jeyabalan. R, Arjunan. K, [7] introduced interval valued fuzzy generalaized semi-preclosed sets. In this paper, we introduce interval valued fuzzy generalized semi-preclosed mappings and interval valued fuzzy generalized semi-preopen mappings and some properties are investigated.

II. Preliminaries

Definition 2.1 *[9] Let* X *be a non empty set. A mapping* $\overline{A} : X \to D[0,1]$ *is called an interval valued fuzzy set (briefly* $IVFS$ *) on* X *, where* $D[0,1]$ *denotes the family of all closed subintervals of [0,1] and* $\overline{A}(x) = [A^-(x), A^+(x)]$ *, for all* $x \in X$ *, where* $A^-(x)$ *and* $A^+(x)$ *are fuzzy sets of* X *such that* $A^-(x) \le A^+(x)$ *, for all* $x \in X$ *.*

Thus $\overline{A}(x)$ *is an interval (a closed subset of [0,1]) and not a number fom the interval [0,1] as in the case of fuzzy set.*
Obviously any fuzzy set A on X is an $IVFS$.

Notation 2.2 D^X *denotes the set of all interval valued fuzzy subsets of a non empty set* X *.*

Definition 2.3 *[9] Let* $\overline{A}$ *and* $\overline{B}$ *be any two* $IVFS$ *of* X *, that is* $\overline{A} = \{< x, [A^-(x), A^+(x)] >: x \in X\}$*, and* $\overline{B} = \{< x, [B^-(x), B^+(x)] >: x \in X\}$*. We define the following relations and operations:*

(i) $\overline{A} \subseteq \overline{B}$ *if and only if* $A^-(x) \le B^-(x)$ *and* $A^+(x) \le B^+(x)$ *, for all* $x \in X$ *.*

(ii) $\overline{A} = \overline{B}$ *if and only if* $A^-(x) = B^-(x)$ *, and* $A^+(x) = B^+(x)$ *, for all* $x \in X$ *.*

(iii) $(\overline{A})^c = \overline{1} - \overline{A} = \{< x, [1 - A^+(x), 1 - A^-(x)] >: x \in X\}$.

(iv) $\overline{A} \cap \overline{B} = \{< x, [\min\{A^-(x), B^-(x)\}, \min\{A^+(x), B^+(x)\}] >: x \in X\}$.

(v) $\overline{A} \cup \overline{B} = \{< x, [\max\{A^-(x), B^-(x)\}, \max\{A^+(x), B^+(x)\}] >: x \in X\}$.

We denote by $\overline{0}_X$ and $\overline{1}_X$ for the interval valued fuzzy subsets $\{< x,[0,0] >: x \in X\}$ and $\{< x,[1,1] >: x \in X\}$ on a nonempty set X respectively.

Definition 2.4 *[9] Let X be a set and $\Im$ be a family of interval vlued fuzzy sets $(IVFSs)$ of X. The family $\Im$ is called an interval valued fuzzy topology $(IVFT)$ on X if and only if $\Im$ satisfies the following axioms:*

(i) $\overline{0}_X, \overline{1}_X \in \Im$,

(ii) If $\{\overline{A}_i : i \in I\} \subseteq \Im$, then $\bigcup_{i \in I} \overline{A}_i \in \Im$,

(iii) If $\overline{A}_1, \overline{A}_2, \overline{A}_3, \ldots, \overline{A}_n \in \Im$, then $\bigcap_{i=1}^{n} \overline{A}_i \in \Im$.

The pair $(X, \Im)$ is called an interval valued fuzzy topological space $(IVFTS)$. The members of $\Im$ are called interval valued fuzzy open sets $(IVFOS)$ in X.

An interval valued fuzzy set $\overline{A}$ in X is said to be interval valued fuzzy closed set $(IVFCS)$ in X if $(\overline{A})^c$ is an $IVFOS$ in X.

Definition 2.5 *[9] Let $(X, \Im)$ be an $IVFTS$ and $\overline{A} = \{< x,[A^-(x), A^+(x)] >: x \in X\}$ be an $IVFS$ in X. Then the interval valued fuzzy interior and interval valued fuzzy closure of $\overline{A}$ denoted by $ivfint(\overline{A})$ and $ivfcl(\overline{A})$ are defined by*

$$ivfint(\overline{A}) = \bigcup \{\overline{G} : \overline{G} \text{ is an IVFOS in } X \text{ and } \overline{G} \subseteq \overline{A}\},$$
$$ivfcl(\overline{A}) = \bigcap \{\overline{K} : \overline{K} \text{ is an IVFCS in } X \text{ and } \overline{A} \subseteq \overline{K}\}.$$

For any $IVFS$ $\overline{A}$ in $(X, \Im)$, we have $ivfcl(\overline{A}^c) = (ivfint(\overline{A}))^c$ and $ivfint(\overline{A}^c) = (ivfcl(\overline{A}))^c$.

Definition 2.6 *An $IVFS$ $\overline{A} = \{< x,[A^-(x), A^+(x)] >: x \in X\}$ in an $IVFTS$ $(X, \Im)$ is said to be an*

(i) *interval valued fuzzy semi-closed set ($IVFSCS$) if $ivfint(ivfcl(\overline{A})) \subseteq \overline{A}$;*

(ii) *interval valued fuzzy preclosed set ($IVFPCS$) if $ivfcl(ivfint(\overline{A})) \subseteq \overline{A}$;*

(iii) *interval valued fuzzy α closed set ($IVF\alpha CS$) if $ivfcl(ivfint(ivfcl(\overline{A}))) \subseteq \overline{A}$;*

(iv) *interval valued fuzzy β closed set ($IVF\beta CS$) if $ivfint(ivfcl(ivfint(\overline{A}))) \subseteq \overline{A}$.*

Definition 2.7 *An $IVFS$ $\overline{A} = \{< x,[A^-(x), A^+(x)] >: x \in X\}$ in an $IVFTS$ $(X, \Im)$ is said to be an interval valued fuzzy generalized closed set $(IVFGCS)$ if $(ivfcl(\overline{A})) = \overline{U}$, whenever $\overline{A} \subseteq \overline{U}$ and $\overline{U}$ in an $IVFOS$.*

Definition 2.8 *An $IVFS$ $\overline{A} = \{< x,[A^-(x), A^+(x)] >: x \in X\}$ in an $IVFTS$ $(X, \Im)$ is said to be an*

(i) *interval valued fuzzy semi-preclosed set (* $IVFSPCS$ *) if there exist on* $IVFPCS$ $\overline{B}$ *, such that* $ivfint\overline{B} \subseteq \overline{A} \subseteq \overline{B}$ *;*

(ii) *interval valued fuzzy semi-preopen set (* $IVFSPOS$ *) if there exist on* $IVFPOS$ $\overline{B}$ *, such that* $\overline{B} \subseteq \overline{A} \subseteq ivfcl(\overline{B})$.

Definition 2.9 *Let* $\overline{A}$ *be an* $IVFS$ *in an* $IVFTS$ $(X,\Im)$*. Then the interval valued fuzzy semi-preinterior of* $\overline{A}$ $(ivfspint(\overline{A}))$ *and the interval valued fuzzy semi-preclosure of* $\overline{A}$ $(ivfspcl(\overline{A}))$ *are defined by*

$$ivfspint(\overline{A}) = \bigcup\{\overline{G} : \overline{G} \text{ is an IVFSPOS in } X \text{ and } \overline{G} \subseteq \overline{A}\},$$

$$ivfspcl(\overline{A}) = \bigcap\{\overline{K} : \overline{K} \text{ is an IVFSPCS in } X \text{ and } \overline{A} \subseteq \overline{K}\}.$$

For any $IVFS$ $\overline{A}$ in $(X,\Im)$, we have $ivfspcl(\overline{A}^c) = (ivfspint(\overline{A}))^c$ and $ivfspint(\overline{A}^c) = (ivfspcl(\overline{A}))^c$.

Definition 2.10 *[7] An* $IVFS$ $\overline{A}$ *in* $IVFTS$ $(X,\Im)$ *is said to be an interval valued fuzzy generalized semi-preclosed set (* $IVFGSPCS$ *) if* $ivfspcl(\overline{A}) \subseteq \overline{U}$ *, whenever* $\overline{A} \subseteq \overline{U}$ *and* $\overline{U}$ *is an* $IVFOS$ *in* $(X,\Im)$.

Definition 2.11 *[7] The complement* $\overline{A}^c$ *of an* $IVFGSPCS$ $\overline{A}$ *in an* $IVFTS$ $(X,\Im)$ *is called an interval valued fuzzy generalized semi-preopen set (* $IVFGSPOS$ *) in* X .

Definition 2.12 *[9] Let* $(X,\Im)$ *and* (Y,σ) *be* $IVFTSs$*. Then a map* $g: X \to Y$ *is called interval valued fuzzy continuous (* IVF *continuous) mapping if* $g^{-1}(\overline{B})$ *is* $IVFOS$ *in* X *for all* $IVFOS$ $\overline{B}$ *in* Y .

Definition 2.13 *An* $IVFTS$ $(X,\Im)$ *is called an interval valued fuzzy semi-pre* $T_{1/2}$ *space* $(IVFSPT_{1/2})$*, if every* $IVFGSPCS$ *is an* $IVFSPCS$ *in* X .

Definition 2.14 *[9] Let* $(X,\Im)$ *and* (Y,σ) *be* $IVFTSs$*. Then a map* $g: X \to Y$ *is called an interval valued fuzzy closed mapping* $(IVFC)$ *mapping if* $g(\overline{A})$ *is an* $IVFCS$ *in* Y *for each* $IVFCS$ $\overline{A}$ *in* X .

Definition 2.15 *[9] Let* $(X,\Im)$ *and* (Y,σ) *be* $IVFTSs$*. Then a map* $g: X \to Y$ *is called an interval valued fuzzy semi-closed mapping* $(IVFSC)$ *mapping if* $g(\overline{A})$ *is an* $IVFSCS$ *in* Y *for each* $IVFCS$ $\overline{A}$ *in* X .

Definition 2.16 *[9] Let* $(X,\Im)$ *and* (Y,σ) *be* $IVFTSs$*. Then a map* $g: X \to Y$ *is called an interval valued fuzzy preclosed mapping* $(IVFPC)$ *mapping if* $g(\overline{A})$ *is an* $IVFPCS$ *in* Y *for each* $IVFCS$ $\overline{A}$ *in* X .

Definition 2.17 *Let* $(X,\Im)$ *and* (Y,σ) *be* $IVFTSs$*. Then a map* $g: X \to Y$ *is called an interval valued fuzzy semi-open mapping (* $IVFSO$ *) mapping if* $g(\overline{A})$ *is an* $IVFSOS$ *in* Y *for each* $IVFOS$ $\overline{A}$ *in* X .

Definition 2.18 *Let* $(X,\Im)$ *and* (Y,σ) *be* $IVFTSs$*. Then a map* $g: X \to Y$ *is called an interval valued fuzzy generalized semi-preopen mapping* $(IVFGSPO)$ *mapping if* $g(\overline{A})$ *is an* $IVFGSPOS$ *in* Y

for each $IVFOS$ $\overline{A}$ *in* X .

Definition 2.19 *Let* $(X,\Im)$ *and* (Y,σ) *be* $IVFTSs$. *Then a map* $g : X \to Y$ *is called an interval valued fuzzy generalized semi-preclosed mapping* $(IVFGSPC)$ *mapping if* $g(\overline{A})$ *is an* $IVFGSPCS$ *in* Y *for each* $IVFCS$ $\overline{A}$ *in* X .

Example 2.20 *Let* $X = \{a,b\}$, $Y = \{u,v\}$ and

$$\overline{K}_1 = \{< a,[0.3,0.4] >,< b,[0.4,0.6] >\},$$
$$\overline{L}_1 = \{< u,[0.1,0.2] >,< v,[0.3,0.4] >\}.$$

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are $IVFT$ on X and Y respectively. Define a mapping $g : (X,\Im) \to (Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an $IVFGSPC$ mapping.

III. Main Results

Theorem 3.1 *Every* $IVFC$ *mapping is an* $IVFGSPC$ *mapping.*

Proof. Let X and Y be two $IVFTSs$. Assume that $g : X \to Y$ is an $IVFC$ mapping. Let $\overline{A}$ be an $IVFCS$ in X. Then $g(\overline{A})$ is an $IVFCS$ in Y. Since every $IVFCS$ is an $IVFGSPCS$, $g(\overline{A})$ is an $IVFGSPCS$ in Y and hence g is an $IVFGSPC$ mapping.

Remark 3.2 *The converse of the above theorem 3.1 need not be true from the following example:* Let $X = \{a,b\}$, $Y = \{u,v\}$ and $\overline{K}_1 = \{< a,[0.3,0.4] >,< b,[0.4,0.6] >\}$, $\overline{L}_1 = \{< u,[0.1,0.2] >,< v,[0.3,0.4] >\}$. Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are $IVFT$ on X and Y respectively.

Define a mapping $g : (X,\Im) \to (Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an $IVFGSPC$ mapping but not an $IVFC$ mapping, since $\overline{K}_1^{\,c}$ is an $IVFCS$ in X but $g(\overline{K}_1^{\,c}) = \{< u,[0.6,0.7] >,< v,[0.4,0.6] >\}$ is not an $IVFCS$ in Y, because $ivfcl$ $(g(\overline{K}_1^{\,c})) = \overline{L}_1^{\,c}$ $\neq g(\overline{K}_1^{\,c})$.

Theorem 3.3 *Every* $IVF\alpha C$ *mapping is an* $IVFGSPC$ *mapping.*

Proof. Let X and Y be two $IVFTSs$. Assume that $g : X \to Y$ is an $IVF\alpha C$ mapping. Let $\overline{A}$ be an $IVFCS$ in X. Then $f(\overline{A})$ is an $IVF\alpha CS$ in Y. Since every $IVF\alpha CS$ is an $IVFGSPCS$, $g(\overline{A})$ is an $IVFGSPCS$ in Y and hence g is an $IVFGSPC$ mapping.

Remark 3.4 *The converse of the above theorem 3.3 need not be true from the following example:* Let $X = \{a,b\}$, $Y = \{u,v\}$ and $\overline{K}_1 = \{< a,[0.3,0.4] >,< b,[0.4,0.6] >\}$, $\overline{L}_1 = \{< u,[0.1,0.2] >,< v,[0.3,0.4] >\}$. Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are $IVFT$ on X and Y respectively.

Define a mapping $g : (X,\Im) \to (Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an $IVFGSPC$ mapping but not an $IVF\alpha C$ mapping, since $\overline{K}_1^{\,c}$ is an $IVFCS$ in X but $g(\overline{K}_1^{\,c})$ $= \{< u,[0.6,0.7] >,< v,[0.4,0.6] >\}$ is not an $IVF\alpha CS$ in Y, because $ivfcl$ $(ivfint$ $(ivfcl$ $(g(\overline{K}_1^{\,c}))))$ $=$ $ivfcl(ivfint(\overline{L}_1^{\,c}))$ $= ivfcl(\overline{K}_1)$ $=$ $\overline{L}_1^{\,c}$ $\not\subset$ $g(\overline{K}_1^{\,c})$.

Theorem 3.5 *Every $IVFSC$ mapping is an $IVFGSPC$ mapping.*

Proof. Assume that $g: X \to Y$ is an $IVFSC$ mapping, where X and Y be two $IVFTSs$. Let $\overline{A}$ be an $IVFCS$ in X. Then $g(\overline{A})$ is an $IVFSCS$ in Y. Since every $IVFSCS$ is an $IVFGSPCS$, $g(\overline{A})$ is an $IVFGSPCS$ in Y and hence g is an $IVFGSPC$ mapping.

Remark 3.6 *The converse of the above theorem 3.5 need not be true from the following example:* Let $X = \{a,b\}$, $Y = \{u,v\}$ and $\overline{K}_1 = \{< a,[0.7,0.8] >,< b,[0.8,0.9] >\}$, $\overline{L}_1 = \{< u,[0.2,0.3] >,< v,[0.3,0.4] >\}$. Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are $IVFT$ on X and Y respectively. Define a mapping $g:(X,\Im) \to (Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an $IVFGSPC$ mapping but not an $IVFSC$ mapping, since $\overline{K}_1^{\,c}$ is an $IVFCS$ in X but $g(\overline{K}_1^{\,c}) = \{< u,[0.2,0.3] >,< v,[0.1,0.2] >\}$ is not an $IVFSCS$ in Y, because $ivfint\ (ivfcl\ (g(\overline{K}_1^{\,c}))) = ivfint(\overline{L}_1^{\,c}) = \overline{L}_1 \not\subset g(\overline{K}_1^{\,c})$.

Theorem 3.7 *Every $IVFPC$ mapping is an $IVFGSPC$ mapping.*

Proof. Assume that $g: X \to Y$ is an $IVFPC$ mapping, where X and Y be two $IVFTSs$. Let $\overline{A}$ be an $IVFCS$ in X. Then $g(\overline{A})$ is an $IVFPCS$ in Y. Since every $IVFPCS$ is an $IVFGSPCS$, $g(\overline{A})$ is an $IVFGSPCS$ in Y and hence g is an $IVFGSPC$ mapping.

Remark 3.8 *The converse of the above theorem 3.7 need not be true from the following example:* Let $X = \{a,b\}$, $Y = \{u,v\}$ and $\overline{K}_1 = \{< a,[0.1,0.3] >,< b,[0.2,0.4] >\}$, $\overline{L}_1 = \{< u,[0.6,0.8] >,< v,[0.6,0.7] >\}$. Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are $IVFT$ on X and Y respectively. Define a mapping $g:(X,\Im) \to (Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an $IVFGSPC$ mapping but not an $IVFPC$ mapping, since $\overline{K}_1^{\,c}$ is an $IVFCS$ in X but $g(\overline{K}_1^{\,c}) = \{< u,[0.7,0.9] >,< v,[0.6,0.8] >\}$ is not an $IVFPCS$ in Y, because $ivfcl\ (ivfint\ (g(\overline{K}_1^{\,c}))) = ivfcl(\overline{L}_1) = \overline{1}_Y \not\subset g(\overline{K}_1^{\,c})$.

Theorem 3.9 *Let $g: X \to Y$ be an $IVFGSPC$ mapping between two $IVFTSs$ X and Y. Then for every $IVFS$ $\overline{A}$ of X, $g(ivfcl(\overline{A}))$ is an $IVFGSPCS$ in Y.*

Proof. Let $\overline{A}$ be any $IVFS$ in X. Then $ivfcl(\overline{A})$ is an $IVFCS$ in X. By hypothesis $g(ivfcl(\overline{A}))$ is an $IVFGSPCS$ in Y.

Theorem 3.10 *Let $\overline{A}$ be an $IVFGCS$ in X. If a mapping $g: X \to Y$ from an $IVFTS$ X onto an $IVFTS$ Y is both IVF continuous and an $IVFGSPC$, then $g(\overline{A})$ is an $IVFGSPCS$ in Y.*

Proof. Let $g(\overline{A}) \subseteq \overline{U}$ where $\overline{U}$ is an $IVFOS$ in Y. Then $\overline{A} \subseteq g^{-1}(g(\overline{A})) \subseteq g^{-1}(\overline{U})$, where $g^{-1}(\overline{U})$ is an $IVFOS$ in X, by hypothesis. Since $\overline{A}$ is an $IVFGCS$, $ivfcl(\overline{A}) \subseteq g^{-1}(\overline{U})$ in X. This implies $g(ivfcl(\overline{A})) \subseteq g(g^{-1}(\overline{U}) = \overline{U}$. But $g(ivfcl(\overline{A}))$ is an $IVFGSPCS$ in Y, since $ivfcl(\overline{A})$ is an $IVFCS$ in X and by hypothesis. Therefore $ivfspcl\ (g(ivfcl(\overline{A}))) \subseteq \overline{U}$. Now $ivfspcl(g(\overline{A})) \subseteq ivfspcl\ (g(ivfcl(\overline{A}))) \subseteq \overline{U}$. Hence $g(\overline{A})$ is an $IVFGSPCS$ in Y.

Theorem 3.11 *A bijective mapping $g: X \rightarrow Y$ from an IVFTS X into an IVFTS Y is an IVFGSPC mapping if and only if for every IVFS $\overline{B}$ of Y and for every IVFOS $\overline{U}$ containing $g^{-1}(\overline{B})$, there is an IVFGSPOS $\overline{A}$ of Y such that $\overline{B} \subseteq \overline{A}$ and $g^{-1}(\overline{A}) \subseteq \overline{U}$.*

Proof. *Necessity*: Let $\overline{B}$ be any IVFS in Y. Let $\overline{U}$ be an IVFOS in X such that $g^{-1}(\overline{B}) \subseteq \overline{U}$. Then $\overline{U}^c$ is an IVFCS in X. By hypothesis $g(\overline{U}^c)$ is an IVFGSPCS in Y. Let $\overline{A} = (g(\overline{U}^c))^c$. Then $\overline{A}$ is an IVFGSPOS in Y and $\overline{B} \subseteq \overline{A}$. Now $g^{-1}(\overline{A}) = g^{-1}(g(\overline{U}^c))^c = (g^{-1}(g(\overline{U}^c)))^c \subseteq \overline{U}$.

Sufficiency: Let $\overline{A}$ be any IVFCS in X. Then $\overline{A}^c$ is an IVFOS in X and $g^{-1}(g(\overline{A}^c)) \subseteq \overline{A}^c$ where $g(\overline{A})$ is an IVFS in Y. By hypothesis, there exists an IVFGSPOS $\overline{B}$ in Y such that $g(\overline{A}^c) \subseteq \overline{B}$ and $g^{-1}\overline{B} \subseteq \overline{A}^c$. Therefore $\overline{A} \subseteq (g^{-1}(\overline{B}))^c$. Hence $\overline{B}^c \subseteq g(\overline{A}) \subseteq g(g^{-1}(\overline{B}))^c \subseteq \overline{B}^c$. This implies that $g(\overline{A}) = \overline{B}^c$. Since $\overline{B}^c$ is an IVFGSPCS in Y, $g(\overline{A})$ is an IVFGSPCS in Y. Hence g is an IVFGPC mapping.

Theorem 3.12 *Let X, Y and Z be IVFTSs. If $g: X \rightarrow Y$ is an IVFC mapping and $h: Y \rightarrow Z$ is an IVFGSPC mapping, then $h \circ g$ is an IVFGSPC mapping.*

Proof. Let $\overline{A}$ be an IVFCS in X. Then $g(\overline{A})$ is an IVFCS in Y, by hypothesis. Since h is an IVFGSPC mapping, $h(g(\overline{A}))$ is an IVFGSPCS in Z. Therefore $h \circ g$ is an IVFGSPC mapping.

Theorem 3.13 *Let $g: X \rightarrow Y$ be a bijection from an IVFTS X to an $IVFSPT_{1/2}$ space Y. Then the following statements are equivalent:*

(i) g *is an IVFGSPC mapping,*
(ii) $ivfspcl\,(g(\overline{A})) \subseteq g(ivfcl(\overline{A}))$ *for each IVFS $\overline{A}$ of X,*
(iii) $g^{-1}\,(ivfspcl(\overline{B})) \subseteq ivfcl\,(g^{-1}(\overline{B}))$ *for every IVFS $\overline{B}$ of Y.*

Proof. $(i) \Rightarrow (ii)$ Let $\overline{A}$ be an IVFS in X. Then $ivfcl(\overline{A})$ is an IVFCS in X. (i) implies that $g(ivfcl(\overline{A}))$ is an IVFGSPC in Y. Since Y is an $IVFSPT_{1/2}$ space, $g(ivfcl(\overline{A}))$ is an IVFSPCS in Y. Therefore $ivfspcl(g(ivfcl(\overline{A}))) = g(ivfcl(\overline{A}))$. Now $ivfspcl(g(\overline{A})) \subseteq ivfspcl(g(ivfcl(\overline{A}))) = g(ivfcl(\overline{A}))$. Hence $ivfspcl(g(\overline{A})) \subseteq g(ivfcl(\overline{A}))$ for each IVFS $\overline{A}$ of X.

$(ii) \Rightarrow (i)$ Let $\overline{A}$ be any IVFCS in X. Then $ivfcl\,(\overline{A}) = \overline{A}$. (ii) implies that $ivfspcl\,(g(\overline{A})) \subseteq g(ivfcl(\overline{A})) = g(\overline{A})$. But $g(\overline{A}) \subseteq ivfspcl(g(\overline{A}))$. Therefore $ivfspcl(g(\overline{A})) = g(\overline{A})$. This implies $g(\overline{A})$ is an IVFSPC in Y. Since every IVFSPCS is an IVFGSPCS, $g(\overline{A})$ is an IVFGSPCS in Y. Hence g is an IVFGSPC mapping.

$(ii) \Rightarrow (iii)$ Let $\overline{B}$ be an IVFS in Y. Then $g^{-1}(\overline{B})$ is an IVFS in X. Since g is onto, $ivfspcl(\overline{B}) = ivfspcl(g(g^{-1}(\overline{B})))$ and (ii) implies $ivfspcl\,(g(g^{-1}(\overline{B}))) \subseteq g(ivfcl(g^{-1}(\overline{B})))$. Therefore we have $ivfspcl\,(\overline{B}) \subseteq g(ivfcl\,(g^{-1}(\overline{B})))$. Now $g^{-1}(ivfspcl(\overline{B})) \subseteq g^{-1}(g(ivfcl(g^{-1}(\overline{B})))) = ivfcl(g^{-1}(\overline{B}))$, since g is one to one. Hence $g^{-1}(ivfspcl(\overline{B})) \subseteq ivfcl(g^{-1}(\overline{B}))$.

$(iii) \Rightarrow (ii)$ Let $\overline{A}$ be any IVFS in X. Then $g(\overline{A})$ is an IVFS in Y. Since f is one to one,

(iii) implies that g^{-1} ($ivfspcl\ (g(\overline{A}))) \subseteq ivfcl(g^{-1}(g(\overline{A}))) = ivfcl(\overline{A})$. Therefore $g(g^{-1}(ivfspcl(g(\overline{A})))) \subseteq g(ivfcl(\overline{A}))$. Since g is onto $ivfspcl(g(\overline{A})) = g(g^{-1}(ivfspcl(g(\overline{A}))))$ $\subseteq g(ivfcl(\overline{A}))$.

Theorem 3.14 *Let $g: X \to Y$ be bijective mapping, where X is an $IVFTS$ and Y is an $IVFSPT_{1/2}$ space. Then the following statements are equivalent:*

(i) g *is an* $IVFGSPC$ *mapping,*

(ii) g *is an* $IVFGSPO$ *mapping,*

(iii) $g(ivfint(\overline{B})) \subseteq ivfcl(ivfint(ivfcl(g(\overline{B}))))$ *for every* $IVFS$ $\overline{B}$ *in* X.

Proof. $(i) \Leftrightarrow (ii)$ is obvious.

$(ii) \Rightarrow (iii)$ Let $\overline{B}$ be an $IVFS$ in X. Then $ivfint(\overline{B})$ is an $IVFOS$ in X. By hypothesis $g(ivfint(\overline{B}))$ is an $IVFGSPOS$ in Y. Since Y is an $IVFSPT_{1/2}$ space, $g(ivfint(\overline{B}))$ is an $IVFSPOS$ in Y. Therefore $g(ivfint\ (\overline{B})) \subseteq ivfcl(ivfint(ivfcl(g(ivfint(\overline{B}))))) \subseteq ivfcl(ivfint(ivfcl(g(\overline{B}))))$.

$(iii) \Rightarrow (i)$ Let $\overline{A}$ be an $IVFCS$ in X. Then $\overline{A}^c$ is an $IVFOS$ in X. By hypothesis, $g(ivfint(\overline{A}^c)) = g(\overline{A}^c) \subseteq ivfcl(ivfint(ivfcl(g(\overline{A}^c))))$. That is $ivfint(ivfcl(ivfint(g(\overline{A})))) \subseteq g(\overline{A})$. This implies $g(\overline{A})$ is an $IVF\beta CS$ in Y and hence an $IVFGSPCS$ in Y. Therefore g is an $IVFGSPC$ mapping.

Theorem 3.15 *Let $g: X \to Y$ be bijective mapping, where X is an $IVFTS$ and Y is an $IVFSPT_{1/2}$ space. Then the following statements are equivalent:*

(i) g is an $IVFGSPC$ mapping,

(ii) $g(\overline{B})$ is an $IVFGSPCS$ in Y for every $IVFCS$ $\overline{B}$ in X,

(iii) $ivfint(ivfcl(ivfint(g(\overline{B})))) \subseteq g(ivfcl(\overline{B}))$ for every $IVFS$ $\overline{B}$ in X.

Proof. $(i) \Leftrightarrow (ii)$ is obvious.

$(ii) \Rightarrow (iii)$ Let $\overline{B}$ be an $IVFS$ in X. Then $ivfcl(\overline{B})$ is an $IVFCS$ in X. By hypothesis $g(ivfcl(\overline{B}))$ is an $IVFGSPCS$ in Y. Since Y is an $IVFSPT_{1/2}$ space, $g(ivfcl(\overline{B}))$ is an $IVFSPCS$ in Y. Therefore $g(ivfcl\ (\overline{B})) \supseteq ivfint(ivfcl(ivfint(g(ivfcl(\overline{B}))))) \supseteq ivfint(ivfcl(ivfint(g(\overline{B}))))$.

$(iii) \Rightarrow (i)$ Let $\overline{A}$ be an $IVFCS$ in X. By hypothesis, $g(ivfcl(\overline{A})) = g(\overline{A}) \supseteq ivfint(ivfcl(ivfint(g(\overline{A}))))$. This implies $g(\overline{A})$ is an $IVF\beta CS$ in Y and hence an $IVFGSPCS$ in Y. Therefore g is an $IVFGSPC$ mapping.

Theorem 3.16 *Let X and Y be $IVFTSs$. A mapping $g: X \to Y$ is an $IVFGSPC$ mapping if $g(ivfspint\ (\overline{A})) \subseteq ivfspint(g(\overline{A}))$ for every $\overline{A}$ in X.*

Proof. Let $\overline{A}$ be an $IVFOS$ in X. Then $ivfint(\overline{A}) = \overline{A}$. Now $g(\overline{A}) = g(ivfint(\overline{A})) \subseteq g(ivfspint(\overline{A})) \subseteq ivfspint\ (g(\overline{A}))$, by hypothesis. But $ivfspint\ (g(\overline{A})) \subseteq g(\overline{A})$. Therfore $g(\overline{A})$ is an $IVFSPOS$ in Y. That is $g(\overline{A})$ is an $IVFGSPOS$ in Y. Hence g is an $IVFGSPC$ mapping, by theorem 3.14.

Theorem 3.17 *Let* X *be an* $IVFTS$ *and* Y *be an* $IVFSPT_{1/2}$ *space. Let* $g: X \to Y$ *be bijection. Then the following statements are equivalent:*

(i) g *is an* $IVFGSPC$ *mapping,*

(ii) $g(ivfint(\overline{A}) \subseteq ivfspint(g(\overline{A}))$ *for each* $IVFS$ $\overline{A}$ *of* X *,*

(iii) $ivfint(g^{-1}(\overline{B})) \subseteq g^{-1}(ivfspint(\overline{B}))$ *for every* $IVFS$ $\overline{B}$ *of* Y.

Proof. $(i) \Rightarrow (ii)$ Let g be an $IVFGSPC$ mapping. Let $\overline{A}$ be any $IVFS$ in X. Then $ivfint(\overline{A})$ is an $IVFOS$ in X. Now $g(ivfint(\overline{A}))$ is an $IVFGSPOS$ in Y, by theorem 3.14. Since Y is an $IVFSPT_{1/2}$ space, $g(ivfint(\overline{A}))$ is an $IVFSPOS$ in Y. Therefore $ivfspint(g(ivfint(\overline{A})) = g(ivfint(\overline{A}))$. Now $g(ivfint(\overline{A})) = ivfspint(g(ivfint(\overline{A}))) \subseteq ivfspint(g(\overline{A}))$.

$(ii) \Rightarrow (iii)$ Let $\overline{B}$ be an $IVFS$ in Y. Then $g^{-1}(\overline{B})$ is an $IVFS$ in X. By (ii), $g(ivfint(g^{-1}(\overline{B}))) \subseteq ivfspint(g(g^{-1}(\overline{B}))) \subseteq ivfspint(\overline{B})$. Now $ivfint(g^{-1}(\overline{B})) \subseteq g^{-1}(g(ivfint(g^{-1}(\overline{B})))) \subseteq g^{-1}(ivfspint(\overline{B}))$.

$(iii) \Rightarrow (i)$ Let $\overline{A}$ be an $IVFOS$ in X. Then $ivfint(\overline{A}) = \overline{A}$ and $g(\overline{A})$ is an $IVFS$ in Y. By (iii), $ivfint(g^{-1}(g(\overline{A}))) \subseteq g^{-1}(ivfspint(g(\overline{A})))$. Now $\overline{A} = ivfint(\overline{A}) \subseteq ivfint(g^{-1}(g(\overline{A}))) \subseteq g^{-1}(ivfspint(g(\overline{A})))$. Therefore $g(\overline{A}) \subseteq g(g^{-1}(ivfspint(g(\overline{A})))) \subseteq ivfspint(g(\overline{A})) \subseteq g(\overline{A})$. Therefore $ivfspint(g(\overline{A})) = g(\overline{A})$ is an $IVFSPOS$ in Y and hence an $IVFGSPOS$ in Y. Thus g is an $IVFGSPC$ mapping, by theorem 3.14.

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