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Fuzzy Semi-Pre-Generalized Super Closed Sets

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Abstract: *In this paper, a new class of sets called fuzzy semi-pre-generalized super closed sets is introduced and its properties are studied and explore some of its properties.*
Keywords : *Fuzzy topology, fuzzy super closure, fuzzy super interior ,fuzzy super closed set, fuzzy super open set, fuzzy super closed set, fuzzy super generalized closed set .*

I. Preliminaries

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by x_β (y)=β for y=x and x(y) =0 for $y \neq x$, $\beta\in[0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x) > 1$.$A \leq B$ if and only if $\rceil(A_qB^c)$.

A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy super open subsets of A.

Defination1.1[5]:- Let (X,τ) fuzzy topological space and $A\subseteq X$ then

1.Fuzzy Super closure $scl(A)=\{x\in X:cl(U)\cap A\neq\phi\}$
2.Fuzzy Super interior $sint(A) =\{x\in X:cl(U)\leq A\neq\phi\}$

Definition 1.2[5]: -A fuzzy set A of a fuzzy topological space (X,τ) is called:
(a) Fuzzy super closed if $scl(A) \leq A$.
(b) Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

Remark 1.1[5]:- Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 1.2[5]:- Let A and B are two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$, then $A \cup B$ is fuzzy super closed.

Remark 1.3[5]:- The intersection of two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$ may not be fuzzy super closed.

Definition 1.3[1,5,6,7]:- A fuzzy set A of a fuzzy topological space (X,τ) is called:
(a)fuzzy semi super open if there exists a super open set O such that $O\leq A\leq cl(O)$.
(b) fuzzy semi super closed if its complement 1-A is fuzzy semi super open.

Remark 1.4[1,5,7]:- Every fuzzy super open (resp. fuzzy super closed) set is fuzzy semi super open (resp. fuzzy semi super closed) but the converse may not be true .

Definition 1.4[5]:- The intersection of all fuzzy super closed sets which contains A is called the semi super closure of a fuzzy set A of a fuzzy topological space (X,τ) . It is denoted by scl(A).

Definition 1.5[3,8,9,10, 11]:- A fuzzy set A of a fuzzy topological space (X,τ) is called:

1.fuzzy g- super closed if $cl(A) \leq G$ whenever $A \leq G$ and G is super open.
2.fuzzy g- super open if its complement 1-A is fuzzy g- super closed.
3.fuzzy sg- super closed if $scl(A) \leq O$ whenever $A \leq O$ and O is fuzzy semi super open.
4.fuzzy sg- super open if if its complement 1-A is sg- super closed.
5.fuzzy gs- super closed if $scl(A) \leq O$ whenever $A \leq O$ and O is fuzzy super open.
6.fuzzy gs- super open if if its complement 1-A is gs- super closed.

Remark 1.5[10,11]:- Every fuzzy super closed (resp. fuzzy super open) set is fuzzy g- super closed (resp. fuzzy g- super open) and every fuzzy g-super closed (resp. fuzzy g-super open) set is fuzzy gs-super closed (resp. gs – super open) but the converses may not be true.

Remark 1.6[10,11]:- Every fuzzy semi super closed (resp. fuzzy semi super open) set is fuzzy sg-super closed (resp. fuzzy sg-super open) and every fuzzy sg-super closed (resp. fuzzy sg-super open) set is fuzzy gs-super closed (resp. gs - super open) but the converses may not be true.

Definition 1.6.[3,8,9,10, 11] A fuzzy set A of (X,τ) is called:

(1) Fuzzy semi super open (briefly, Fs- super open) if A ≤ cl(int(A)) and a fuzzy semi super closed (briefly, Fs-super closed) if int(cl(A))≤ A .

(2) Fuzzy pre super open (briefly, Fp- super open) if A ≤ int(cl(A)) and a fuzzy pre super closed (briefly, Fp-super closed) if cl(int(A)) ≤ A .

(3) Fuzzy α super open (briefly, Fα- super open) if A ≤ IntCl(Int(A)) and a fuzzy α- super closed (Briefly, Fα-super closed) if cl (int(cl(A))) ≤ A .

(4) Fuzzy semi-pre super open (briefly, Fsp- super open) if A ≤ cl(int(cl(A)) and a fuzzy semi-pre super closed (briefly, Fsp- super closed) if int(cl(int(A))) ≤ A]. By FSPO (X, τ), we denote the family of all fuzzy semi-pre super open sets of fts X .

The semi closure (resp α- super closure , semi-pre super closure of a fuzzy set A of (X, τ) is the intersection of all Fs- super closed (resp. Fα- super closed, Fsp- super closed) sets that contain A and is denoted by scl(A) (resp. α cl(A) and spcl(A)).

Definition 1.7. [3,8,9,10, 11]:- A fuzzy set A of (X, τ) is called:

(1) Fuzzy generalized super closed (briefly, Fg-super closed) if cl(A) ≤ H, whenever A ≤ H and H is fuzzy super open set in X;

(2) Generalized fuzzy semi super closed (briefly, gFs- super closed) if scl(A) ≤H, whenever A ≤ H and H is Fs-super open set in X.

(3) Fuzzy generalized semi super closed (briefly, Fgs- super closed) if scl(A) ≤ H, whenever A ≤ H and H is fuzzy super open set in X;

(4) Fuzzy α generalized super closed (briefly, Fαg- super closed) if α cl(A)≤H, whenever A ≤ H and H is fuzzy super open set in X;

(5) Fuzzy generalized α- super closed (briefly, Fg_- super closed) if α cl(A) ≤ H, whenever A ≤ H and H is Fα-super open set in X;

(6) Fuzzy generalized semi-pre super closed (briefly, Fgsp- super closed) if spcl(A) ≤ H, whenever A ≤ H and H is fuzzy super open set in X.

Definition 1.8. [3,8,9,10, 11]:- A fuzzy point $x_p \in A$ is said to be quasi-coincident with the fuzzy set A denoted by x_pqA iff p + A(x) > 1. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by A_qB iff there exists x ∈ X such that A(x) + B(x) > 1. If A and B are not quasi-coincident then we write A_qB. Note that A ≤ B , Aq(1−B).

Definition 1.9. [3,8,9,10, 11]:- A fuzzy topological space (X, τ) is said to be fuzzy semi connected (briefly, Fs-connected) iff the only fuzzy sets which are both Fs- super open and Fs- super closed sets are 0 and 1.

II. Fspg-Super closed sets

Definition 2.1.:- A fuzzy set A of (X, τ) is called fuzzy semi-pre-generalized super closed (briefly, Fspg- super closed) if spCl(A) ≤H, whenever A ≤ H and H is Fs- super open in X. By FSPGC (X, τ), we denote the family of all fuzzy semi-pre-generalized super closed sets of fts X.

Lemma 2.1.:- Every Fp- super closed, gFs- super closed, Fsp- super closed sets are Fspg- super closed and every Fspg- super closed set is Fgsp- super closed but the converse may not be true in general. For,

Example 2.1.:- Let X = {a, b} and Y = {x, y, z} and fuzzy sets A,B,E,H,K and M be defined by; A(a) = 0.3, A(b) = 0.4; B(a) = 0.4, B(b) = 0.5; E(a) = 0.3, E(b) = 0.7; H(a) = 0.7, H(b) = 0.6; K(x) = 0.1, K(y) = 0.2, K(z) = 0.7; M(x) = 0.9, M(y) = 0.2, M(z) = 0.5. Let τ = {0, A, 1}, σ = {0,E, 1} and γ = {0,K, 1}. Then B is Fspg- super closed in (X, τ) but not Fp- super closed; M is Fspg- super closed in (Y,) but not gFs- super closed because: If we consider a fuzzy set T(x) = 0.9, T(y) = 0.2, T(z) = 0.7, then clearly scl(M) ≰T where as M ≤ T and T is fuzzy semi super open in (Y,σ) and H is Fgsp- super closed in (X, τ) but neither Fspg- super closed because: If we consider a fuzzy set L(a) = 0.8, L(b) = 0.7, then clearly spcl(H) ≰ L where as H ≤ L and L is fuzzy semi super open in (X, τ) nor Fsp- super closed because int(cl(int(H))) ≰ H.

Theorem 2.1.:- If A is fuzzy semi super open and Fspg- super closed in (X, τ), then A is a Fsp super closed in (X, τ).

Proof.:- Since A ≤ A and A is fuzzy semi super open and Fspg- super closed, then spcl(A) ≤ A. Since A ≤spcl(A), we have A = spcl(A) and thus A is a Fsp- super closed set in X.

Theorem 2.2.:- A fuzzy set A of (X,τ) is Fspg- super closed if AqE)⇒ spcl(A)qE, for every Fs- super closed set E of X.

Proof. (Necessity.):- Let E be a Fs- super closed set of X an AqE. Then A ≤1 − E and 1 − E is Fs-open in X which implies that spcl(A) ≤1 − E as A is Fspg- super closed. Hence, spcl(A)qE.

(Sufficiency.):- Let H be a Fs- super open set of X such that A ≤ H. Then Aq(1 − H) and 1−H is Fs- super closed in X. By hypothesis, spcl(A)q(1−H) implies spcl(A) ≤ H.Hence, A is Fspg- super closed in X. _

Theorem 2.3.:- Let A be a Fspg- super closed set of (X, τ) and xp be a fuzzy point of X such that xpq spcl(A) then spcl(xp)qA.

Proof.:- If spcl(x_p)qA then A≤ 1−spcl(x_p) and so spcl(A) ≤ 1−spcl(x_p)≤1−xp because 1−spcl(x_p) is Fs- super open and A is Fspg- super closed in X. Hence, x_pq spcl(A), a contradiction.

Theorem 2.4.:- If A is a Fspg- super closed set of (X, τ) and A ≤ B≤spcl(A), then B is a Fspg- super closed set of (X, τ).

Proof.:- Let H be a Fs- super open set of (X, τ) such that B ≤ H. Then A ≤ H. Since A is Fspg- super closed, it follows that spcl(A) ≤ H. Now, B ≤ spcl(A) implies spcl(B)≤spcl(spcl(A)) = spcl(A). Thus, spcl(B) ≤ H. This proves that B is also a Fspg super closed set of (X, τ).

Definition 2.2.:- A fuzzy set A of (X, τ) is called fuzzy semi-pre-generalized super open (briefly, Fspg- super open) iff (1−A) is Fspg- super closed in X. That is, A is Fspg- super open iff E ≤ sp int(A) whenever E ≤ A and E is a Fs- super closed set in X. By FSPGO (X, τ), we denote the family of all fuzzy semi-pre-generalized super open sets of fts X.

Leema2.2:- Every Fp- super open, gFs- super open, Fsp- super open sets are Fspg- super open and every Fspg- super open set is Fgsp- super open but not conversely.

Theorem 2.5.:- FSPSO(X, τ) ≤ FSPGSO(X, τ).

Proof. :-Let A be any fuzzy semi-pre super open set in X. Then, 1 − A is Fsp- super closed and hence Fspg- super closed by Leema.1. This implies that A is Fspg- super open. Hence, FSPSO (X, τ) ≤ FSPGSO(X, τ).

Theorem 2.6.:- Let A be Fspg- super open in X and sp Int(A) ≤ B ≤ A, then B is Fspg super open.

Proof.:- Suppose A is Fspg- super open in X and sp IntA(A) ≤ B ≤ A. Then 1 − A is Fspg- super closed and 1−A≤1−B≤ spcl(1−A). Then 1−B is Fspg- super closed set by Theorem 2.4. Hence, B is Fspg- super open set in X.

Proof:- Obvious

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