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Common Fixed Point Theorems for Generalisation of R-Weak Commutativity

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Abstract: *The main purpose of this paper is to obtain fixed point theorems for R-weak commutativity which generalizes theorem 1 of R.P.Pant [2].*
Key Words*: and Phrases. Fixed point, coincidence point, compatible maps, non-compatible, R-weak commuting maps.*

I. Introduction

In 1986 Jungck [1] generalized the concept of weakly commuting mappings by introducing the notion of compatible maps. Since then the study of common fixed points of generalized contractions satisfying compatibility or some other commutativity conditions have emerged as an area of research activity. The central question concerning the common fixed points of generalized contractions may be formulated as given the self maps A_i,B_i,S_i,T_i $\forall$ i of a metric space (X,d) satisfying a contractive condition what assumptions on commutativity and the contractive condition guarantee the existence of a common fixed point. For compatible maps satisfying the contractive condition.

(1) $d(A_ix,B_iy) < M_{ii}(x,y)=\max\{d(S_ix,T_iy),d(A_ix,S_ix),d(B_iy,T_iy), [d(S_ix,B_iy) + d(A_ix,T_iy)]/2\}$ $\forall$ i

(2) $d(A_ix,B_iy) \leq \phi\, (M_{ii}(x,y))$ where $\phi:R_+ \longrightarrow R_+$ is an upper semi-continuous function such that $\Phi(t)<t$,for each t>0. And (3) there exists a function $\delta\ (0, \infty) \longrightarrow (0, \infty)$, which is non decreasing or lower semi-continuous, such that $\in \leq M_{ii}(x, y) < \in + \delta(\in)$ implies that $d(A_ix,B_iy) < \in$.

Key Words and Phrases. Fixed point, coincidence point, compatible maps, non-compatible, R-weak commuting maps.

II. Preliminaries

Before proving our results, we need the following definitions and known results in this sequel.

Definition 2.1([2]).Two self maps A and S of a metric space(X,d) are called compatible if $\lim_{n\to\infty} d(AS_{x_n}, SA_{x_n}) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n$=$\lim_{n\to\infty} Sx_n$=t for some t in X.

Definition2.2 ([2]). Two self maps A and S of a metric space(X, d) are defined to be R-weakly commuting at a point x in X if $d\,(AS_x, SA_x) \leq Rd\,(A_x,S_x)$ for some R>0.The maps A and S are called point wise R-weakly commuting on X if given x in X there exists R>0 such that $d\,(AS_x, SA_x) \leq Rd\,(A_x,S_x)$.

Remark 2.3.

- It is obvious that maps A and S are point wise R-weakly commuting on X <==>they commute at their coincidence points.
- If A and S commute at their coincidence, we can define $R=\max\{1, d\,(AS_x, SA_x)/d\,(A_x,S_x)\}$ when $A_x \neq S_x$, while R can be chosen arbitrarily when x is a coincidence point. The converse of this is obvious. Thus A and S can fail to be point wise R-weakly commuting only if they possess a coincidence point at which they do not commute.
- Compatible maps are necessarily point wise R-weakly commuting since compatible maps commute at their coincidence points.

R.P.Pant proved the following theorems.

Theorem 2.4 (R.P.Pant [1]).Let (A, S) and (B, T) be point wise R-weakly commuting pairs of self mappings of a metric space(X,d) satisfying (i)$AX \subset TX, BX \subset SX$,

(ii) $d\,(A_x,B_y)<M(x,y)=\max\{d(S_x,T_y),d(A_x,S_x),d(B_y,T_y),[d(A_x,T_y)+d(B_y,S_x)]/2\}$ whenever M(x,y)>0. Suppose that one of the pairs(A,S) or (B,T) is compatible and the other is Non compatible. If the mapping in the compatible pair be continuous then A, B, S and T have a unique common fixed point.

Theorem 2.5 (R.P.Pant [2]). Let $\{A_i\}$, i=1, 2, 3,….. S and T be self-mappings of a metric space (X, d) such that $A_iX \subset SX$ when i>1,$A_1X \subset TX$ and (i) Pairs (A_1, S) and (A_i,T), i>1, are point wise R-weakly commuting with atleast one pair non compatible,

(ii) d $(A_1x,A_iy) < M_{1i}(x,y)=\max\{d(S_x,T_y),d(A_1x,S_x),d(A_iy,T_y),[d(A_1x,T_y)+d(A_iy,S_x)]/2\}$.
Also let ϕ : $R_+ \to R_+$ denote a function such that Φ (t) $<t$ for each $t>0$.Whenever $M_{1i}(x,y)>0$ and i>1.(iii)
d $(A_1x,A_2y) \le \phi$ $(M_{12}(x,y))$.If the range of one of the mappings is a complete subspace of X then all the A_i, S and T $\forall$ i have a unique common fixed point.

III. Main Results

In this section we prove common fixed point theorem for sequence of mappings that generalizes the theorem 2.5.

Theorem 3.1.Let $\{A_i\},\{B_i\},\{S_i\},\{T_i\}$ $\forall$ i=1,2,3,…. be self-mappings of a metric space (X, d) such that $B_iX\subset S_iX$, $A_iX\subset T_iX$ $\forall$ i and (i) Pairs (A_i, S_i) and (B_i,T_i) $\forall$ i are Point wise R-weakly commuting with atleast one pair non compatible,
(ii) $d(A_ix,B_iy) < M_{ii}(x,y)=\max\{d(S_ix,T_iy),d(A_ix,S_ix),d(B_iy,T_iy), [d(S_ix,B_iy) + d(A_ix,T_iy)]/2\}$ $\forall$ i whenever $M_{ii}(x,y)>0$. If the range of one of the mappings is a Complete subspace of X then all the A_i,B_i,T_i and S_i $\forall$ i have a unique common fixe point.

Proof: Suppose that T_i is non-compatible with B_i $\forall$ i.
Then there exists a sequence $\{z_n\}$in x such that $\lim_{n\to\infty} B_iz_n = \lim_{n\to\infty} T_iz_n$ =t for some t inX. $\forall$ i. But $\lim_{n\to\infty} d(B_iT_iz_n, T_iB_iz_n)$ is either non zero or does not exist. Since, $B_iX\subset S_iX$ $\forall$ i corresponding to each z_n there exists x_n in X such that $B_iz_n=S_ix_n$ $\forall$ i. Thus $B_iz_n=S_ix_n\to t$ and $T_iz_n\to t$ as $n\to\infty$.We claim that $A_ix_n\to t$ as $n\to\infty$.If not, then by virtue of (ii) for sufficiently large values of n we get $d(A_ix_n,B_iz_n) \le M_{ii}(x_n,z_n)$=Max $\{d(S_ix_n,T_iz_n),d(A_ix_n,S_ix_n),d(B_iz_n,T_iz_n), [d(S_ix_n,B_iz_n) + d(A_ix_n,T_iz_n)]/2\}$. $\forall$ i.
$= d(A_ix_n,S_ix_n)= d(A_ix_n,B_iz_n)$. Which is a contradication.
Hence $A_ix_n\to t$. Also, Since $A_iX\subset T_iX$ $\forall$ i
For each x_n there exists y_n in X such that $A_ix_n=T_iy_n$ $\forall$ i and $A_ix_n=T_iy_n\to t$.
We show that $B_iy_n\to t$ $\forall$ i. If not, then using (ii) for sufficiently large values of n, we get $d(A_ix_n,B_iy_n) < M_{ii}(x_n,y_n)$ = Max $\{d(S_ix_n,T_iy_n),d(A_ix_n,S_ix_n),d(B_iy_n,T_iy_n), [d(S_ix_n,B_iy_n) + d(A_ix_n,T_iy_n)]/2\}$. $\forall$ i.
$=d(A_ix_n,B_iy_n)$ $\forall$ i Which is a contradiction. Thus $A_ix_n\to t,S_ix_n\to t,T_iy_n\to t,B_iy_n\to t$ $\forall$ i where $T_iy_n=A_ix_n$ $\forall$ i.
Next, suppose that S_i $\forall$ i is a noncompatible with A_i $\forall$ i.
Then there exists a sequence $\{x_n\}$in X such that $\lim_{n\to\infty} A_ix_n = \lim_{n\to\infty} S_ix_n$ =t for some t in X. $\forall$ i. But $\lim_{n\to\infty} d(A_iS_ix_n, S_iA_ix_n)$ $\forall$ i is either non zero or does not exist.
Since $A_iX\subset T_iX$ $\forall$ i, corresponding to each x_n there exists y_n in X such that $A_ix_n=T_iy_n$ $\forall$ i and $A_ix_n=T_iy_n\to t$.By using (ii) and we have $\lim_{n\to\infty} A_iy_n$=t $\forall$ i.
Thus we get sequences $\{x_n\}$ and $\{y_n\}$ in X such that $A_ix_n\to t,S_ix_n\text{-}\to t,T_iy_n\to t$ and $A_iy_n\to t$ $\forall$ i.
where $T_iy_n = A_ix_n$ $\forall$ i.
Now, suppose that S_i $\forall$ i, the range of S_i $\forall$ i is a complete subspace of X.Then, Since $\lim_{n\to\infty} S_ix_n$ = t $\forall$ i, there exists a point u in X such that $t=S_iu$ $\forall$ i
Therefore, $\lim_{n\to\infty} A_ix_n=\lim_{n\to\infty} B_iy_n=\lim_{n\to\infty} T_iy_n=\lim_{n\to\infty} S_ix_n=S_iu$ $\forall$ i
$d(A_iu,B_iy_n) < M_{ii}(x,y) = \max\{ d(S_iu,T_iy_n), d(A_iu,S_iu), d(B_iy_n,T_iy_n), [d(S_iu,B_iy_n)+d(A_iu,T_iy_n)]/2\}$ $\forall$ i.
$=\text{Max}\{ d(A_iu,B_iy_n),0\} = d(A_iu,B_iy_n)$ $\forall$ i.
Therefore, $d(A_iu,B_iy_n) < d(A_iu,B_iy_n)$ $\forall$ i. Which is a contradiction.
Hence $A_iu=S_iu$ $\forall$ i .
Since $A_iX\subset T_iX$ $\forall$ i, there exists w in X such that $A_iu=T_iw$ $\forall$ i. If $A_iu\neq B_iw$ for all i,uing (ii)
We obtain $d(A_iu,B_iw) < M_{ii}(u,w)$ = Max $\{d(S_iu,T_iw),d(A_iu,S_iu),d(B_iw,T_iw), [d(S_iu,B_iw) + d(A_iu,T_iw)]/2\}$. $\forall$ i.
$= \max\{ d(B_iw,A_iw),[d(A_iu,B_iw)+0]/2\}= d(A_iu,B_iw)$.
$d(A_iu,B_iw)< d(A_iu,B_iw)$ $\forall$ i. Which is a contradiction
Hence, $S_iu=A_iu=T_iw=B_iw$ $\forall$ i.
Next let us assume that T_iX $\forall$ i is a complete subspace of X.
Then since $\lim_{n\to\infty} T_iy_n$= t $\forall$ i there exists a point w in X such that $t=T_iw$ $\forall$ i
If $B_iw\neq T_iw$ using (ii) for sufficiently large values of n,

We get $d(A_ix_n,B_iw) \leq M_{ii}(x,y) = \max\{d(S_ix_n,T_iw), d(A_ix_n,S_ix_n), d(B_iw,T_iw),[d(S_ix_n,B_iw)+d(A_ix_n,T_iw)]/2\}$
On letting $n\to\infty$, we have $d(T_iw,B_iw)< d(T_iw,B_iw)$ Which is a contradiction. $\forall$ i.
Hence $T_iw=B_iw$ $\forall$ i.
Since $B_iX\subset S_iX$ $\forall$ i, there exists u in X such that $T_iw=A_iw=S_iu$ $\forall$ i
using (ii) we get $T_iw=B_iw=S_iu=A_iu$ $\forall$ i
Again using (ii) we get $S_iu = A_iu =T_iw=B_iw$ $\forall$ i
Thus irrespective of whether S_iX $\forall$ i is assumed complete or T_iX $\forall$ i is assumed to be so.
we get u u,w in X such that $A_iu=S_iu=T_iw=B_iw$ $\forall$ i
Point wise R-weak commutativity of A_i and S_i $\forall$ i implies that there exists $R_I>0$ such that
$d(A_iS_iu,S_iA_iu) \leq R_I\, d(A_iu,S_iu)=0$
That is $A_iS_iu=S_iA_iu$ $\forall$ i and $A_iA_iu=A_iS_iu=S_iA_iu=S_iS_iu$ $\forall$ i
Similarly, for every i, there exists $R_i>0$ such that $d(B_iT_iw,T_iB_iw) \leq R_i\, d(B_iw,T_iw)=0$,that is
$B_iwT_iw= T_iw B_iw$ $\forall$ i and $B_iwB_iw= B_iwT_iw= T_iw B_iw= T_iwT_iw$ $\forall$ i
If $A_iA_iu\neq A_iu$ $\forall$ i, using (ii) weget $d(A_iA_iu, A_iu) =d(A_iA_iu,B_iw) <M_{ii}(A_iu,w)=d(A_iA_iu,B_iw)$ $\forall$ i
Which is a contradiction. Hence $A_iu= A_iA_iu=S_iA_iu$ $\forall$ i and A_iu is a common fixed point of A_i and S_i $\forall$ i.
Similarly, if $B_iB_iw \neq B_iw$ $\forall$ i using (ii) we have $d(B_iw\, B_iB_iw) =d(A_iu, B_iB_iw) <M_{ii}(u,B_iw) =d(A_iu,B_iB_iw)$, $\forall$ i
which is a contradiction.
Hence $B_iw= B_iB_iw= T_iB_iw$ $\forall$ i that is $B_iw=A_iu$ is a common fixed point of T_i and B_i $\forall$ i
Uniqueness. Suppose u, v are fixed point of A_i,B_i,T_i and S_i $\forall$ i
Then $A_iu=S_iu=B_iu=T_iu=u$ $\forall$ i and
$A_iv=S_iv=B_iv=T_iv=v$ $\forall$ i
$d(u,v)=d(A_iu,B_iv)<\max\{d(S_iu,T_iv),d(A_iu,S_iu),d(B_iv,T_iv),[d(S_iu,B_iv)+d(A_iu,T_iv)]/2\}$
$= \max\{d(u,v),0,0,[d(u,v)+d(u,v)]/2\}=\max\{d(u,v),d(u,v)\}=d(u,v)$
= >< = when $u\neq v$.
Therefore u = v.
The proof is similar when B_iX is assumed complete for some i.
Since, $A_iX\subset T_iX$ and $B_iX\subset S_iX$ $\forall$ i.
Therefore proof is complete.

Theorem 3.2. Let $\{A_i\},\{B_i\},\{S_i\},\{T_i\}$ $\forall$ i=1,2,3,…. be self-mappings of a metric space (X, d) such that $A_iX\subset T_iX$,$B_iX\subset S_iX$ $\forall$ i and (i) Pairs (A_i, S_i) and (B_i,T_i) $\forall$ i are Point wise R-weakly commuting with atleast one non pair compatible, one non Compatible.
(ii) $d(A_ix,B_iy)<M_{ii}(x,y)=\max\{d(S_ix,T_iy),d(A_ix,S_ix),d(B_iy,T_iy), d(S_ix,B_iy) ,d(A_ix,T_iy)\}$ $\forall$ i whenever $M_{ii}(x,y)>0$.
If one of the mappings in the Compatible pair is continuous then all the A_i,B_i,S_i andT_i $\forall$ i have a unique common fixed point.
Proof. Let B_i and T_i $\forall$ i be a non compatible mappings and A_i and S_i $\forall$ i be continuous compatible mappings. Then non compatible of B_i and T_i $\forall$ i implies that there exists some sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} B_ix_n=\lim_{n\to\infty} T_ix_n=t$ $\forall$ i for some t in X
While $\lim_{n\to\infty} d(B_iT_ix_n, T_iB_ix_n)$ $\forall$ i is either non zero or nonexistent. Since $B_iX\subset S_iX$ $\forall$ i
Corresponding to each x_n there exists a y_n in X such that $B_ix_n = S_iy_n$ $\forall$ i.
Thus $B_ix_n\to t$, $T_ix_n\to t$ and $S_iy_n\to t$ $\forall$ i.
We claim that $A_iy_n\to t$ $\forall$ i. If not, then there exists a subsequence $\{A_iy_m\}$ of $\{A_iy_n\}$ $\forall$ i
a number r>0 and a positive integer M such that for each m≥M, we have $d(A_iy_m,t)\geq r$, $d(A_iy_m, B_ix_m)\geq r$ $\forall$ i and
$d(A_iy_m, B_ix_m)<\max\{d(S_iy_m,T_ix_m),d(A_iy_m,S_iy_m),d(B_ix_m,T_ix_m), d(S_iy_m,B_ix_m) , d(A_iy_m,T_ix_m)\}$ $\forall$ i
$= \max\{d(A_iy_m,B_ix_m), d(A_iy_m,B_ix_m)\} = d(A_iy_m,B_ix_m)$ $\forall$ i. which is a contradiction.
Hence $\lim_{n\to\infty} A_iy_n=t$, $\lim_{n\to\infty} S_iy_n=t$, $\lim_{n\to\infty} B_ix_n=t$, and $\lim_{n\to\infty} T_ix_n=t$ $\forall$ i,
Where $S_iy_n=B_ix_n$ $\forall$ i. Since, A_i and S_i $\forall$ i are continuous, we get $\lim_{n\to\infty} S_iA_iy_n=S_it$ $\forall$ i
and $\lim_{n\to\infty} A_iS_iy_n=A_it$ $\forall$ i compatibility of A_i and S_i $\forall$ i implies that $\lim_{n\to\infty} d(A_iS_iy_n, S_iA_iy_n)=0$ $\forall$ i. That is, $d(A_it,S_it)=0$ $\forall$ i.
Thus $A_it=S_it$, $\forall$ i Since $A_iX\subset T_iX$ $\forall$ i, there exists some point w in X such that $A_it=T_iw$ $\forall$ i
Now, if $T_iw\neq B_iw$ $\forall$ i.

$d(A_it,B_iw) < \max\{d(S_it,T_iw),d(A_it,S_it),d(B_iw,T_iw),d(S_it,B_iw),d(A_it,T_iw)\}\ \forall\ i$
$d(A_it,B_iw) < \max\{d(B_iw, A_it), d(A_it,B_iw)\}= d(B_iw, A_it)$
Therefore, $d(A_it,B_iw) < d(A_it,B_iw)\ \forall$ i.Which is a contradiction.
Hence $B_iw=T_iw\ \forall$ i and $S_it=A_it=T_iw=B_iw\ \forall$ i
Point wise R-weak commutativity of B_i and $T_i\ \forall$ i implies that there exists R>0 such that
$d(B_iT_iw,T_iB_iw)\le R\ d(B_iw,T_iw)=0\ \forall$ i .
That is, $B_iT_iw= T_iB_iw\ \forall$ i. More over $B_iB_iw=B_iT_iw= T_i B_iw =T_iT_iw\ \forall$ i
Similarly, compatibility of A_i and $S_i\ \forall$ i implies that $A_i S_it= S_iA_it$ and
$A_iA_it= S_iA_it= S_iS_it\ \forall$ i. Now if $A_it\neq A_iA_it\ \forall$ i, using (ii) we get $d(A_it, A_i A_it) =d(A_iA_it,B_iw)$
$<M_{ii}(A_it, w)= d(A_iA_it,B_iw)\ \forall$ i. Which is a contradiction.
Hence, $A_it= A_iA_it= S_iA_it\ \forall$ i and $A_it\ \forall$ i is a common fixed point of A_i and $S_i\ \forall$ i
Similarly, $B_iw(=A_it)\ \forall$ i is a common fixed point of B_i and $T_i\ \forall$ i.
Uniqueness. Suppose u,v are fixed points of A_i,B_i,S_i and $T_i\ \forall$ i.
Then $A_iu=S_iu=B_iu=T_iu=u\ \forall$ i and
$A_iv=S_iv=B_iv=T_iv=v\ \forall$ i $d(u,v)=d(A_iu,B_iv)<\max\{d(S_iu,T_iv),d(A_iu,S_iu),d(B_iv,T_iv),d(S_iu,B_iv),d(A_iu,T_iv)\}\ \forall$ i
$= \max\{d(u,v),d(u,u),d(v,v),d(u,v),d(u,v)\}=d(u,v)$
$d(u,v)<d(u,v)$
= >< = when $u\neq v$.
Therefore,u=v.

The proof is similar when A_i and $S_i\ \forall$ i are assumed noncompatible and B_i and $T_i\ \forall$ i are assumed continuous compatible mappings.
Hence the theorem.

Remark 3.3. If follows from the above proof that the assumption of the theorem that one of pairs, say $(B_i,T_i)\ \forall$ i is non compatible can be weakened in the following way: There exists a sequence $\{x_n\}$ such that
$d(B_ix_n,T_ix_n) \to 0\ \forall$ i
(Equivalently, for any $\in>0$, B_i and $T_i\ \forall$ i have an $\in$-coincidence point $x_\in$, that is $d((B_ix_\in,T_ix_\in) <\in\ \forall$ i) and the sequence$\{B_ix_n\}$ is convergent[then, automatically $\{T_ix_n\}\ \forall$ i converges].

Example3.4. Let X = [2 20) with the d be the usual metric on X.
Define $A_i, B_i, S_i,T_i : X \to X$, i=1,2,3,…… by
$A_ix = 2$ for each x,
$S_ix = x$ if $x <8$ $S_ix = 8$ if $x > 8\ \forall$ i
$B_ix = 2$, if x = 2 or >5 $B_ix =8$ if $2 <x< 4$ $B_ix =3+ x$ if $4 <x < 5$,
$T_i2=2$ $T_ix = 12 + x$ if $2<x<4$ $T_ix=9+x$ if $4 <x < 5$ $T_ix = x -3$ if $x>5$:
Then A_i,B_i,S_i and $T_i\ \forall$ i satisfy all the conditions of the above theorem and have a unique common fixed point x = 2. It may be noted in this example that A_i and $S_i\ \forall$ i are continuous compatible mappings while B_i and $T_i\ \forall$ i are non-compatible point wise R-weakly commuting mappings. B_i and $T_i\ \forall$ i are point wise R-weakly commuting since they commute at their coincidence points. To see that B_i and $T_i\ \forall$ i are noncompatible, let us consider a decreasing sequence $\{x_n\}$ in X such that $x_n \to 5$. Then $B_ix_n = 2,\ \forall$ i
$T_ix_n = x_n -3 \to 2$, $T_iB_ix_n = T_i2=2\ \forall$ i
and $B_iT_ix_n = B_i(x_n -3) = 8.\ \forall$ i Hence B_i and $T_i\ \forall$ i are noncompatible. A_i,B_i,S_i and $T_i\ \forall$ i satisfy the contractive condition (1) but do not satisfy the contractive conditions (2) and (3). To show that (1) holds observe that $d(A_ix\ B_iy) = 0$ for y = 2 or >5, $\forall$ i and $d(A_ix\ B_iy) < d(B_iy\ T_iy) <M_{ii}(x\ y)$ if $2 < y <5$. $\forall$ i
To show that condition (2) is not satisfied, put x = 8 and $y_n = 5 -1/n$. Then
$d(A_i8, B_iy_n) =1+ y_n \to 6$ and $M_{ii}(8, y_n) = 6,\ \forall$ i and we see that $\phi(t)$ cannot be defined at
t = 6. Therefore, (2) does not hold.
Hence condition (3) is not satisfied either, because, as shown in [3], conditions (2) and (3) are equivalent. In fact, the function $\delta(\in)$ of condition (3) is also undefined at $\in = 6$. To see this, let x = 8, $y_n = 2+1/n$, then
$d(A_i8, B_iy_n) =6$ and $M_{ii}(8,y_n) = 6+1/n$, and hence $\delta(\in)$ satisfying (3) cannot be defined at $\in = 6$.

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