IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728,p-ISSN: 2319-765X, Volume 7, Issue 5 (Jul. - Aug. 2013), PP 21-29
www.iosrjournals.org

Effect of Heat Generation on Quasi- Static Thermal Stresses in a Solid Sphere

S.P. Pawar[1], K.C. Deshmukh[2], G.D. Kedar[3]
[1]S. N. Mor and Smt. G. D. Saraf Science College Tumsar Dist- Bhandara (M. S.), India.
[2]Department of Mathematics, R.T.M. Nagpur University, (MS.) India.
[2]Department of Mathematics, Kamla Nehru Mahavidhyalaya, Nagpur, (M. S.) India.

Abstract: *In this paper, a general analysis of one dimensional non-steady state temperature distribution and stresses under thermal load in a solid sphere subjected to different types of heat sources is developed. The article deals with comparative study of the effect of varying heat generation on displacement and thermal stresses. The heat conduction equation solved by integral transforms technique with convective thermal boundary condition and arbitrary initial and surrounding temperature. The results are obtained in a trigonometric series and are studied numerically and are illustrated graphically.*
Keywords: *Thermal stresses, heat sources, integral transform.*

I. Introduction

The study of the problems of determination of thermal stresses in different solids under the different thermal condition and heat sources become a subject of extensive research all around the globe. Heat conduction in spherical objects is an important problem in engineering practices. Transient thermal stresses in a sphere are discussed by a number of authors. Although the thermoelasticity has been well understood for more than a century, early studies are focused on the theoretical approach. The history of literature on these problems found in the texts by Parkus [1], Boley, Wiener [2], Nowacki [3], Noda [4], Carslaw and Jaeger [5]. During the last two decades increased attention has been given to transient problems, especially to those involving cylindrical and spherical geometries with heat generation. The generation of heat has a significant effect on the temperature profile and its effect on thermal stresses in solids in the Engineering fields and life sciences.

Carslaw and Jeager [5] studied the use of sources and sinks in cases of variable temperature in a sphere, Ozisik [6] discussed many homogeneous and non homogeneous heat conduction boundary value problems with heat sources, Cheung et al [7] studied the transient problem in a sphere with local heating, Takeuti et al [8] studied the transient thermal stresses of a hollow sphere due to rotating heat source, Hetnarski [9] discussed the stresses in a long cylinder due to rotating line source, Nasser M. EI-Maghraby [10,11] deal with problems of thermoelasticity with heat sources. Deshmukh et al [12] studied the determination of displacement and thermal stresses in a thin hollow circular disk due to internal heat generation and integral transform is used to solve the heat distribution and stresses are obtained in a Bessel's functions. Deshmukh et al [13] studied the thermal deflection which is built in-edge in a thin hollow disk subjected to the activity of heat source which changes its place on the plate surface with time. Recently Kedar and Deshmukh [14] determined thermal stresses in a thin clamped hollow disk under unsteady temperature field due to point heat source.

In this paper, the one dimensional quasi-static uncoupled thermoelastic problem of a solid sphere with heat generation is considered. The aim is to obtain the mathematical model for predicting the results about the temperature profile and stresses with considering independently different types of heat sources within a body and assuming arbitrary initial and surrounding temperatures. The special cases are studied with instantaneous point, volume and spherical heat sources. This is a new approach to have knowledge of comparative study of heat distribution and produced stresses in a sphere due to internal heat sources. Integral transform technique is to obtain temperature distribution. This is a novel approach of study of thermal stresses which is useful in engineering field where different types of sources are to be used.

II. Formulation Of The Problem

Consider the solid sphere defined by $0 \leq r \leq a$. Initially the sphere is kept at arbitrary temperature$F(r)$. For time $t > 0$ heat generated within the sphere at the rate of $g(r,t)\, J/sm^3$ and heat is dissipated by convection from the boundary at $r = a$ to the medium at temperature$f(t)$. The sphere is homogeneous and isotropic. The temperature distribution, displacement and thermal stresses are to be determined and analyse graphically.

The transient temperature distribution is governed by [6] the following equation,

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{g\,(r,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{in } 0 \leq r < a\,, \ t > 0 \tag{1}$$

Boundary and initial condition

$k\frac{\partial T}{\partial r}+ hT = f(t)$ at $r = a,$ $t > 0$ (2)

$T = F(r)$ in $0 \le r \le a$ at $t = 0$ (3)

A new dependent variable $U(r,t)$ is defined as

$U(r,t) = rT(r,t)$ (4)

Then the problem (1-3) is transformed as

$\frac{\partial^2 U}{\partial r^2} + \frac{rg\ (r,t)}{k} = \frac{1}{\alpha}\frac{\partial U}{\partial t}$ in $0 \le r < a$, $t > 0$ (5)

$U = 0\ \ at\ r = 0$ in $0 \le r \le a$ (6)

$\frac{\partial U}{\partial r} + MU = \frac{af(t)}{k}$, at $r = a,\ t > 0$ (7)

$U = rF(r)$ in $0 \le r \le a$, at $r = 0$ (8)

where, $H = \frac{h}{k}$, $M = \left(H - \frac{1}{a}\right)$ (9)

where, k is thermal conductivity, h is heat transfer coefficient and α is thermal diffusivity of the material.

The temperature is symmetric with respect to centre of sphere, a function of r that is the radial distance only. One dimensional problem of thermoelasticity means spherically symmetric problem [4], in which the shearing stresses and strains vanish and strain and stress components in spherical coordinate θ and ∅ direction are identical

$\sigma_{\theta\theta} = \sigma_{\emptyset\emptyset}$, $\in_{\theta\theta} = \in_{\emptyset\emptyset}$ (10)

$\sigma_{r\theta} = \sigma_{\theta\emptyset} = \sigma_{\emptyset r} = 0$ (11)

$\in_{r\theta} = \in_{\theta\emptyset} = \in_{\emptyset r} = 0$ (12)

The equilibrium equation without body force in spherical coordinates [4] is reduces to

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}\{2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\phi\phi}\} = 0$$

$\frac{d\sigma_{rr}}{dr} + \frac{2}{r}\ (\sigma_{rr} - \sigma_{\theta\theta}\ ) = 0$ (13)

Stress strain relation or Hooke's relations are

$\sigma_{rr} = 2\mu \in_{rr} + \lambda e - \beta\tau$ (14)

$\sigma_{\theta\theta} = \sigma_{\phi\phi} = 2\mu \in_{\theta\theta} + \lambda e - \beta\tau$ (15)

where, strain dilation $e = \in_{rr} + \in_{\theta\theta} + \in_{\emptyset\emptyset} = \in_{rr} + 2 \in_{\theta\theta}$ (16)

σ_{rr} , $\sigma_{\theta\theta}$ and $\sigma_{\phi\phi}$ are the stresses in the radial and tangential direction and $\in_{rr}$ and $\in_{\theta\theta}$ are strains in radial and tangential direction, τ is the temperature change, e is the strain dilation and λ and μ are the Lamé constants related to the modulus of elasticity E and the Poisson's ratio ν as,

$\lambda = \frac{\upsilon E}{(1-\upsilon)(1-2\upsilon)}$, $\mu = \frac{E}{2(1-\upsilon)}$ (17)

The strain component in terms of radial displacement $u_r = u$ is

$\in_{rr} = \frac{du}{dr}$ $\in_{\theta\theta} = \in_{\emptyset\emptyset} = \frac{u}{r}$ (18)

The boundary condition on traction free surface is

$\sigma_{rr} = 0$ at $r = a$ (19)

Now with equations (10-19) one can obtain the displacement and thermal stresses as [4]

$u = \frac{\alpha}{(1-\upsilon)}\ [(1+\upsilon)\frac{1}{r^2}\int_0^r \tau r^2\ dr + 2\ (1-2\upsilon)\frac{r}{a^3}\int_0^a \tau r^2\ dr]$ (20)

$\sigma_{rr} = \frac{\alpha E}{(1-\upsilon)}\ [\frac{2}{a^3}\int_0^a \tau r^2\ dr - \frac{2}{r^3}\int_0^r \tau r^2\ dr]$ (21)

$\sigma_{\theta\theta} = \sigma_{\emptyset\emptyset} = \frac{\alpha E}{(1-\upsilon)}\ [\frac{2}{a^3}\int_0^a \tau r^2\ dr + \frac{1}{r^3}\int_0^r \tau r^2\ dr - \tau]$ (22)

The equations (1-22) constitutes the Mathematical formulation of the problem

III. Analytic Solutions

Following the general procedure of Ozisik [6], we develop the Fourier integral transform of $U(r,t)$ over the variable r in problem (4-8) and the inverse formula as

(Integral Transform) $\overline{U}\ (\beta_m, r) = \int_{r'=0}^{a} K(\beta_m\ r')\ U(r',t)dr'$ (23)

(Inverse Formula) $U(r,t) = \sum_{m=1}^{\infty} K(\beta_m, r)\ \overline{U}(\beta_m, r)$ (24)

Where, the summation is taken over all Eigen values.

On applying the above integral transform and inverse formula to the problem (5-8), one obtains, the expression for the temperature function of a non-homogeneous boundary problem of heat conduction in a solid sphere as,

$T(r,t) =$
$\frac{2}{r}\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t} sin(\beta_m r)\left(\frac{\beta_m^2+M^2}{a(\beta_m^2+M^2)+M}\right)\left[\int_{r'=0}^{a} r' F(r) sin(\beta_m r) + \int_{t'=0}^{t} e^{\alpha\beta_m^2 t}\left[\left\{\frac{\propto}{k}\int_{r'=0}^{a} r' g(r',t') sin(\beta_m r) dr'\right\} +\right.\right.$
$\propto aksin\beta maf(t')dt'$ (25)

$$K(\beta_m, r) = \frac{sin\beta_m r}{\sqrt{N}} = \sqrt{2}\left[\frac{\beta_m^2 + M^2}{a(\beta_m^2 + M^2) + M}\right]^{\frac{1}{2}} sin\beta_m r \quad (26)$$

$$N = \frac{1}{2}\left\{\frac{a(\beta_m^2 + M^2) + M}{\beta_m^2 + M^2}\right\}$$

$M = H - \frac{1}{a}$, a finite value

$$H = \frac{h}{k} \quad (27)$$

β_m are the positive roots of the transcendental equation $\beta_m cot\beta_m a = -M$

The roots of this transcendental equation are real if $a(H - \frac{1}{a}) > -1$

$\rightarrow H > 0$ (28)

The temperature change is obtained as

$\tau = T(r,t) - F(r) =$
$\frac{2}{r}\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t} sin(\beta_m r)\left(\frac{\beta_m^2+M^2}{a(\beta_m^2+M^2)+M}\right)\left[\int_{r'=0}^{a} r' F(r) sin(\beta_m r) + \int_{t'=0}^{t} e^{\propto\beta_m^2 t}\left[\left\{\frac{\propto}{k}\int_{r'=0}^{a} r' g(r',t') sin(\beta_m r) dr'\right\} +\right.\right.$
$\propto aksin\beta maf(t')dt' \; - \; F(r)$ (29)

Using Equations (20-22), Displacement and stresses are obtained as,

$u =$
$\frac{\propto}{(1-\upsilon)}\left[(1-\upsilon)\frac{1}{r^2}\left(\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t}\left(\frac{sin(\beta_m r) - r\beta_m cos(\beta_m r)}{\beta_m^2 N}\right)L - \int_0^r F(r) r^2 dr\right) +\right.$
21−2υ ra3m=1∞e−∝βm2tsinβma−aβmcosβmaβm2NL−0aF(r)r2dr (30)

$$\sigma_{rr} = \frac{2\propto E}{1-\upsilon}\begin{bmatrix}\frac{1}{a^3}\left(\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t}\left(\frac{sin(\beta_m a) - a\beta_m cos(\beta_m a)}{\beta_m^2 N}\right)L - \int_0^a F(r) r^2 dr\right) - \\ \frac{1}{r^3}\left(\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t}\left(\frac{sin(\beta_m r) - r\beta_m cos(\beta_m r)}{\beta_m^2 N}\right)L - \int_0^r F(r) r^2 dr\right)\end{bmatrix} \quad (31)$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \frac{\propto E}{1-\upsilon}\begin{bmatrix}\frac{2}{a^3}\left(\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t}\left(\frac{sin(\beta_m a) - a\beta_m cos(\beta_m a)}{\beta_m^2 N}\right)L - \int_0^a F(r) r^2 dr\right) + \\ \frac{1}{r^3}\left(\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t}\left(\frac{sin(\beta_m r) - r\beta_m cos(\beta_m r)}{\beta_m^2 N}\right)L - \int_0^r F(r) r^2 dr\right) - \\ \left[\frac{1}{r}\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t}\frac{sin(\beta_m r)}{N}L - F(r)\right]\end{bmatrix} \quad (32)$$

where,

$$L = \left[\int_{r'=0}^{a} r' F(r) sin(\beta_m r) + \int_{t'}^{t} e^{-\propto\beta_m^2 t}\begin{bmatrix}\left\{\frac{\propto}{k}\int_{r'=0}^{a} r' g(r',t') sin(\beta_m r) dr'\right\} \\ +\frac{\propto a}{k} sin(\beta_m a) f(t')\end{bmatrix} dt'\right] \quad (33)$$

IV. Results and Discussion

The exact analytical solutions for temperature, displacement and thermal stresses are obtained in the previous part. The mathematical software MATLAB is used for further numerical calculation and graphical analysis.

For special cases we assume the initial temperature $F(r) = 0$, therefore $\tau = T(r,t)$ and for simplicity take the ambient temperature $f(t) = t$. The numerical solutions are presented for following material properties,

Thermal diffusivity $\alpha = 112.34 \times 10^{-6} m^2 s^{-1}$
Thermal conductivity $k = 386 W/mk$
Specific heat $c_\rho = 383\, J/kgK$
Poisson's Ratio $\upsilon = 0.35$

Setting the radius of the sphere $r = 1m$ and $M = -0.4$, the roots of the transcendental equation $\beta_m cot\beta_m a = -M$ are as [6]

$\beta_1 = 0.7593, \beta_2 = 4.5379, \beta_3 = 7.7511, \beta_4 = 10.9225, \beta_5 = 14.0804, \beta_6 = 17.2324$

Then temperature change eq. (29) reduces to,

$$\tau(r,t) = \frac{2}{r}\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t} sin(\beta_m r)\left(\frac{\beta_m^2 + M^2}{a(\beta_m^2 + M^2) + M}\right)\left[\int_{r'=0}^{a} r' F(r) sin(\beta_m r) + \int_{t'=0}^{t} e^{\propto\beta_m^2 t'}\left[\left\{\frac{\propto}{k}\int_{r'=0}^{a} r' g(r',t') sin(\beta_m r) dr'\right\} + \frac{\propto a}{k} sin(\beta_m a) f(t')\right] dt'\right] \quad (34)$$

Following cases are independently discussed with different types of sources,

Case 1

Let heat source is instantaneous constant volume source of strength $g_i J/m^2$ that releases its heat spontaneously at $t = 0$ i.e. single explosion takes place within the sphere and the energy released throughout the solid sphere. It is related with volume heat source by the relation as [6]

$g(r',t') = g_i \delta(t'-0)$, therefore using (30, 31, 32 and 34)

$$\tau(r,t) = \frac{2\propto}{rk}\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t} sin(\beta_m r)\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[g_i \frac{[sin\beta_m - \beta_m cos\beta_m]}{\beta_m^3} + sin\beta_m \left\{\frac{(\propto\beta_m^2 t - 1)e^{\propto\beta_m^2 t}+1}{\alpha^2\beta_m^4}\right\}\right] \quad (35)$$

$$\boldsymbol{u = \frac{2\alpha^2}{(1-v)k}\sum_{m=1}^{\infty}\left[\frac{(1-v)[sin(\beta_m r) - \beta_m r cos(\beta_m r)]}{r^2\beta_m^2} + \frac{2(1-2v)r(sin\beta_m - \beta_m cos\beta_m)}{\beta_m^2}\right]}$$
$$\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[g_i \frac{(sin\beta_m - \beta_m cos\beta_m)}{\beta_m^2} + sin\beta_m \left\{\frac{(\propto\beta_m^2 t-1)e^{\propto\beta_m^2 t}+1}{\alpha^2\beta_m^4}\right\}\right] e^{-\propto\beta_m^2 t} \quad (36)$$

$$\sigma_{rr} = \frac{4\alpha^2 E}{(1-v)k}\sum_{m=1}^{\infty}\left[\frac{[sin\beta_m - \beta_m cos\beta_m]}{\beta_m^2} - \frac{(sin(\beta_m r) - \beta_m r cos(\beta_m r))}{r^3\beta_m^2}\right]$$
$$\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[g_i \frac{(sin\beta_m - \beta_m cos\beta_m)}{\beta_m^2} + sin\beta_m \left\{\frac{(\propto\beta_m^2 t-1)e^{\propto\beta_m^2 t}+1}{\alpha^2\beta_m^4}\right\}\right] e^{-\propto\beta_m^2 t} \quad (37)$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \frac{2\alpha^2 E}{(1-v)k}\sum_{m=1}^{\infty}\left[\frac{2[sin\beta_m - \beta_m cos\beta_m]}{\beta_m^2} + \frac{(sin(\beta_m r) - \beta_m r cos(\beta_m r))}{r^3\beta_m^2} - \frac{sin((\beta_m r)}{r}\right]$$
$$\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[g_i \frac{(sin\beta_m - \beta_m cos\beta_m)}{\beta_m^2} + sin\beta_m \left\{\frac{(\propto\beta_m^2 t-1)e^{\propto\beta_m^2 t}+1}{\alpha^2\beta_m^4}\right\}\right] e^{-\propto\beta_m^2 t} \quad (38)$$

Case2

The heat source is instantaneous point heat source of strength g_{pti} (J) situated at the centre of the sphere and releases its heat spontanteniously at time $t = 0$. this source is related with volumetric heat source [6] by

$$g(r',t') = \frac{g_{pti}}{4\pi'^2}\delta(t'-0)\delta(r'-0) \quad (39)$$

$$T(r,t) = \frac{2\propto}{rk}\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t} sin(\beta_m r)\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[\frac{\beta_m g_{pti}}{4\pi} + sin\beta_m \left\{\frac{(\propto\beta_m^2 t-1)e^{\propto\beta_m^2 t}+1}{\propto^2\beta_m^4}\right\}\right] \quad (40)$$

$$u = \frac{2\alpha^2}{(1-v)k}\sum_{m=1}^{\infty}\left[\frac{(1-v)[sin(\beta_m r) - \beta_m r cos(\beta_m r)]}{r^2\beta_m^2} + \frac{2(1-2v)r(sin\beta_m - \beta_m cos\beta_m)}{\beta_m^2}\right]$$
$$\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[\frac{\beta_m g_{pti}}{4\pi} + sin\beta_m \left\{\frac{(\propto\beta_m^2 t-1)e^{\propto\beta_m^2 t}+1}{\alpha^2\beta_m^4}\right\}\right] e^{-\propto\beta_m^2 t} \quad (41)$$

$$\sigma_{rr} = \frac{4\alpha^2 E}{(1-v)k}\sum_{m=1}^{\infty}\left[\frac{[sin\beta_m - \beta_m cos\beta_m]}{\beta_m^2} - \frac{(sin(\beta_m r) - \beta_m r cos(\beta_m r))}{r^3\beta_m^2}\right]$$
$$\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[\frac{\beta_m g_{pti}}{4\pi} + sin\beta_m \left\{\frac{(\propto\beta_m^2 t-1)e^{\propto\beta_m^2 t}+1}{\alpha^2\beta_m^4}\right\}\right] e^{-\propto\beta_m^2 t} \quad (42)$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \frac{2\alpha^2 E}{(1-v)k}\sum_{m=1}^{\infty}\left[\frac{2[sin\beta_m - \beta_m cos\beta_m]}{\beta_m^2} + \frac{(sin(\beta_m r) - \beta_m r cos(\beta_m r))}{r^3\beta_m^2} - \frac{sin((\beta_m r)}{r}\right]$$
$$\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[\frac{\beta_m g_{pti}}{4\pi} + sin\beta_m \left\{\frac{(\propto\beta_m^2 t-1)e^{\propto\beta_m^2 t}+1}{\alpha^2\beta_m^4}\right\}\right] e^{-\propto\beta_m^2 t} \quad (43)$$

Case3

The heat source is instantaneous spherical source of radius r_1 of total strength $g_{sphi} J$ situated concentrically inside the sphere and releases its heat spontaneously at time $t = \tau'$ then as [6]

$$g(r',t') = \frac{g_{sphi}}{4\pi r'^2}\delta(t'-\tau')\delta(r'-r_1)$$

$$T(r,t) = \frac{2\propto}{rk}\sum_{m=1}^{\infty} e^{-\propto\beta_m^2 t} sin(\beta_m r)\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[\frac{g_{sphi}}{4\pi r_1} e^{\propto\beta_m^2\tau'} sin(\beta_m r_1) + \right.$$

sinβm(∝βm2t−1)e∝βm2t+1∝2βm4

(44)

$$u = \frac{2\alpha^2}{(1-\upsilon)k}\sum_{m=1}^{\infty}\left[\frac{(1-\upsilon)[\sin(\beta_m r)-\beta_m r\cos(\beta_m r)]}{r^2\beta_m^2} + \frac{2(1-2\upsilon)r(\sin\beta_m - \beta_m\cos\beta_m)}{\beta_m^2}\right]$$
$$\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[\frac{g_{sphi}}{4\pi r_1}e^{\propto\beta_m^2\tau'}\sin(\beta_m r_1) + \sin\beta_m\left\{\frac{(\propto\beta_m^2 t-1)e^{\propto\beta_m^2 t}+1}{\alpha^2\beta_m^4}\right\}\right]e^{-\propto\beta_m^2 t} \tag{45}$$

$$\sigma_{rr} = \frac{4\alpha^2 E}{(1-\upsilon)k}\sum_{m=1}^{\infty}\left[\frac{[\sin\beta_m - \beta_m\cos\beta_m]}{\beta_m^2} - \frac{(\sin(\beta_m r)-\beta_m r\cos(\beta_m r))}{r^3\beta_m^2}\right]$$
$$\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[\frac{g_{sphi}}{4\pi r_1}e^{\propto\beta_m^2\tau'}\sin(\beta_m r_1) + \sin\beta_m\left\{\frac{(\propto\beta_m^2 t-1)e^{\propto\beta_m^2 t}+1}{\alpha^2\beta_m^4}\right\}\right]e^{-\propto\beta_m^2 t} \tag{46}$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \frac{2\alpha^2 E}{(1-\upsilon)k}\sum_{m=1}^{\infty}\left[\frac{2[\sin\beta_m - \beta_m\cos\beta_m]}{\beta_m^2} + \frac{(\sin(\beta_m r)-\beta_m r\cos(\beta_m r))}{r^3\beta_m^2} - \frac{\sin((\beta_m r)}{r}\right]$$
$$\left(\frac{\beta_m^2+M^2}{(\beta_m^2+M^2)+M}\right)\left[\frac{g_{sphi}}{4\pi r_1}e^{\propto\beta_m^2\tau'}\sin(\beta_m r_1) + \sin\beta_m\left\{\frac{(\propto\beta_m^2 t-1)e^{\propto\beta_m^2 t}+1}{\alpha^2\beta_m^4}\right\}\right]e^{-\propto\beta_m^2 t} \tag{47}$$

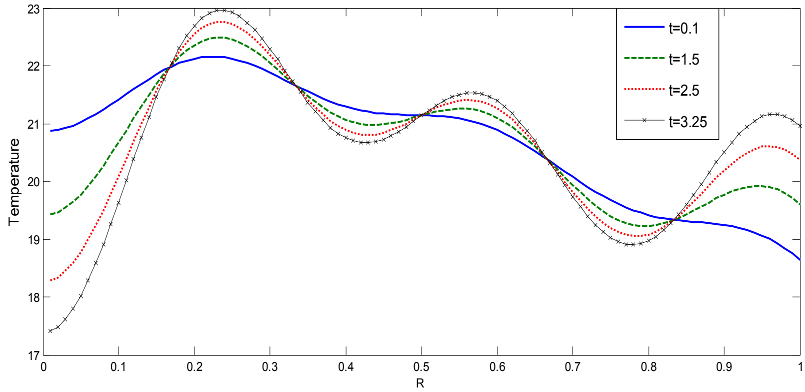

Figure 1: Temperature case 1

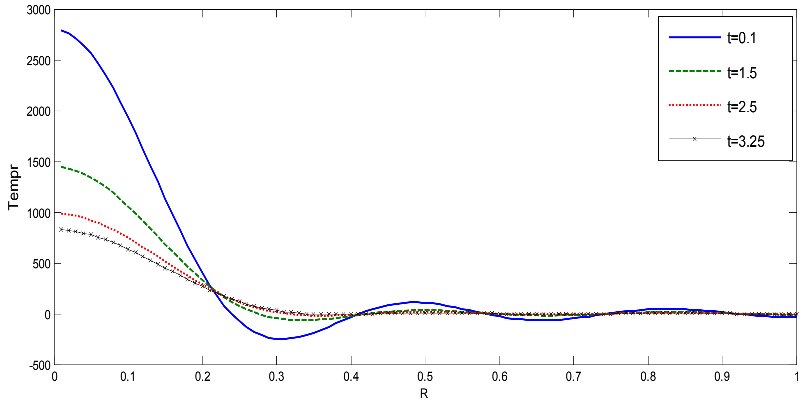

Figure2: Temperature case 2

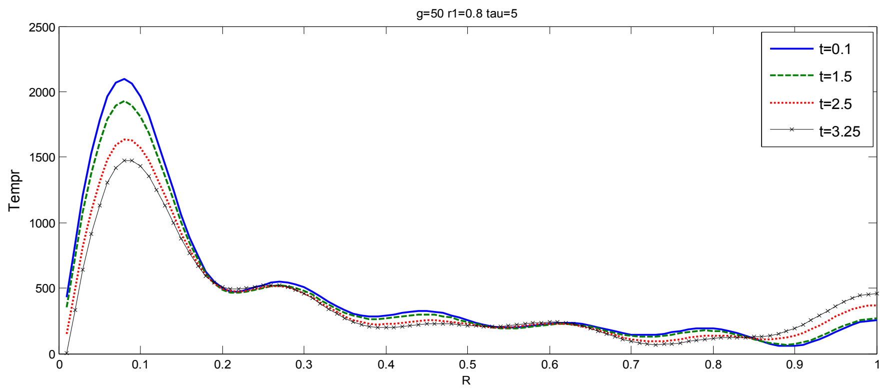

Figure 3: Temperature case 3

Fig 1, 2 and 3 represent the temperature variation for the instantaneous constant volume, instantaneous point and instantaneous spherical source respectively. In every case has the source of constant magnitude $g = 50$ but the nature is different. The temperature variation along the radial direction is shown in the graphs. In case 1 the temperature decreases along the radial direction and there are some radii where it has the constant value and independent of time. Temperature is lowest near the centre of the sphere and time passes first it immediately increases and has local minimum and maximum peaks. For the curve associated with time $t = 0.1$ the temperature decreases along the radial direction. In case 2 the temperature is highest about the centre and it agree with the instantaneous point source at the centre of the sphere. The temperature decreases along the radial direction. It is observed that for very small time it changes the direction also. For the spherical source employed at $r = 0.8$ and $t = \tau' = 1$, the temperature decreases along the radial direction. It has the maximum values at about the radius $r = 0.1$ and minimum value at the centre of the sphere. The temperature distribution changes with the change in the nature of the source.

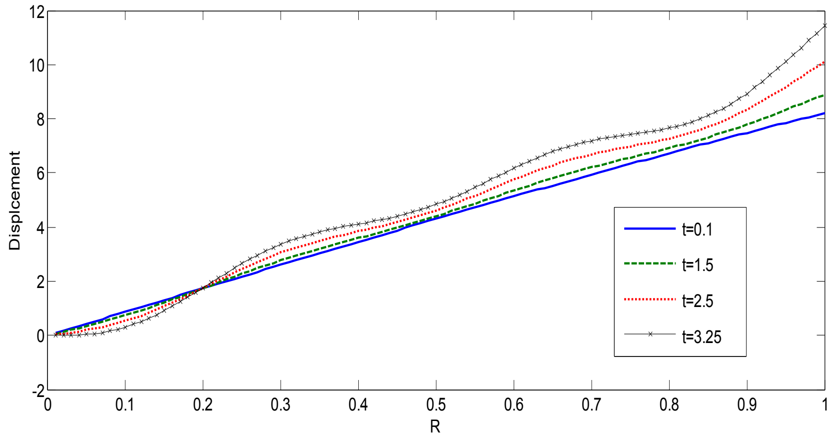

Figure 4: Displacement Case 1

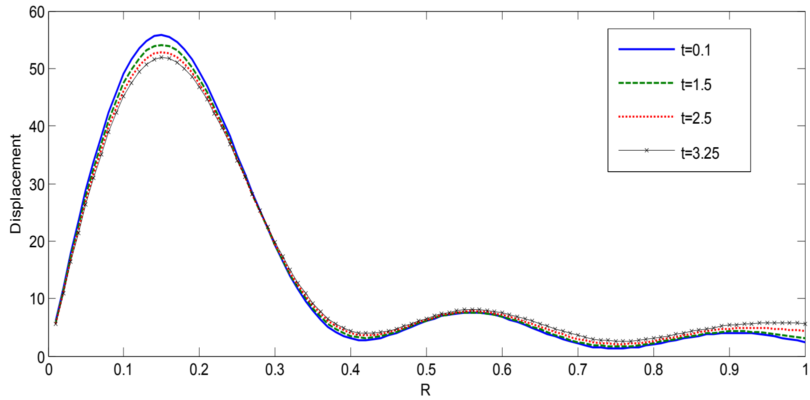

Figure 5: Displacement case2

Fig 4, 5 and 6 shows the change in the displacement along the radius. In case 1 it is seen that the displacement increases along the radial direction. For a very small time the increase is linear. As time passes there is a variation in the temperature distribution and maximum displacement occur near the surface of the sphere and the displacement at the centre is very minor.. For the instantaneous point source at the centre, fig 5 shows the greater displacement about the centre and highest values occur at about the radius $r = 0.15$, while for the spherical source the maximum values of the shifts towards the surface. The displacement is minor at the centre of the sphere in case 1 and 3.In case 3 the displacement is independent of the time at about $r = 0.2m$. For case 1 and 3 the displacement is large on the surface but for case 2 conditions is reversed and it agree with the point heat source at the centre.

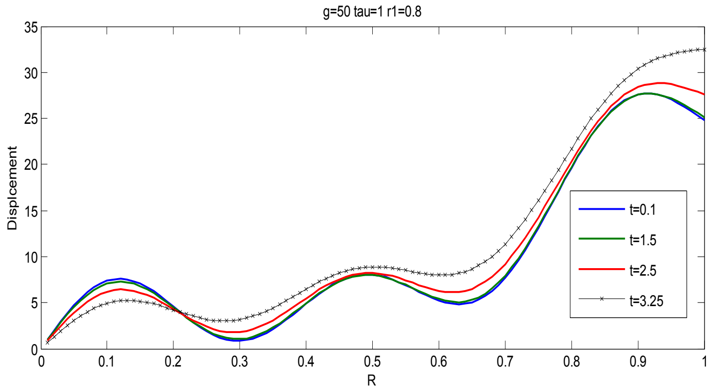

Figure 6: Displacement case 3

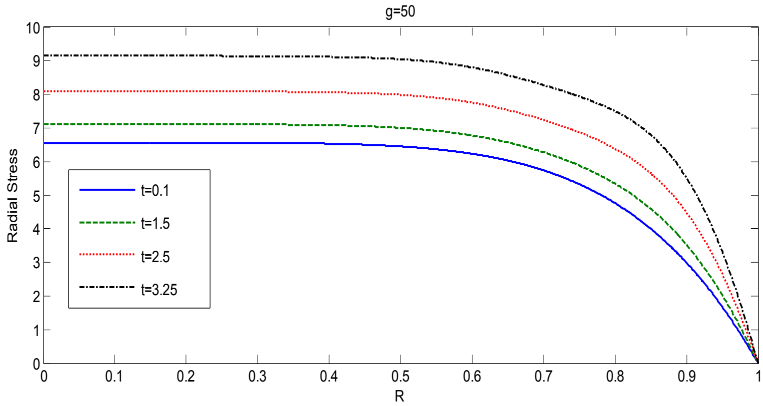

Figure 7: Radial stress case 1

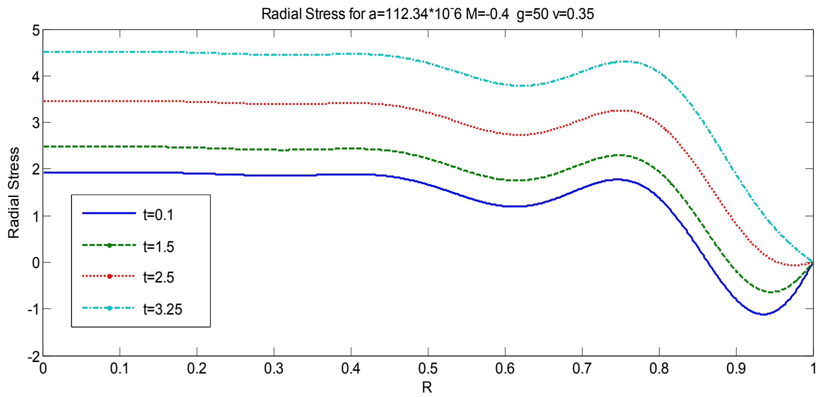

Figure 8: Radial stress case 2

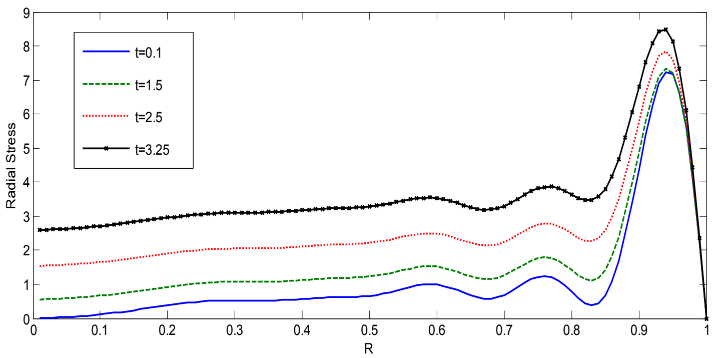

Figure 9: Radial Stress case 3

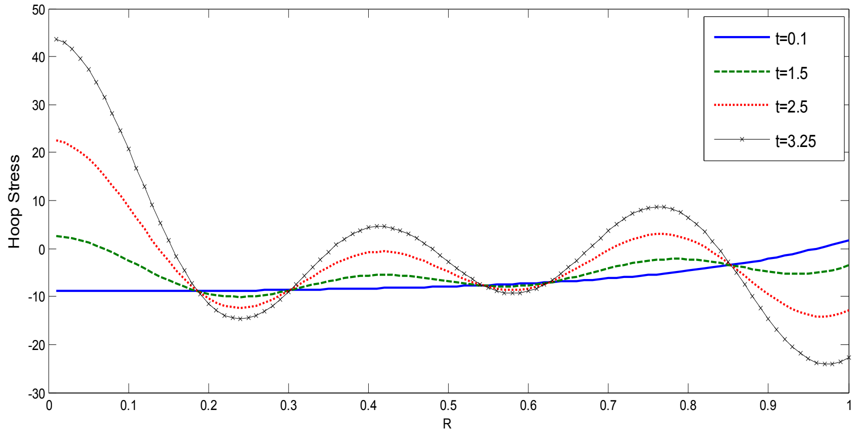

Figure 10: Hoop stress case1

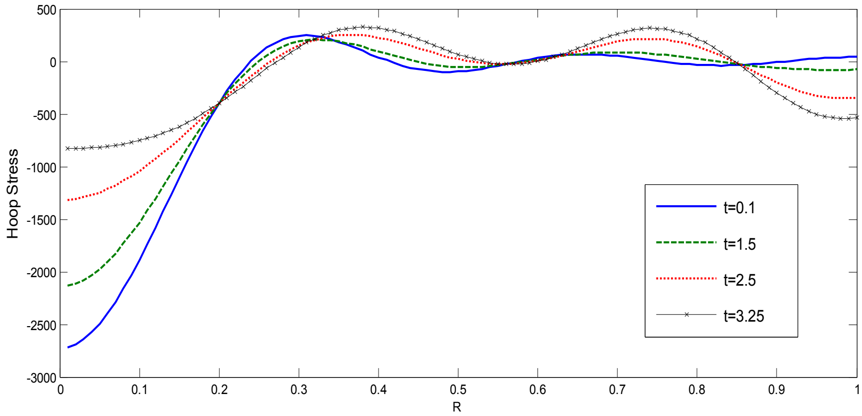

Figure 11: Hoop stress case 2

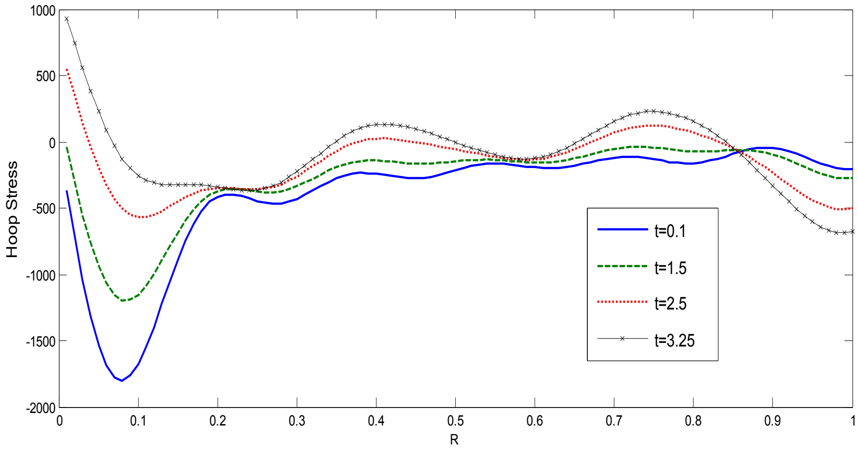

Figure 12: Hoop stress case 3

Fig 7, 8 and 9 shows the variation of radial stresses along the radial direction for three different types of heat generation within the sphere respectively. From the graph it is very much clear that the stresses on the surface are null as per the assumed mechanical condition induced for the radial stresses on the surface of sphere. For instantaneous volume heat source within the sphere the stresses are compressive inside the sphere and the values decreases gradually on the surface. The tension is large and as good as constant for $r < 0.5$ and the nature of the variation is minor. For case 2 the nature of the stresses is compressive as well as tensile. There is large compression on the surface for case 2, while there is tension on the surface for case 3.

Fig 10, 11 and 12 shows the variation of tangential stresses along the radial direction. For instantaneous volumetric source the stresses are compressive for very small time. As time passes the nature of the stresses abruptly changes, centre of the sphere is under tension for case 1, and there are certain radii where the stress is independent of time while the surface has got compression. The nature of the stresses continuously changes from tensile to compression and compression to tensile and stress values are highest about the centre and gradually decreases on the surface. For the point heat source, the centre is under compression and stresses increases along the radial direction with variation. Like case 1, the there are certain radii where the stresses are constant. For the spherical source the variation of tangential stress is shown in fig 12.

V. Conclusions

In this study the analytical solutions are obtained for temperature distribution, displacement and stresses under thermal load with arbitrary initial and ambient temperature and analysis is made by employing three different heat source cases and results are obtained independently. In the analysis instantaneous point, volume and spherical source are used to make the comparative study. In observations it is found that there is a total change in the temperature and thermal stresses profile along the radius with change in the nature of the sources. This model can be applied to spherical structures and to design useful structural applications. The proposed method may be readily extended to solve a wide range of physical engineering problems with change in the form of arbitrary initial and surrounding temperature. The numerical results are discussed as special cases.

Acknowledgement

The authors are thankful to University Grants Commission, New Delhi to provide the partial financial assistance under major/minor research project scheme.

References

[1] Parkus H., Instaionäre Wärmespannungen, Sprnger,Wien, (1959).
[2] Boley B.A. and Weiner J.H., Theory of thermal stresses, Wiley, New York, (1960).
[3] Nowacki W., The state of stress in thick circular plate due to temperature field, Bull Sci. Acad. Polon Sci. Tech., 5, 227, (1957).
[4] Noda N., Hetnarski R.B., Tanigawa Y., Thermal Stresses, 2nd Ed. Talor and Francis, New York, 302, (2003).
[5] Carslaw H.S. and Jeager J.C., Conduction of heat in solids, Clarendon Press , 2nd Ed (1959).
[6] Ozisik M.N., Boundary Value Problem Of Heat Conduction, International text book Company, Scranton, Pennsylvania (1968).
[7] Cheung J.B., Chen T. S. and Thirumalai. K , Transient thermal stresses in a sphere by Local heating, J. Appl. Mech., 41(4), 930-934 (1974).
[8] Takiuti Y. and Tanigawa Y., Transient thermal stresses of a hollow sphere due to rotating heat Source, J. Therm. Stresses, 5(3-4), 283-298 (1982).
[9] Hetnarski R.B., Stresses in a long cylinder due to rotating line source of heat, AIAA, J., 7 (3), 419-423, (1969).
[10] Nasser M. , EI-Maghraby, Two dimensional problem in generalized themoelasticity with heat sources , J. of Thermal Stresses , 27, 227-239, (2004).
[11] Nasser M. , EI-Maghraby, Two dimensional problem for thick plate with heat sources In generalized themoelasticity , J. of Thermal Stresses , 28, 1227-1241, (2005).
[12] Kulkarni V.S., Deshmukh K.C., Warbhe S.D., Quasi-Static Thermal Stresses Due to Heat Generation in a Thin Hollow Circular Disk, J. of Thermal stresses, 31(8), 698- 705, (2008).
[13] Deshmukh K.C., Khandait M.V. and Kulkarni V.S., Thermal stresses due to temperature Distribution in a hollow disk heated by a moving heat source, Far East J. of Applied Mathematics, 66(1), 25-37, (2012).
[14] Kedar G.D. and Deshmukh K.C., Determination of thermal stresses in a thin clamped hollow disk under unsteady temperature field due to point heat source, IOSR J. of Mathematics, 4(6), 14-19, (2013).