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Some Remarks on Complement of Fuzzy Graphs

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Abstract: *Fuzzy Graphs are having numerous applications in problems like Network analysis, Clustering, Pattern Recognition and Neural Networks. The analysis of properties of fuzzy graphs has facilitated the study of many complicated networks like Internet. In this paper we study the structures of complement of many important fuzzy graphs such as Fuzzy cycles, Blocks etc. The complement of fuzzy graphs with a certain structural property is studied in this paper.*
Keywords*: Fuzzy Relations, Fuzzy Graphs, Complement of fuzzy graph, Fuzzy cycle, Fuzzy Blocks, Connectivity in fuzzy graphs.*

I. Introduction

One of the remarkable mathematical inventions of the 20th century is that of Fuzzy sets by Lotfi A. Zadeh in 1965[9]. His aim was to develop a mathematical theory to deal with uncertainty and imprecision. The advantage of replacing the classical sets by Zadeh's fuzzy sets is that it gives greater accuracy and precision in theory and more efficiency and system compatibility in applications. So in systems with imprecision, a fuzzy set model is more efficient than a classic model. The distinction between sets and fuzzy sets is that the sets divide the universal set into two subsets, namely, members and non-members while fuzzy set assigns a membership value to each element of the universal set ranging from zero to one. That is partial memberships are allowed in the latter. Also, fuzzy sets can be used effectively to studynatural variables and qualities like intelligence, beauty, consistency, etc. Zadeh's paper Fuzzy Sets also paved the way to a new philosophical thinking of Fuzzy Logic, which now, is an essential concept in artificial intelligence. Gradually the idea of fuzziness crept into other areas of science and technology. The concept of fuzziness and fuzzy relations led to the development of fuzzy graphs.

A relation represents the presence or absence of association, interaction or interconnectedness between elements of two or more sets. Degrees or strengths of relation or interaction between elements can be made to vary [2]. Degrees of association can be represented by membership grades in a fuzzy relation in the same way as degrees of memberships of objects represented in the fuzzy set. Applications of fuzzy relations are widespread and important; especially in the field of clustering analysis, neural networks, computer networks, pattern recognition, decision making and expert systems. In each of these, the basic mathematical structure is that of a fuzzy graph.

Rosenfeld introduced the notion of fuzzy graphin the year 1975[8]. Fuzzy analogs of many structures in crisp graph theory, like bridges, cut nodes, connectedness, trees and cycles, etc were developed after that. Fuzzy trees were characterized by Sunitha and Vijayakumar [5]. The authors have characterized fuzzy trees using its unique maximum spanning tree. A sufficient condition for a node to be a fuzzy cut node is also established. Center problems in fuzzy graphs, blocks in fuzzy graphs and properties of self complementary fuzzy graphs were also studied by the same authors. They have obtained a characterization for blocks in fuzzy graphs using the concept of strongest paths [7]. Bhutani and Rosenfeld have introduced the concepts of strong arcs, fuzzy end nodes and geodesics in fuzzy graphs [1] .The authors have defined the concepts of strong arcs and strong paths in fuzzy graphs. Sunil Mathew and Sunitha have identified the different types of strong arcs in a fuzzy graph. They have named the different types of strong arcs as α - strong, β - strong and δ-arc.[4]

II. Preliminaries

The following basic definitions are taken from[3]. A fuzzy graph is a pair $G:(\sigma, \mu)$, where σ is a fuzzy subset of a set V and$\mu$is a fuzzy relation on σ, i.e$\mu(u,v) \le \sigma(u) \wedge \sigma(v)$, $\forall u,v\in V$. We assume that V is finite and non-empty, μ is reflexive and symmetric. In all the examples σ is chosen suitably. Also we denote the underlying crisp graph by $G^*: (\sigma^*, \mu^*)$, where$\sigma^* = \{u\in V / \sigma(u) > 0\}$ and $\mu^* = \{(u,v) \in VXV : \mu(u,v) > 0\}$. $H = (\tau,\upsilon)$ iscalledapartialfuzzysub graphofGif$\tau\le\sigma$and$\upsilon\le\mu$. WecallH= (τ,υ) a spanning fuzzy sub graph of $G = (\sigma,\mu)$ if $\tau = \sigma$.A path P of length n is a sequence of distinct nodes u_0, u_1, …$u_n$such that $\mu(u_{i-1}, u_i) > 0$ and degree of membership of a weakest arc is defined as its strength. If $u_0= u_n$and $n\ge3$,then P is called a cycle and it is fuzzy cycle if there is more than one weak arc.

The strength of connectedness between two nodes u,v is defined as the maximum of strengths of all paths between uand v and is denoted by $CONN_G(u, v)$. An arc (u, v) is called a bridge in G if the removal of (u,v) reduces the strength of connectedness between some pair of nodes in G. A connected fuzzy graph is called a fuzzy tree if it contains a spanning sub graph F which is a tree such that, for all edges (u, v) not in F, $\mu(u, v) < CONN_F(u, v)$. A node u in a fuzzy graph G is called a fuzzy cut node if its removal from G reduces the strength of connectedness between some other pair of nodes not involving u. A fuzzy graph G is called a fuzzy block if it does not have fuzzy cut nodes.

Mordeson first introduced the complement of a fuzzy graph. This definition was later modified by Sunitha and Vijayakumar [6]. The complement of a fuzzy graph G: (σ,μ) is $G^c = (\sigma^c,\mu^c)$ where $\sigma^c = \sigma$ and $\mu^c(u, v) = \wedge[\sigma(u), \sigma(v)] - \mu(u, v)$. The advantage of this definition was that, for every fuzzy graph G, $[G^c]^c = G$.

III. About G^c

It can be observed that the class of fuzzy graphs is so wide and it is difficult to understand and analyze the structural properties of fuzzy graphs. It was noticed that there are fuzzy graphs, which are connected, but their complements become disconnected. There may be cases when a fuzzy graph will be a fuzzy tree or fuzzy cycle and this structural property may not be inherited by its complement. In this paper we try to understand and analyze the inherent structure of complement of fuzzy graphs.

Throughout this paper we take the membership value of nodes as 1 unless otherwise specified.

Theorem-1

If G:(σ, μ) is a fuzzy cycle with 5 or more vertices then $(G^c)^*$ will be a block.

Proof

If G is a crisp graph then G is a block if and only if,for any three nodes u,v,w in G there exist a path joining any two of them not involving the third. Let u,v,w be three arbitrary nodes in Supp(G^c). These vertices are also present in G.

Case-1

Suppose there does not exist an edge between u and v in G, then there will be a u − v edge in $(G^c)^*$ and this edge will serve as a path between u and v not involving w. So Supp(G^c) will be a block.

Case-2

If there exist an edge (u,v) in G and if this edge is also present in $(G^c)^*$ then again this edge will be a u−v path not involving w and then $(G^c)^*$ will be a block.

Case-3

Now suppose u and v are adjacent in G but arc (u,v) is not present in G^c. G is a cycle having 5 or more than 5 nodes. Each node in G is adjacent to exactly two nodes. Hence we can choose a node v_1 in $(G^c)^*$ such that both u and v are not adjacent to v_1 in G.

Then $(G^c)^*$ will contain the arcs (u,v_1) and (v,v_1). Then $(u,v_1)(v_1,v)$ will be a u − v path not involving w in $(G^c)^*$.

Therefore $(G^c)^*$ is a block.

Remark

If G is fuzzy cycle with 5 or more nodes, G^c may or may not be a fuzzy block [Figure 1].

Example-1

n=6,Gisafuzzycycle. Supp(G^c)isablockbutG^c isnotafuzzy block.

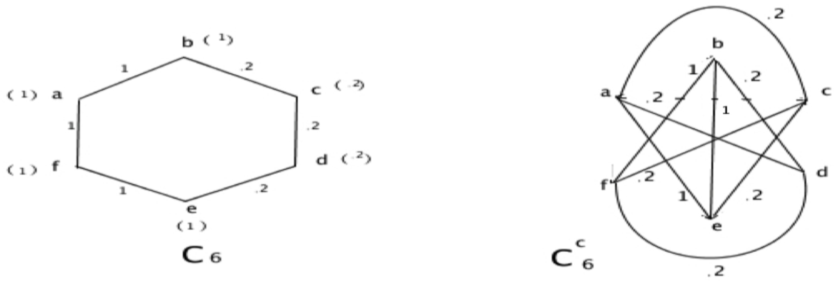

Figure1: eisafuzzycut nodeinC_6^c,soC_6^c isnotafuzzyblock. Notethat(C_6^c)* isa block

Theorem-2

Let G: (σ, μ) be fuzzy graph such that for any three nodes u, v, w in G, there exist a fourth node x such that at least two among u,v,w are not adjacent with x, then $(G^c)^*$is a block.

Proof

Consider three nodes u, v, w, in $(G^c)^*$. Then u, v, w are also nodes of G. Then we can find a fourth node x in G such that at least two among u, v, w are not adjacent with x. Let the non-adjacent nodes be u and v. Then arcs (u,x) and (v,x) will be present in $(G^c)^*$. Hence we get a path between u and v in $(G^c)^*$ not involving w. Hence $(G^c)^*$ will be a block.

Theorem-3

Let G:(σ, μ) be a fuzzy graph such that $\sigma(u) = m, \forall u \in V$, where $m \in [0, 1]$ and for any three nodes u, v, w in G, there exist a fourth node x such that at least two among u, v, w are not adjacent with x, then G^c is a fuzzy block.

Proof

Suppose u,v, and w be three nodes in G^c. If we can obtain a strongest path connecting any two of these nodes not involving the third then G^c will be fuzzy block.

Suppose x be an arbitrary node in G different from u, v, w.Then at least two nodes among u,v, and w will not be adjacent to x. Let the non-adjacent nodes be u and v.

Since u and v are not adjacent to x in G, these nodes will be adjacent to x in G^c. Since $\sigma(u) = m, \forall u \in V$, the arcs (u,x) and (v,x) will have membership values m each and these arcs will be strong arcs in G^c.

Thus (u,x)(x,v) will be a u−v path of strength of m in G^c. This path will be a strongest u − v path in G^c not involving w.

∴ G^c will be a fuzzy block.

Remark

All the results stated above are only sufficient and are not necessary.

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