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MHD Oscillatory Flow of Elastico Viscous Blood through Porous Medium in a Stenosed Artery

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Abstract: *The analysis of the effect of MHD oscillatory flow of elastico Viscous blood through porous medium in a stenosed artery has been presented. Here we assume that the blood behaves as a elastico-viscous fluid(Rivlin-Ericksen type). The expressions for velocity of blood, instantaneous flow rate, wall shear stress and resistive impedance have been obtained. Results obtained have been discussed graphically.*

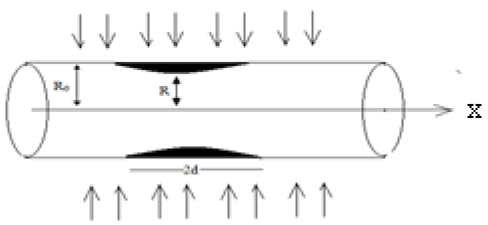
I. Introduction

Recently, biomagnetic fluid dynamics has emerged as a new field for the study of the fluid dynamical behavior of biological fluids. The pumping mechanism of the heart gives arise to a pressure gradient which produces an oscillatory flow in the blood vessel. Womersley[13] worked on the oscillatory motion of a viscous fluid in the rigid tube under a simple harmonic pressure gradient; Daly[2] studied the pulsatile flow through canine femoral arteries with lumen constrictions; Newman *et. al.*[8] worked on the oscillatory flow in a rigid tube with stenosis; Halder[4] has studied the oscillatory flow of blood in a stenosed artery; Bhattacharya[1] have also studied a simple blood flow problem in the presence of magnetic field; Mazumdar *et. al.*[7] have studied some effects of magnetic field on a Newtonian fluid through a circular tube; Tiwari[11] investigated the effect of magnetic field on a simple in an arterial region; Kumar[5] discussed the oscillatory MHD flow of blood in a stenosed artery; Quadrio and Sibilla[9] have study a numerical simulation of turbulent flow in a pipe oscillating around its axis; Rathod *et. al.*[10] have discussed the steady blood flow with periodic body acceleration and magnetic field through an exponentially diversing vessel; Liang *et. al.*[6] have studied the oscillating motions of slug flow in capillary tubes; Kumar and Singh[4] have discussed the oscillatory flow of blood in a stenosed artery in the presence of magnetic field. Tripathi and Singh[12] worked on oscillatory flow of blood through porous medium in a stenosed artery in presence of magnetic field. We consider the problem of Tripathi et.al. with Elastico-Viscous(Rivlin-Ericksen) fluid under the same conditions.

The purpose of present problem is to study the effect of elastico-viscous blood through porous medium in a stenosed artery.

II. Mathematical Model

Consider an oscillatory but laminar fully developed flow of blood through an artery with mild stenosis which is assumed to be Non Newtonian(Rivlin Ericksen). The artery is of constant radius preceeding and following the stenosis. The viscosity and density of the fluid is assumed to be constant. It is also assumed that constriction develops symmetrically caused by deposition of cardiac plaque in the lumen of the artery. The geometry of stenosis is given by.



Geometry of Stenosis

$$\frac{R(x)}{R_0} = 1 - \frac{\varepsilon}{2R_0\left(1 + \cos\frac{\pi x}{d}\right)} \qquad ---- \qquad (1)$$

Governing equation of flow in the tube are the equation of continuity

$$\frac{\partial u}{\partial x} = 0 \qquad ---- \qquad (2)$$

Equation of motion

$$0 = -\frac{\partial p}{\partial r} \qquad ---- \qquad (3)$$

And

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu\left(1 + \lambda\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \left(\sigma B_0^2 + \frac{\mu}{K}\right)u \qquad ---- \qquad (4)$$

Where R(x) = the radius of the artery in the stenotic region
B_0 = Electro-Magnetic Induction
σ = conductivity of fluid
R_0 = Radius of normal artery
2d = Length of stenosis
K = Permeability of porous medium
ε = Maximum height of stenosis
μ = Viscosity
P = Fluid pressure
u = Velocity in axial direction
ρ = Density of the fluid
λ = Visco- elastic coefficient.
Suppose that
$\frac{\varepsilon}{R_0} << 1$
Subject to boundary conditios

$$u = 0 \; on \; r = R, no \; slip \; at \; the \; wall \qquad --- \qquad (5)$$

$$\frac{\partial u}{\partial r} = 0 \; on \; r = 0, symmetry \; about \; the \; axis \qquad ---- \qquad (6)$$

Solution of the problem:

The simple solution of the oscillatory motion of a Non-Newtonian (Rivlin- Ericksen) will be obtained in this section under a pressure gradient which varies with respect to time. A transformation is defined by $y = \frac{r}{R_0}$ is introduced in equation (4) and boundary (5) and (6) we get

$$\left(1 + \lambda\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial y^2} + \frac{1}{y}\frac{\partial u}{\partial y}\right) - \frac{\rho R_0^2}{\mu}\frac{\partial u}{\partial t} = \frac{R_0^2}{\mu}\frac{\partial p}{\partial x} + R_0^2\left(\frac{\sigma B_0^2}{\mu} + \frac{1}{K}\right) \qquad ----- \; (7)$$

$$u = 0 \; on \; y = \frac{R}{R_0} \qquad ---- \qquad (8)$$

$$\frac{\partial u}{\partial y} = 0 \; on \; y = 0 \qquad (9)$$

Let the solution for u and p be set in the forms

$$u(y,t) = \bar{u}(y)e^{i\omega t} \qquad ----- \qquad (10)$$

$$-\frac{\partial p}{\partial x} = Pe^{i\omega t} \qquad ---- \; (11)$$

and

$$\frac{d^2\bar{u}}{dy^2} + \frac{1}{y}\frac{d\bar{u}}{dy} - k^2\bar{u} = +\frac{R_0^2 P}{\mu(1 + i\lambda\omega)} \qquad ----- \qquad (12)$$

Where $k^2 = i\beta_1^2$

And $\beta_1^2 = \frac{\frac{\rho R_0^2\omega}{\mu} - iR_0^2\left(\frac{\sigma}{\mu}B_0^2 + \frac{1}{K}\right)}{(1 + i\lambda\omega)}$ $\qquad --- (13)$

The solution of equation (12) with boundary conditions (8) and (9) is

$$\bar{u}(y) = \frac{PR_0^2}{i\mu\beta_1^2(1 + i\lambda\omega)}\left[1 - \frac{J_0(i^{\frac{3}{2}}\beta_1 y)}{J_0(i^{\frac{3}{2}}\beta_1\frac{R}{R_0})}\right] \qquad ----(14)$$

Where J_0 is Bessel function of zero order with complex argument, then the resulting expression for the axial velocity in the artery is given by;

$$\bar{u}(r,t)=\frac{PR_0^2}{i\mu\beta_1^2(1+i\lambda\omega)}\left[1-\frac{J_0(i^{\frac{3}{2}}\beta_1 y)}{J_0(i^{\frac{3}{2}}\beta_1\frac{R}{R_0})}\right]e^{i\omega t}\quad ----(15)$$

The volumetric flow rate Q is given by

$$Q=2\pi\int_0^R urdr\quad ---------------(16)$$

Which gives on integration

$$Q=\frac{\pi PR_0^4}{i\mu\beta_1^2(1+i\lambda\omega)}\left(\frac{R}{R_0}\right)\left[\left(\frac{R}{R_0}\right)-\frac{2J_1(i^{\frac{3}{2}}\beta_1\frac{R}{R_0})}{i^{\frac{3}{2}}J_0(i^{\frac{3}{2}}\beta_1\frac{R}{R_0})}\right]e^{i\omega t}\quad ---\ (17)$$

The shear stress at the wall r=R is defined by

$$\tau_R=\mu\left(\frac{\partial u}{\partial r}\right)_{r=R}$$

Which gives

$$\tau_R=\frac{PR_0\ e^{i\omega t}}{\beta_1}i^{3/2}\frac{J_1(i^{\frac{3}{2}}\beta_1\frac{R}{R_0})}{J_0(i^{\frac{3}{2}}\beta_1\frac{R}{R_0})}\quad ----(18)$$

Now

$$\frac{\tau_R}{Q}=-i\frac{\mu\beta_1^2}{\pi R_0^3}\frac{J_1(i^{\frac{3}{2}}\beta_1\frac{R}{R_0})}{i\beta_1\left(\frac{R}{R_0}\right)J_0\left(i^{\frac{3}{2}}\beta_1\frac{R}{R_0}\right)^2-2\left(\frac{R}{R_0}\right)J_1(i^{\frac{3}{2}}\beta_1\frac{R}{R_0})}\quad ----(19)$$

If τ_R is normalized with the steady flow solution given by

$$\tau_N=\frac{4\mu Q_0}{\pi {R^3}_0}\quad --(20)$$

Where Q_0 is the steady flow volume rate.

The resistive impedance to flow is defined by

$$z=\frac{\partial p/\partial x}{Q}$$

$$z=\frac{i\mu\beta_1^2}{\pi R_0^4}\frac{\left(\frac{R}{R_0}\right)}{\left\{\frac{R}{R_0}-\frac{2J_1\left(i^{3/2}\beta_1\frac{R}{R_0}\right)}{i^{3/2}\beta_1 J_0\left(i^{3/2}\beta_1\frac{R}{R_0}\right)}\right\}}\quad -----\ (21)$$

III. Deduction

If λ is taken as zero then results agree with Anil Tripathi, K.K. Singh (2012).

IV. Discussion

From figure 1 it is clear that blood velocity is going to decreases as the radius of artery is going to increase. From figure 2 it is clear that axial velocity of blood(Rivlin- Ericksen fluid) is going to decrease for higher values of visco elastic coefficient(λ).

We have also calculated instantaneous flow rate vs λ for different values of stenosis heights. From figure 3 it is observed that instantaneous flow rate is going to decrease as λ is gong to increase.

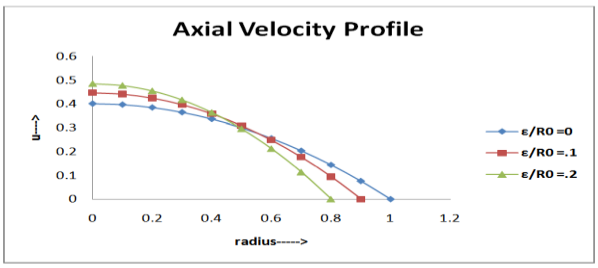


Figure :-1 variation of axial velocity for different values of stenosis heights

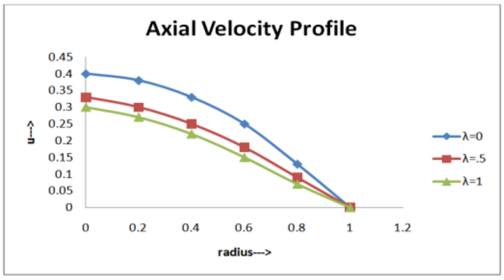


Figure :-2 Variation of axial velocity for different values of λ(visco elastic coefficient)

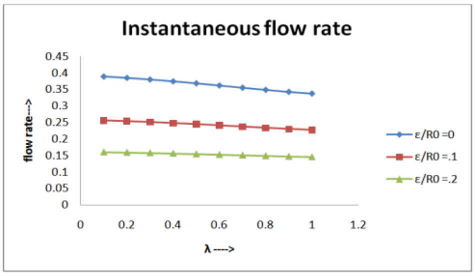


Figure :-3 variation of instantaneous flow rate with respect to λ for different values of stenosis heights

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