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A Study on Optimization using Stochastic Linear Programming

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Abstract: *The self Help Group (SHG) is group of rural poor who have organized themselves into a group for eradicationof poverty. The members of the group belong to families below the poverty line. This will help the families of occupational groups like agricultural labourers, marginal farmers, designers and artisans marginally above the poverty line, or who may have been excluded from the Below Poverty Line (BPL) list to become members of the Self Help Group. A self help group consists of two categories. One named as magalier thittam and another is non- magalier thittam. The factors of Self help group categories are random in nature. These factors can be handled using stochastic linear programming problem (SLPP). Here the data is collected from Tuticorin district. The optimization technique such as two stage programming and chance constrained programming can be adopted for SLPP. In this paper chance constrained programming (CPP) is used to obtain optimal solution.*
Keywords: *SLPP, CCP, LP, SHG.*

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I. Need for Stochastic Programming Problem

Linear programming (LP) constitutes a set of Mathematical Methods specially designed for the modeling of certain kinds of constrained optimization problems. LP problems deal with the optimization of a function which consists of many decision variables. The coefficients and the decision variables of optimization function of a LP model are known in advance. However, for many actual problems, the decision variables may not be known accurately for a variety of reasons. The fundamental reason is that the decision variables may represent information about the future (e.g. product demand or price for a future time period) and simply cannot be known with certainty. So we can apply SLPP. SLPP deals with situations where some or all of the parameters of the optimization problem are described by stochastic (or random or probabilities) variables rather than by deterministic quantities. This paper attempts to find an optimal solution for the following problem.

Optimization Problem dealing with SHG

- SHG (11) is the group of women either working together or living in the neighbourhood, engaged in similar line of activity ranging from 12-20 members.
- The disabled persons are allowing to this group, the group shall not consist of more than one member from the same family and a person should not be a member of more than one group.
- These groups should operate a group account preferably in their service area bank branch and the group should develop financial management norms covering the loan sanction procedure, repayment schedule and interest rates.
- The group should maintain simple basic records such as Minutes book, Attendance register Loan ledger, General ledger, Cash book, Bank passbook and individual passbooks.
- The members in the group meetings should take all the loaning decisions through a participatory decision making process.
- The group should be able to enhance priorities for the loan applications, fix repayment schedules, fix appropriate rate of interest for the loans advanced and closely monitor the repayment of the loan installments from the borrower.

II. Details of the Problem

According to 2001 Census the total population of the study area was 68,263 which are spread over 146 sq.km. The Density of population per Sq.km is 266. Among 68,263 populations, 33,774 belong to male and 34,489 belong to female. The sexratio of this block is 1,021. The concerned details of population status in Tuticorin Block is shown in

Table 1

Serial No	Gender	Number of Period in %
1	Male	33,774 (49.48)
2	Female	34,489 (50.52)
	Total	68,263 (100)

A SHG consists of two categories viz. magalier thittam (part I) and non- magalier thittam (part II). The members of SHG in Tuticorin district are farmers, designers, small shopkeepers, sales men etc. The data (working time required per unit) is collected from designers, who are engaging in embroidery mechanism, designing glass works and fancy works. the items required on different decorative works for each group are not known precisely (as they vary from worker to worker) but are known to follow normal distribution with mean (μ) and standard deviation (σ) as indicated in the table 2

Details of Magalier Thittam and Non Magalier Thittam

Typesof Textiles designing	Working time required per unit (minutes)		Maximum time available per week (minutes)	
	Part I Magalier Thittam	Part II Non Magalier Thittam	Mean	SD
Embroidery	$\overline{a_{11}}$ =7.14σ_{a11}=3.18	$\overline{a_{12}}$ =4.16σ_{a12}=2.67	$\overline{b_1}$ =6000	σ_{b1}=600
Design with glass work	$\overline{a_{21}}$ =5.2σ_{a21}=1.9	$\overline{a_{22}}$ =6.4σ_{a12}=2.7	$\overline{b_2}$ =4800	σ_{b2}=400
Fancy work	$\overline{a_{31}}$ =3.2σ_{a31}=1.92	$\overline{a_{32}}$ =3σ_{a32}=1.58	$\overline{b_3}$ =3600	σ_{b3}=200
Profit per Unit	C_1=180	C_2=250		

III. Stochastic Linear Programming Problem

A SLPP [5] [6] [8] can be stated as follows:

Maximize or Minimize $f(x)= c^{T}x$

Subject to

$$A_i^T x = \sum_{j=1}^{n} = a_{ij}x_j \geq b_i,\ i= 1,2,\ldots,m$$

and $x_j \geq 0$, j=1,2,3…..n

Where $c_j a_{ij}$ and b_i are random variables (the decision variables x_j are assumed to be deterministic for simplicity) with known probability distributions. Several methods are available for solving the SLPP. However, this paper considers the CCP alone.

Chance Constrained Programming Technique

The Chance Constrained programming [1] [2] [3] [4] [6] [7] was originally developed by Charnes and Cooper. It belongs to the major approaches for dealing random parameters in optimization problems. Typical areas of application are engineering and finance where uncertainties like product demand, meteorological or demographic conditions, current exchange rates etc.

In CCP, the SLPP is stated as follows:

$$\text{Maximize or Minimize } f(x)= \sum_{j=1}^{n} = c_j x_i$$

Subject

$$P\left(\sum_{j=1}^{n} a_{ij}x_j \leq b_i \geq p_i, \right) \quad i= 1,2,\ldots, m \qquad (2.1.1.)$$

and $x_j \geq 0$, j= 1,2,3….n

Where $c_j a_{ji}$ and b_i are random variables and p_i are specified probabilities.

We shall first consider special cases where only c_j or a_{ji} or b_i are random variables before considering the general case in which $c_j a_{ji}$ and b_i and b_i are all random variables. We shall further assume that all the random variables are normally distributed with known mean and standard deviation.

When c_j, a_{ji}, b_i are random variables: As the random variables c_j, j= 1,2…n, appear only in the objective function, the new objective function is

Maximize or Minimize $F(x)= k_1 \overline{f} + k_2 \sqrt{\text{var}(f)}$

subject $p[h_i \leq 0] \geq p_i$, i = 1,2…., m (2.12)

Where $\overline{f} = \sum_{j=1}^{n} \overline{c}_j x_j$, k_1 and k_2 are nonnegative constants whose values indicate the relative importance of $\overline{f}$ and standard deviation of f for minimization or maximization.

h_i is anew random variables defined as

$$h_i = \sum_{j=1}^{n} a_{ij}x_j - b_i = \sum_{k=1}^{n+1} q_{ik}y_k q_{ik} = a_{ik}q_{i,n+1} = b_i \qquad y_k = x_k, k=12,..y_{n+1}=-1$$

it will follow normal distribution, the mean and the Variance of h_i are given by

$$\overline{h}_i = \sum_{k=1}^{n+1} \overline{q}_{ik} y_k = \sum_{k=1}^{n+1} \overline{a}_{ij} x_j - \overline{b}_i \text{ and } Var(h_i) = Y^T V_i Y$$

$Var(h_i) = Y^T V_i Y$

$$y = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ y_{n+1} \end{Bmatrix} \quad V_i = \begin{bmatrix} var(q_{i1}) & Cov(q_{i1}, q_{i2}) & .. & Cov(q_{i1}, q_{i,n+1}) \\ Cov(q_{i2}, q_{i1}) & var(q_{i2}) & .. & Cov(q_{i2}, q_{i,n+1}) \\ . & . & .. & . \\ . & . & .. & . \\ Cov(q_{i,n+1}, q_{i1}) & Cov(q_{i,n+1}, q_{i2}) & .. & var(q_{i'n+1}) \end{bmatrix}$$

This can be written as

$$Var(h_i) = \left[\sum_{k=1}^{n+1} x_k^2 \, Var(a_{ik} + 2) \sum_{i=k+1}^{n} x_k x_1 Cov(a_{ik}, a_{i1}) + Var(b_i) - 2 \sum_{k=1}^{n} x_k Cov(a_{ik}, b_i) \right]$$

Thus the constraints in (2.1.2) can be restated as

$$p\left[\frac{h_i - \overline{h}_i}{\sqrt{Var(h_i)}} \le \frac{-\overline{h}_i}{\sqrt{Var(h_i)}} \right] \ge p_i \quad 1=1,2\ldots,m \qquad (2.1.3)$$

The constraints of equation (2.1.3) can be stated as

$$\phi \left(-\frac{\overline{h}_i}{\sqrt{Var(h_i)}} \right) \ge \phi(e_i)$$

i= 1,2…..,m

The SLPP can be stated as

$$\text{Maximize or Minimize } F(X) = k_1 \sum_{j=1}^{n} \overline{c}_j x_j + k_2 \sqrt{X^T V X}, \; k_1 \ge 0, k_2 \ge 0$$

Subject to

$$\overline{h}_i + e_i \sqrt{Var(h_i)} \le 0, \; i=1,2,\ldots,m$$

and

$$x_j \ge 0, \; j=1,2\ldots,n$$

Implementation of this Technique

By denoting the number of SHG part I and part II manufactured per week as x_1 and x_2 respectively, the problem can be stated as follows:

Maximize

$$f = k_1(180\,x_1 + 200)k_2 \sqrt{400x_1^2 + 2500x_2^2}$$

Subject to constraints

$$7.14x_1 + 4.6\,x_2 + 2.33\sqrt{10.11x_1^2 + 7.13x_2^2 + 360000} - 6000 \le 0$$

$$5.2x_1 + 6.2x_2 + 2.33\sqrt{3.61x_1^2 + 7.29x_2^2 + 160000} - 4800 \le 0$$

$3.2x_1+3x_2+2.33\sqrt{3.686x_1^2+2.49x_2^2+40000}-3600\leq 0$

and $x_1\geq 0$, $x_2\geq 0$

This problem can be solved by using any of the nonlinear programming methods. Here the Kuhn Tucker conditions are followed.

$$\frac{\delta f}{\delta x_1}-\lambda_1\frac{\delta h}{\delta x_1}=0$$

$$\frac{\delta f}{\delta x_2}-\lambda_2\frac{\delta h}{\delta x_2}=0$$

$$\frac{\delta f}{\delta x_3}-\lambda_3\frac{\delta h}{\delta x_3}=0$$

$\lambda_i h^i=0$, $h^i\leq 0$, $\lambda_i\geq 0$

λ is the Lagrangian multipliers.

The Kuhn Tucker conditions related to the problem is shown as Maximize

$f-k_1(180x_1+200x_2)+k_2$

subjects to constraints

$h^1(x)=7.14x_1+4.6x2+2.33\sqrt{10.11x_1^2+7.13x_2^2+360000}-6000\leq 0$

$h^2(x)=5.2x_1+6.2x_2+2.33\sqrt{3.61x_1^2+7.29x_2^2+160000}-4800\leq 0$

$h^3(x)=3.2x_1+3x_2+2.33\sqrt{3.686x_1^2+2.49x_2^2+40000}-3600\leq 0$

and $x_1\geq 0$, $x_2\geq 0$

Hence, the solution of the SLPP is obtained as $x_1=128$ and $x_2=173$. The maximization yields the two categories of magalier thittam & non magalier thittam are 96182.61.

IV. Conclusion and Suggestions

In this paper, SLPP is used to optimize the number of groups in SHGof Tuticorin district. To solve the SLPP, the CCP is adopted and theoptimal solution is obtained. This chance constrained programmingpermits the constraints to be violated by a specified probability; whereas other techniques like two stage programming technique do not permit any constraint to be violated. In this aspect, this technique)is suitable for various aspects (like production, demand...).

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