IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728,p-ISSN: 2319-765X, Volume 7, Issue 1 (May. - Jun. 2013), PP 63-70
www.iosrjournals.org

On Fuzzy γ - Semi Open Sets and Fuzzy γ - Semi Closed Sets in Fuzzy Topological Spaces

R.Usha Parameswari[1], K.Bageerathi[2],
[1,2]*Department of Mathematics, Govindammal Aditanar College for Women, Tiruchendur-628215, India.*

Abstract: *The aim of this paper is to introduce the concept of fuzzy γ- semi open and fuzzy γ- semi closed sets of a fuzzy topological space. Some characterizations are discussed, examples are given and properties are established. Also, we define fuzzy γ- semi interior and fuzzy γ- semi closure operators. And we introduce fuzzy γ- t-set, γ-SO extremely disconnected space analyse the relations between them.*
MSC 2010*: 54A40, 03E72.*
Key words: *Fuzzy γ - open, fuzzy γ - closed, fuzzy γ- semi open, fuzzy γ- semi closed, fuzzy γ- semi interior and fuzzy γ- semi closure, γ- t-set and fuzzy topology.*

I. Introduction

The concepts of fuzzy sets and fuzzy set operations were first introduced by L.A.Zadeh [6] in his paper. Let X be a non empty set and I be the unit interval [0,1]. A Fuzzy set in X is a mapping from X in to I. In 1968, Chang [3] introduced the concept of fuzzy topological space which is a natural generalization of topological spaces. Our notation and terminology follow that of Chang. Azad introduced the notions of fuzzy semi open and fuzzy semi closed sets. And T.Noiri and O.R.Sayed[5] introduced the notion of γ-open sets and γ-closed sets. Swidi Oon[4] studied some of its properties.

Through this paper (X, τ) (or simply X), denote fuzzy topological spaces. For a fuzzy set A in a fuzzy topological space X, cl(A), int(A), A^C denote the closure, interior, complement of A respectively. By 0_x and l_x we mean the constant fuzzy sets taking on the values 0 and 1, respectively.

In this paper we introduce fuzzy γ-semi open sets and fuzzy γ-semi closed sets its properties are established in fuzzy topological spaces. The concepts that are needed in this paper are discussed in the second section. The concepts of fuzzy γ-semi open and fuzzy γ-semi closed sets in fuzzy topological spaces and studied their properties in the third and fourth section respectively. Using the fuzzy γ- semi open sets, we introduce the concept of fuzzy γ- SO extremely disconnected space. The section 5 and 6 are dealt with the concepts of fuzzy γ-semi interior and γ-semi closure operators. In the last section, we define fuzzy γ-t-sets and discuss the relations between this set and the sets defined previously.

II. Preliminaries

In this section, we give some basic notions and results that are used in the sequel.

Definition 2.1: A fuzzy set A of a fuzzy topological space X is called:

1) fuzzy semi open (semi closed) **[2]** if there exists a fuzzy open (closed) set U of x such that $U \le A \le cl\ U$ ($int\ U \le A \le U$).
2) fuzzy strongly semi open (strongly semi closed) **[4]** if $A \le int(cl(int A))$ ($A \ge cl(int(cl A)))$.
3) fuzzy γ-open (fuzzy γ-closed) **[5]** if $A \le (int(cl A)) \vee cl(int(A))$ ($A \ge (cl(int(A))) \wedge (int (cl(A)))$).

Definition 2.2[7]: If λ is a fuzzy set of X and μ is a fuzzy set of Y, then $(\lambda \times \mu)(x, y)= \min \{ \lambda(x), \mu(y)\}$, for each X×Y.

Definition 2.3[2]: An fuzzy topological space (X, τ_1) is a product related to an fuzzy topological space (Y, τ_2) if for fuzzy sets A of X and B of Y whenever $C^c \not\ge A$ and $D^c \not\ge B$ implies $C^c \times 1 \vee 1 \times D^c \ge A \times B$, where $C \in \tau_1$ and $D \in \tau_2$, there exist $C_1 \in \tau_1$ and $D_1 \in \tau_2$ such that $C_1^c \ge A$ or $D_1^c \ge B$ and $C_1^c \times 1 \vee 1 \times D_1^c = C^c \times 1 \vee 1 \times D^c$.

Lemma 2.4 [2]: Let X and Y be fuzzy topological spaces such that X is product related to Y. Then for fuzzy sets A of X and B of Y,

1) $cl (A \times B) = cl (A) \times cl (B)$
2) $int (A \times B) = int (A) \times int (B)$

Lemma 2.5[1]: For fuzzy sets λ, μ, υ and ω in a set S, one has
$(\lambda \wedge \mu) \times (\upsilon \wedge \omega) = (\lambda \times \omega) \wedge (\mu \times \upsilon)$

Remark 2.6[5]:
1. Any union of fuzzy γ-open sets in a fuzzy topological space X is a fuzzy γ-open set.
2. Any intersection of fuzzy γ-closed sets is fuzzy γ-closed set.
3. Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a collection of fuzzy γ-open sets in a fuzzy topological space X. Then $\vee_{\alpha\in\Delta} A_\alpha$ is fuzzy γ-open.

Definition 2.7[5]: Let A be any fuzzy set in the fuzzy topological space X. Then we define γ-cl (A) = $\wedge$ {B: B $\geq$ A, B is fuzzy γ-closed} and
γ-int (A) = $\vee$\{B: B $\leq$ A, B is fuzzy γ-open in X\}.

Properties 2.8[5]: Let A be any fuzzy set in the fuzzy topological space X. Then
a) γ-cl $(A^c) = (\gamma\text{-int } (A))^c$
b) γ-int $(A^c) = (\gamma\text{-cl}(A))^c$

Properties 2.9[5]: Let A and B be any two fuzzy sets in a fuzzy topological space X. Then
1) γ-int (0) = 0, γ-int (1) = 1.
2) γ-int (A) is fuzzy γ-open in X.
3) γ-int(γ-int (A)) = γ-int (A).
4) if A $\leq$ B then γ-int (A) $\leq$ γ-int (B).
5) γ-int (A$\wedge$B) = γ-int (A) $\wedge$ γ-int (B).
6) γ-int (A$\vee$B) $\geq$ γ-int (A) $\vee$ γ-int (B).

Properties 2.10[5]: Let A and B be any two fuzzy sets in a fuzzy topological space X. Then
1) γ-cl(0) = 0, γ-cl(1) = 1.
2) γ-cl(A) is fuzzy γ-closed in X.
3) γ-cl(γ-cl(A)) = γ-cl(A).
4) if A $\leq$ B then γ-cl(A) $\leq$ γ-cl(B).
5) γ-cl(A$\vee$B) = γ-cl(A) $\vee$ γ-cl(B).
6) γ-cl (A$\wedge$B) $\leq$$\gamma$-cl (A) $\wedge$ γ-cl(B).

III. Fuzzy γ-Semi Open Sets

In this section we introduce the concept of fuzzy γ-semi open sets in a fuzzy topological space.

Definition 3.1: Let A be a fuzzy subset of a fuzzy topological space (X,τ). Then A is called fuzzy γ-semi open set of X if there exist a fuzzy γ-open set γ-O such that γ-O $\leq$ A $\leq$ cl (γ-O).

Theorem 3.2: Let (X,τ) be a fuzzy topological space. Let A and B be any two fuzzy subsets of X and γ-int (A)$\leq$ B $\leq$$\gamma$-cl (A). If A is a fuzzy γ-semi open set then so is B.

Proof:

Let A and B be a fuzzy subsets of X and γ-int (A) $\leq$ B $\leq$ γ-cl(A). Let A be fuzzy γ-semi open set. By Definition 3.1, there exists a fuzzy γ-open set γ-O such that γ-O $\leq$ A $\leq$ cl (γ-O), it follows that γ-O $\leq$ γ-int (A) $\leq$ A $\leq$ γ-cl(A) $\leq$ cl (γ-O) and hence γ-O $\leq$ B $\leq$ cl (γ-O). Thus B is a fuzzy γ-semi open set.

Theorem 3.3: Let (X, τ) be a fuzzy topological space. Then a fuzzy subset A of a fuzzy topological space (X, τ) is fuzzy γ-semi open if and only if A $\leq$ cl (γ-int(A)).

Proof:

Let A $\leq$ cl(γ-int(A)). Then for γ-O = γ-int(A), we have γ-int(A) $\leq$ A. Therefore γ-int(A) $\leq$ A $\leq$ cl (γ-int(A)). Conversely, let A be a fuzzy γ-semi open. By Definition 3.1, there exists a fuzzy γ-open set γ-O such that γ-O $\leq$ A $\leq$ cl (γ-O). But γ-O $\leq$ γ-int (A). Thus cl (γ-O) $\leq$ cl (γ-int (A)). Hence A$\leq$ cl(γ-O) $\leq$ cl (γ-int(A)).

Remarks 3.4: It is obvious that every fuzzy γ-open is fuzzy γ-semi open and every fuzzy open set is fuzzy γ-semi open but the separate converses may not be true as shown by the following example.

Example 3.5: Let X =\{a, b, c\} and τ = \{0, 1,\{$a_{.2}$, $b_{.3}$, $c_{.5}$\}, \{$a_{.4}$, $b_{.7}$, $c_{.3}$\}, \{$a_{.2}$, $b_{.3}$, $c_{.3}$\},\{$a_{.4}$, $b_{.7}$, $c_{.5}$\}\}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c =\{0, 1, \{$a_{.8}$, $b_{.7}$, $c_{.5}$\}, \{$a_{.6}$, $b_{.3}$, $c_{.7}$\}, \{$a_{.8}$, $b_{.7}$, $c_{.7}$\}, \{$a_{.6}$, $b_{.3}$, $c_{.5}$\}\}. Let A = \{$a_{.4}$, $b_{.6}$, $c_{.6}$\}. Then cl(int(A))= \{$a_{.6}$, $b_{.3}$, $c_{.5}$\} and int(cl(A)) = \{$a_{.4}$, $b_{.7}$, $c_{.5}$\}.Therefore int(cl(A)) $\vee$ cl(int(A)) = \{$a_{.6}$, $b_{.7}$, $c_{.5}$\}. By Definition 2.1(3), A is not fuzzy γ-open. Now let γ-int(A) = \{$a_{.2}$, $b_{.6}$, $c_{.6}$\}. Then A $\leq$ cl (γ-int(A)) =\{$a_{.8}$, $b_{.7}$, $c_{.7}$\}. Thus A is fuzzy γ-semi open.

The next example shows that every fuzzy γ-semi open set need not be fuzzy open.

Example 3.6:

Let X = \{a, b, c\} and τ = \{0, 1,\{$a_{.1}$, $b_{.2}$, $c_{.3}$\}, \{$a_{.5}$, $b_{.1}$, $c_{.4}$\}, \{$a_{.1}$, $b_{.1}$, $c_{.3}$\},\{$a_{.5}$, $b_{.2}$, $c_{.4}$\}\}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = \{0, 1, \{$a_{.9}$, $b_{.8}$, $c_{.7}$\}, \{$a_{.5}$, $b_{.1}$, $c_{.6}$\}, \{$a_{.9}$, b.$_9$, c.$_7$}, {a.$_5$, b.$_8$, c.$_6$}}. Let A = {a.$_5$, b.$_3$, c.$_5$}. Then γ-int(A) = {a.$_5$, b.$_3$, c.$_4$} and cl(γ-int(A)) = {a.$_5$, b.$_8$, c.$_6$}. It shows that $A \le$ cl (γ-int (A)). By using Theorem 3.3, A is fuzzy γ-semi open. But A is not a fuzzy open set.

It is clear that every fuzzy semi open is fuzzy γ-semi open but the converse need not be true as shown by the following example.

Example 3.7: Let X = {a, b} and τ = {0, 1, {a.$_2$, b.$_3$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {a.$_8$, b.$_7$}}. Let A = {a.$_1$, b.$_2$}. Then γ-int(A) = {a.$_1$, b.$_1$} and cl(γ-int(A)) = {a.$_8$, b.$_7$}. It shows that $A \le$ cl (γ-int (A)). By using Theorem 3.3, A is fuzzy γ-semi open. Now cl(int(A)) = {0}. That shows A $\not\leq$ cl (int (A)) . Hence A is not a fuzzy semi open set.

Proposition 3.8: Let (X, τ) be a fuzzy topological space. Then the union of any two fuzzy γ-semi open sets is a fuzzy γ-semi open set.

Proof:

Let A_1 and A_2 be the two fuzzy γ-semi open sets. By Theorem 3.3, $A_1 \le$ cl (γ-int (A_1)) and $A_2 \le$ cl (γ-int (A_2)). Therefore $A_1 \vee A_2 \le$ cl(γ-int (A_1)) $\vee$ cl(γ-int(A_2)) = cl(γ-int (A_1) $\vee$ γ-int(A_2)). By using Properties 2.9(6), $A_1 \vee A_2 \le$ cl(γ-int ($A_1 \vee A_2$)). Hence $A_1 \vee A_2$ is fuzzy γ-semi open.

The following example shows that the intersection of any two fuzzy γ-semi open sets need not be fuzzy γ-semi open set.

Example 3.9: Let X ={a, b} and τ = {0, 1,{a.$_2$, b.$_4$}, {a.$_3$, b.$_5$}}. Then (X, τ) be a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {a.$_8$, b.$_6$}, {a.$_7$, b.$_5$}}. Let A = {a.$_8$, b.$_9$} and γ-int (A) = {a.$_8$, b.$_8$}. Then we get cl(γ-int(A))= {1}.Thus by Theorem 3.3, A is fuzzy γ-semi open.

Let B = {a.$_9$, b.$_7$}. Then γ-int (B) = {a.$_9$, b.$_5$} and we get cl(γ-int(B))= {1}.Thus by Theorem 3.3, B is fuzzy γ-semi open. Now $A \wedge B$ = {a.$_8$, b.$_7$} and γ-int ($A \wedge B$) = {a.$_6$, b.$_5$}. Then cl(γ-int ($A \wedge B$)) = {a.$_7$, b.$_5$}. Thus $A \wedge B$ is not less than or equal to cl(γ-int ($A \wedge B$)). Therefore $A \wedge B$ is not fuzzy γ-semi open.

Theorem 3.10: Let (X, τ) be a fuzzy topological space and let $\{A_\alpha\}_{\alpha\in\Delta}$ be a collection of fuzzy γ-semi open sets in a fuzzy topological space X. Then $\vee_{\alpha\in\Delta} A_\alpha$ is fuzzy γ-semi open.

Proof:

Let Δ be a collection of fuzzy γ-semi open sets of a fuzzy topological space (X, τ). Then by using Theorem 3.3, for each $\alpha\in\Delta$, $A_\alpha \le$ cl(γ-int(A_α)). Thus $\vee_{\alpha\in\Delta} A_\alpha \le \vee_{\alpha\in\Delta}$ cl(γ-int(A_α)). Since $\vee$cl(A_α)$\le$cl($\vee A_\alpha$), $\vee_{\alpha\in\Delta} A_\alpha \le$ cl($\vee_{\alpha\in\Delta}$ (γ-int (A_α)). By using Remark 2.6(3), $\vee_{\alpha\in\Delta} A_\alpha \le$ cl(γ-int $(\vee_{\alpha\in\Delta} A_\alpha)$). Thus the arbitrary union of fuzzy γ-semi open sets is fuzzy γ-semi open.

Theorem 3.11: Let (X, τ) and (Y, σ) be any two fuzzy topological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of fuzzy γ-open set A_1 of X and a fuzzy γ-open set A_2 of Y is fuzzy γ-open set of the fuzzy product space X $\times$ Y.

Proof:

Let A_1 be a fuzzy γ-open subset of X and A_2 be a fuzzy γ-open subset of Y. Then by Definition 2.1(3), we have $A_1 \le$ int(cl (A_1))$\vee$ cl(int(A_1)) and

$A_2 \le$ int(cl (A_2)) $\vee$ cl(int(A_2)). Now $A_1 \times A_2 \le$ (int(cl (A_1)) $\vee$ cl(int(A_1))) $\times$(int(cl (A_2)) $\vee$ cl(int(A_2))). By using Definition 2.2,

$A_1 \times A_2 \le \min\{$ (int(cl (A_1)) $\vee$ cl(int(A_1))), (int(cl (A_2)) $\vee$ cl(int(A_2)))$\}$

= (int(cl (A_1)) $\vee$ cl(int(A_1))) $\wedge$ (int(cl (A_2)) $\vee$ cl(int(A_2)))

= (int(cl (A_1)) $\wedge$ int(cl (A_2)) $\vee$ (cl(int(A_1))$\wedge$cl(int(A_2)))

= (int(cl ($A_1 \times A_2$))) $\vee$ (cl(int($A_1 \times A_2$)))

Therefore $A_1 \times A_2$ is fuzzy γ-open in the fuzzy product space X $\times$ Y.

Theorem 3.12: Let (X, τ) and (Y, σ) be any two fuzzy topological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of a fuzzy γ-semi open set A_1 of X and a fuzzy γ-semi open set A_2 of Y is fuzzy γ-semi open set of the fuzzy product space X $\times$ Y.

Proof:

Let A_1 be a fuzzy γ-semi open subset of X and A_2 be a fuzzy γ-semi open subset of Y. Then by using Theorem 3.3, we have $A_1 \le$ cl(γ-int (A_1)) and $A_2 \le$ cl (γ-int (A_2)). This implies that, $A_1 \times A_2 \le$ cl(γ-int (A_1)) $\times$ cl (γ-int (A_2)).By Lemma 2.4(1), $A_1 \times A_2 \le$ cl (γ-int (A_1) $\times$ (γ-int (A_2))

By Theorem 3.11, $A_1 \times A_2 \leq$ cl (γ-int ($A_1 \times A_2$)).Therefore $A_1 \times A_2$ is fuzzy γ-semi open set in the fuzzy product space $X \times Y$.

IV. Fuzzy γ – semi closed sets

In this section we introduce the concept of fuzzy γ-semi closed sets in a fuzzy topological space.

Definition 4.1: Let A be a fuzzy subset of a fuzzy topological space (X, τ). Then A is called fuzzy γ-semi closed set of X if there exist a fuzzy γ -closed set γ-c such that int (γ-c) $\leq A \leq \gamma$-c.

Theorem 4.2 : Let A and B be any two fuzzy subset of a fuzzy topological space (X, τ) and γ-int(A) $\leq$ B $\leq \gamma$-cl(A). If A is a fuzzy γ-semi closed set then so is B.

Proof:

Let A be a fuzzy subset of X and γ-int (A) $\leq B \leq \gamma$-cl(A). If A is a fuzzy γ-semi closed set, then by Definition 4.1, there exists a fuzzy γ-closed set γ-c such that int(γ-c) $\leq A \leq \gamma$-c. It follows that int (γ-c) $\leq \gamma$-int (A) $\leq A \leq \gamma$-cl (A)$\leq$ γ-c and hence int (γ-c) $\leq B \leq \gamma$-c. Thus B is fuzzy γ-semi closed.

Theorem 4.3: A fuzzy subset A of a fuzzy topological space (X, τ) is fuzzy γ-semi closed if and only if A $\geq$ int (γ -cl(A)).

Proof :

Let A $\geq$ int (γ-cl(A)). Then for γ-c = γ-cl(A), we have A $\leq \gamma$-cl(A). Therefore int (γ-cl(A)) $\leq A \leq \gamma$ -cl(A). Conversely let A be fuzzy γ-semi closed. Then by Definition 4.1, there exists a fuzzy γ-closed set γ-c such that int(γ-c) $\leq A \leq \gamma$-c. But γ-cl(A) $\leq \gamma$-c and int (γ-cl(A)) $\leq$ int (γ-c) and thus int (γ-cl(A)) $\leq$ int (γ-c) $\leq$ A. Hence A $\geq$ int (γ-cl(A)).

Proposition 4.4 : Let (X, τ) be a fuzzy topological space and A be a fuzzy subset of X. Then A is fuzzy γ-semi closed if and only if A^c is fuzzy γ-semi open.

Proof :

Let A be a fuzzy γ-semi closed subset of X. Then by Theorem 4.3, A $\geq$ int (γ-cl(A)). Taking complement on both sides, we get $A^c \leq (\text{int } (\gamma\text{-cl}(A)))^c$ = cl(γ-cl (A))^c . By using Properties 2.8(b), $A^c \leq$ cl(γ-int (A^c)). By Theorem 3.3, we have A^c is fuzzy γ-semi open.

Conversely let A^c is fuzzy γ-semi open. By Theorem 3.3, $A^c \leq$ cl (γ-int(A^c)).Taking complement on both sides we get, A $\geq$ (cl(γ-int(A^c))^c = int (γ-int(A^c))^c . By using Properties 2.8(a), A $\geq$ int (γ-cl(A)). By Theorem 4.3, we have A is fuzzy γ-semi closed.

Remark 4.5: It is obvious that every fuzzy γ-closed set is fuzzy γ-semi closed and every fuzzy closed set is fuzzy γ-semi closed but the separate converses may not be true as shown by the following example.

Example 4.6: Let X = {a, b, c} and τ = {0, 1,{$a_{.1}$, $b_{.7}$, $c_{.5}$}, {$a_{.2}$, $b_{.1}$, $c_{.2}$}, {$a_{.1}$, $b_{.1}$, $c_{.2}$},{$a_{.2}$, $b_{.7}$, $c_{.5}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c ={0, 1, {$a_{.9}$, $b_{.3}$, $c_{.5}$}, {$a_{.8}$, $b_{.9}$, $c_{.8}$}, {$a_{.9}$, $b_{.9}$, $c_{.8}$}, {$a_{.8}$, $b_{.3}$, $c_{.5}$}}. Let A = {$a_{.4}$, $b_{.5}$, $c_{.3}$} then cl(int(A))= {$a_{.8}$, $b_{.3}$, $c_{.5}$} and int(cl(A))= {$a_{.2}$, $b_{.7}$, $c_{.5}$}.Then int(cl(A))$\wedge$ cl(int(A)) = {$a_{.2}$, $b_{.3}$, $c_{.5}$}. By Definition 2.1(3), A is not fuzzy γ-closed. Now let γ-cl(A) = {$a_{.5}$, $b_{.5}$, $c_{.5}$}. Then A $\geq$ int (γ-cl(A)) = {$a_{.2}$, $b_{.1}$, $c_{.2}$}. Thus A is fuzzy γ-semi closed.

The next example shows that every fuzzy γ-semi closed need not be fuzzy closed.

Example 4.7: Let X ={a, b, c} and τ = {0, 1,{$a_{.1}$, $b_{.7}$, $c_{.5}$}, {$a_{.2}$, $b_{.1}$, $c_{.2}$}, {$a_{.1}$, $b_{.1}$, $c_{.2}$},{$a_{.2}$, $b_{.7}$, $c_{.5}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.9}$, $b_{.3}$, $c_{.5}$}, {$a_{.8}$, $b_{.9}$, $c_{.8}$}, {$a_{.9}$, $b_{.9}$, $c_{.8}$}, {$a_{.8}$, $b_{.3}$, $c_{.5}$}}. Let A = {$a_{.4}$, $b_{.5}$, $c_{.3}$}. Then γ-cl(A))= {$a_{.5}$, $b_{.5}$, $c_{.5}$}and int(γ -cl(A)) = {$a_{.2}$, $b_{.1}$, $c_{.2}$}. That shows A $\geq$ int (γ-cl(A)), it follows that A is fuzzy γ-semi closed. But A is not a fuzzy closed set.

It follows that every fuzzy semi closed set is fuzzy γ-semi closed but the converse may not be true as shown by the following example.

Example 4.8: Let X = {a, b} and τ = {0, 1, {$a_{.8}$, $b_{.7}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.2}$, $b_{.3}$}}. Let A = {$a_{.4}$, $b_{.5}$}. Then γ-cl(A) = {$a_{.5}$, $b_{.5}$} and int(γ-cl(A)) = {0}. It shows that A $\geq$ int (γ-cl (A)). By using Theorem 4.3, A is fuzzy γ-semi closed.Now int(cl(A)) = {1}. That shows A $\ngeq$ int (cl (A)). Hence A is not a fuzzy semi closed set.

Theorem 4.9: Let (X, τ) be a fuzzy topological space. Then the intersection of two fuzzy γ-semi closed sets is fuzzy γ-semi closed set in the fuzzy topological space (X, τ).

Proof:

Let A_1 and A_2 be two fuzzy γ-semi closed sets. By Theorem 4.3, we have $A_1 \geq$ int(γ-cl(A_1)) and $A_2 \geq$ int(γ-cl(A_2)). Therefore $A_1 \wedge A_2 \geq$ int(γ-cl (A_1)) $\wedge$ int ((γ-cl (A_2)) = int(γ-cl (A_1) $\wedge$ (γ-cl A_2)). By using Properties 2.10(6), $A_1 \wedge A_2 \geq$ int(γ-cl ($A_1 \wedge A_2$)). Hence $A_1 \wedge A_2$ is fuzzy γ-semi closed.

The union of two fuzzy γ-semi closed sets is need not be fuzzy γ-semi closed set in the fuzzy topological space X as shown by the following example.

Example 4.10: Let X = {a, b, c} and τ = {0, 1,{$a_{.5}$, $b_{.2}$, $c_{.7}$}, {$a_{.7}$, $b_{.8}$, $c_{.3}$},{$a_{.5}$, $b_{.2}$, $c_{.3}$}, {$a_{.7}$, $b_{.8}$, $c_{.7}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.5}$, $b_{.8}$,

$c_{.3}$}, {$a_{.3}$, $b_{.2}$, $c_{.7}$}, {$a_{.5}$, $b_{.8}$, $c_{.7}$},{$a_{.3}$, $b_{.2}$, $c_{.3}$}}. Let A = {$a_{.6}$, $b_{.7}$, $c_{.6}$} and γ-cl(A) = {$a_{.6}$, $b_{.8}$, $c_{.5}$}. Then we get int(γ-cl(A))= {$a_{.5}$, $b_{.2}$, $c_{.3}$}.Thus by Theorem 4.3, A is fuzzy γ-semi closed. Let B = {$a_{.5}$, $b_{.8}$, $c_{.5}$} and γ-cl(B) = {$a_{.5}$, $b_{.8}$, $c_{.6}$}. Then we get int(γ-cl(B))= {$a_{.5}$, $b_{.2}$, $c_{.3}$}. Thus by Theorem 4.3, B is fuzzy γ-semi closed. Now $A \vee B$ = {$a_{.6}$, $b_{.8}$, $c_{.6}$} and γ-cl ($A \vee B$) = {$a_{.6}$, $b_{.8}$, $c_{.8}$}. Then int(γ-cl ($A \vee B$)) = {$a_{.5}$, $b_{.2}$, $c_{.7}$}. Thus $A \vee B$ is not greater than or equal to int(γ-cl ($A \vee B$)). Therefore $A \vee B$ is not fuzzy γ-semi closed.

Theorem 4.11: Let (X, τ) be a fuzzy topological space and let $\{A_\alpha\}_{\alpha\in\Delta}$ be a collection of fuzzy γ-semi closed sets in a fuzzy topological space X. Then $\wedge_{\alpha\in\Delta} A_\alpha$ is fuzzy γ-semi closed for each $\alpha\in\Delta$.

Proof:

Let Δ be a collection of fuzzy γ-semi closed sets of a fuzzy topological space (X, τ). Then by Theorem 4.3, for each $\alpha\in\Delta$, $A_\alpha \geq$ int (γ-cl(A_α)). Then $\wedge_{\alpha\in\Delta} A_\alpha \geq \wedge_{\alpha\in\Delta}(\text{int}(\gamma - \text{cl}\,(A_\alpha))) \geq \text{int}\ \wedge_{\alpha\in\Delta}(\gamma - \text{cl}(A_\alpha))$. By using Properties 2.6, $\wedge_{\alpha\in\Delta} A_\alpha \geq$ int(γ-cl($\wedge_{\alpha\in\Delta}(A_\alpha)$)). Thus arbitrary intersection of fuzzy γ-semi closed set is fuzzy γ-semi closed.

Theorem 4.12: Let (X, τ) and (Y, σ) be any two fuzzy topological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of fuzzy γ-closed set A_1 of X and a fuzzy γ-closed set A_2 of Y is fuzzy γ-closed set of the fuzzy product space X $\times$ Y.

Proof:

Let A_1 be a fuzzy γ-closed subset of X and A_2 be a fuzzy γ-closed subset of Y. Then by Definition 2.1, we have $A_1 \geq$ int(cl (A_1))$\wedge$ cl(int(A_1)) and $A_2 \geq$ int(cl (A_2))$\wedge$ cl(int(A_2)). Now $A_1 \times A_2 \geq$ (int(cl (A_1))$\wedge$ cl(int(A_1))) $\times$(int(cl (A_2))$\wedge$ cl(int(A_2))). By using Lemma 2.5,

$A_1 \times A_2 \geq$ (int (cl ($A_1 \times A_2$)) $\wedge$ cl(int ($A_1 \times A_2$))). Therefore $A_1 \times A_2$ is fuzzy γ-closed in the fuzzy product space X $\times$ Y.

Theorem 4.13: Let (X, τ) and (Y, σ) be any two fuzzy topological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of fuzzy γ-semi closed set A_1 of X and a fuzzy γ-semi closed set A_2 of Y is fuzzy γ-semi closed set of the fuzzy product space X $\times$ Y.

Proof:

Let A_1 be a fuzzy γ-semi closed subset of X and A_2 be a fuzzy γ-semi closed subset of Y. Then by Theorem 4.3, we have $A_1 \geq$ int(γ-cl (A_1)) and $A_2 \geq$ int(γ-cl(A_2)). Now $A_1 \times A_2 \geq$ int(γ-cl (A_1)) $\times$ int (γ-cl(A_2)). By using Lemma 2.4(2), $A_1 \times A_2 \geq$ int(γ-cl (A_1) $\times$ γ-cl (A_2)). By using the Theorem 4.12, we get $A_1 \times A_2 \geq$ int(γ-cl(A1 $\times$ A2)).

Therefore $A_1 \times A_2$ is fuzzy γ-semi closed in the fuzzy product space X $\times$ Y.

V. Fuzzy γ-semi interior

In this section we introduce the concept of fuzzy γ-semi interior and their properties in a fuzzy topological space.

Definition 5.1: Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset A of X, the fuzzy γ-semi interior of A (briefly γ-sint (A)) is the union of all fuzzy γ-semi open sets of X contained in A. That is, γ-sint (A) = $\vee$ {B: B $\leq$ A, B is fuzzy γ-semi open in X}

Proposition 5.2 : Let (X, τ) be a fuzzy topological space. Then for any fuzzy subsets A and B of a fuzzy topological X we have

(i) γ-sint (A) $\leq$ A
(ii) A is fuzzy γ-semi open $\Leftrightarrow$ γ-sint(A) = A
(iii) γ-sint(γ-sint (A)) = γ-sint (A)
(iv) if A $\leq$ B then γ-sint (A) $\leq$ γ-sint (B)

Proof:

(i) follows from Definition 5.1.
(ii) Let A be fuzzy γ-semi open.
Then A $\leq$ γ-sint (A). By using (i) we get A = γ-sint (A).
Conversely assume that A = γ-sint (A). By using Definition 5.1, A is fuzzy γ-semi open. Thus (ii) is proved.
(iii) By using (ii) we get γ-sint(γ-sint (A)) = γ-sint (A) . This proves (iii).
(iv) Since A $\leq$ B, by using (i) γ-sint (A)$\leq$ A $\leq$ B. That is γ-sint (A)$\leq$ B.
By (iii), γ-sint (γ-sint (A))$\leq$ γ-sint(B). Thus γ-sint (A) $\leq$ γ-sint (B). This proves (iv).

Theorem 5.3: Let (X, τ) be a fuzzy topological space. Then for any fuzzy subset A and B of a fuzzy topological space, we have

(i) γ-sint ($A\wedge B$) = (γ-sint A) $\wedge$ (γ-sint B)

(ii) γ-sint $(A\vee B) \geq (\gamma\text{-sint } A) \vee (\gamma\text{-sint } B)$

Proof:

Since $A\wedge B \leq A$ and $A\wedge B \leq B$, by using Proposition 5.2(iv), we get γ-sint $(A\wedge B) \leq \gamma$-sint(A) and γ-sint $(A\wedge B) \leq \gamma$-sint (B). This implies that γ-sint $(A \wedge B) \leq (\gamma$-sint A) $\wedge$ (γ-sint B) --- (1).

By using Proposition5.2(i), we have γ-sint(A) $\leq$ A and γ-sint(B) $\leq$ B. This implies that γ-sint (A) $\wedge$ γ-sint(B) $\leq A \wedge B$. Now applying Proposition 5.2(iv),

we get γ-sint ((γ-sint (A) $\wedge$ γ-sint (B)) $\leq$ γ-sint $(A\wedge B)$.

By (1), γ-sint(γ-sint (A)) $\wedge$ γ-sint (γ-sint (B)) $\leq$ γ-sint $(A \wedge B)$. By Proposition 5.2(iii), γ-sint (A) $\wedge$ γ-sint (B) $\leq$ γ-sint $(A \wedge B)$ -----------------------(2). From (1) and (2), γ-sint $(A \wedge B)$ = γ-sint(A) $\wedge$ γ-sint (B). This implies(i).

Since $A \leq A\vee B$ and $B\leq A\vee B$, by using Proposition 5.2(iv), we have γ-sint (A) $\leq$ γ-sint $(A \vee B)$ and γ-sint (B) $\leq$ γ-sint $(A \vee B)$. This implies that γ-sint (A) $\vee$ γ-sint (B) $\leq$ γ-sint $(A \vee B)$. Hence (ii).

The following example shows that the equality need not be hold in Theorem 5.3(ii).

Example 5.4: Let X = {a, b, c} and τ = {0, 1, $\{a_{.5}, b_{.3}, c_{.7}\}$, $\{a_{.2}, b_{.4}, c_{.4}\}$, $\{a_{.2}, b_{.3}, c_{.4}\}$, $\{a_{.5}, b_{.4}, c_{.7}\}$}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, $\{a_{.5}, b_{.7}, c_{.3}\}$, $\{a_{.8}, b_{.6}, c_{.6}\}$, $\{a_{.8}, b_{.7}, c_{.6}\}$, $\{a_{.5}, b_{.6}, c_{.3}\}$}. Consider A = $\{a_{.4}, b_{.3}, c_{.4}\}$ and B = $\{a_{.3}, b_{.7}, c_{.4}\}$. Then γ-sint (A) = $\{a_{.3}, b_{.3}, c_{.4}\}$ and γ-sint (B) = $\{a_{.2}, b_{.4}, c_{.4}\}$. That implies γ-sint (A) $\vee$ γ-sint (B) = $\{a_{.3}, b_{.4}, c_{.4}\}$. Now $A\vee B$ = $\{a_{.4}, b_{.7}, c_{.4}\}$, it follows that γ-sint$(A\vee B)$ = $\{a_{.4}, b_{.5}, c_{.4}\}$. Then γ-sint $(A\vee B) \not\leq$ γ-sint (A) $\vee$ γ-sint (B). Thus γ-sint $(A\vee B) \neq$ γ-sint (A) $\vee$ γ-sint (B).

VI. Fuzzy γ-semi closure

In this section we introduce the concept of fuzzy γ-semi closure in a fuzzy topological space.

Definition 6.1: Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset A of X, the fuzzy γ-semi closure of A (briefly γ-scl (A)) is the intersection of all fuzzy γ-semi closed sets contained in A. That is, γ-scl (A) = $\wedge${ B: B $\geq$ A, B is fuzzy γ-semi closed}.

Proposition 6.2: Let (X, τ) be a fuzzy topological space. Then for any fuzzy subsets A of X, we have

i. (γ-sint (A))^c = γ-scl (A^c) and
ii. (γ-scl (A))^c = γ-sint (A^c)

Proof:

By using Definition 5.1, γ-sint (A) = $\vee$\{B: B$\leq$ A, B is fuzzy γ-semi open\}. Taking complement on both sides, we get $[\gamma\text{-sint }(A)]^c$ = (sup{B : B $\leq$ A, B is fuzzy γ-semi open})^c = inf{B^c : $B^c \geq A^c$, B^c is fuzzy γ-semi closed }. Replacing B^c by c, we get $[\gamma\text{-sint }(A)]^c$ = $\wedge$\{c: c $\geq A^c$, c is fuzzy γ-semi closed \}. By Definition 6.1, $[\gamma\text{-sint }(A)]^c$ = γ-scl(A^c). This proves(i).

By using(i), $[\gamma\text{-sint }(A^c)]^c$ = γ-scl $(A^c)^c$ = γ-scl (A). Taking complement on both sides, we set γ-sint (A^c)= $[\gamma\text{-scl }(A)]^c$. Hence proved (ii).

Proposition 6.3: Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset A and B of a fuzzy topological space X, we have

(i) A $\leq$ γ-scl (A).
(ii) A is fuzzy γ-semi closed $\Leftrightarrow$ γ-scl (A) = A.
(iii) γ-scl (γ-scl (A)) = γ-scl (A).
(iv) if A $\leq$ B then γ-scl (A) $\leq$ γ-scl (B).

Proof:

(i) The proof of (i) follows from the Definition 6.1.
(ii) Let A be fuzzy γ-semi closed subset in X. By using Proposition 4.4, A^c is fuzzy γ-semi open. By using Proposition 6.2(ii), γ-sint(A^c) = A^c $\Leftrightarrow$ $[\gamma\text{-scl }(A)]^c$ = A^c $\Leftrightarrow$ γ-scl (A) = A. Thus proved (ii).
(iii) By using (ii), γ-scl(γ-scl (A)) = γ-scl (A). This proves (iii).
(iv) Suppose A $\leq$ B. Then $B^c \leq A^c$. By using Proposition 5.2(iv),
γ-sint (B^c)$\leq$ γ-sint (A^c). Taking complement on both sides, we get
$[\gamma\text{-sint }(B^c)]^c \geq [\gamma\text{-sint }(A^c)]^c$. By proposition 6.2(ii),
γ-scl (B) $\geq$ γ-scl(A). This proves (iv).

Proposition 6.4: Let A be a fuzzy set in a fuzzy topological space X. Then int(A)$\leq$ sint(A) $\leq$ γ-int(A) $\leq$ γ-sint(A) $\leq$ A $\leq$ γ-scl(A) $\leq$ γ-cl(A) $\leq$ scl(A) $\leq$ cl(A).

Proof: It follows from the Definitions of corresponding operators.

Proposition 6.5: Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset A and B of a fuzzy topological space X, we have

(i) γ-scl $(A\vee B) = \gamma$-scl $(A) \vee \gamma$-scl (B) and

(ii) γ-scl $(A\wedge B) \leq \gamma$-scl $(A) \wedge \gamma$-scl (B).

Proof:

Since γ-scl $(A\vee B) = \gamma$-scl$[(A \vee B)^c]^c$, by using Proposition 6.2(i), we have γ-scl $(A\vee B) = [\gamma$-sint $(A\vee B)^c]^c = [\gamma$-sint $(A^c\wedge B^c)]^c$. Again using Proposition 5.3(i), we have γ-scl $(A\vee B) = [\gamma$-sint $(A^c)\wedge \gamma$-sint $(B^c)]^c = [\gamma$-sint $(A^c)]^c \vee[\gamma$-sint $(B^c))]^c$. By using Proposition 6.2(i), we have γ-scl $(A\vee B) = \gamma$-scl $(A^c)^c \vee \gamma$-scl $(B^c)^c = \gamma$-scl $(A) \vee \gamma$-scl (B). Thus proved (i).

Since $A \wedge B \leq A$ and $A \wedge B \leq B$, by using Proposition 6.3(iv), γ-scl $(A\wedge B) \leq \gamma$-scl(A) and γ-scl $(A\wedge B) \leq \gamma$-scl (B). This implies that γ-scl $(A\wedge B) \leq \gamma$-scl $(A) \wedge \gamma$-scl (B). This proves(ii).

The following example shows that γ-scl $(A\wedge B)$ need not be equal to γ-scl$(A) \wedge \gamma$-scl(B).

Example 6.6: Let $X = \{a, b, c\}$ and $\tau =\{0, 1, \{a_{.1}, b_{.7}, c_{.5}\}, \{a_{.2}, b_{.1}, c_{.2}\}, \{a_{.1}, b_{.1}, c_{.2}\}, \{a_{.2}, b_{.7}, c_{.5}\}\}$. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is $\tau^c = \{0, 1, \{a_{.9}, b_{.3}, c_{.5}\}, \{a_{.8}, b_{.9}, c_{.8}\}, \{a_{.9}, b_{.9}, c_{.8}\}, \{a_{.8}, b_{.3}, c_{.5}\}\}$. Consider $A = \{a_{.4}, b_{.5}, c_{.3}\}$ and $B =\{a_{.9}, b_{.5}, c_{.5}\}$. Then γ-scl$(A) = \{a_{.5}, b_{.5}, c_{.5}\}$ and γ-scl $(B) = \{a_{.9}, b_{.5}, c_{.2}\}$. Also γ-scl $(A) \wedge \gamma$-scl $(B) = \{a_{.5}, b_{.5}, c_{.2}\}$. Now $A \wedge B = \{a_{.4}, b_{.5}, c_{.2}\}$ and γ-scl $(A\wedge B) = \{a_{.4}, b_{.5}, c_{.3}\}$. Thus γ-scl $(A) \wedge \gamma$-scl $(B) \neq \gamma$-scl $(A\wedge B)$.

Theorem 6.7: Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset A and B of X we have,

(i) γ-scl $(A) \geq A \vee \gamma$-scl $(\gamma$-sint $(A))$.

(ii) γ-sint $(A) \leq A\wedge\gamma$-sint $(\gamma$-scl $(A))$.

(iii) int $(\gamma$-scl $(A)) \leq$ int $(cl (A))$.

(iv) int $(\gamma$-scl $(A)) \geq$ int $(\gamma$-scl$(\gamma$-sint $(A)))$.

Proof:

(i) By Proposition 6.3(i), $A\leq \gamma$-scl (A) ------------ (1).
Again using Proposition 5.2(i), γ-sint $(A) \leq A$.
Then γ-scl $(\gamma$-sint $(A) \leq \gamma$- scl (A) ------- (2).
By (1) & (2) we have, $A \vee \gamma$-scl $(\gamma$-sint $(A)) \leq \gamma$-scl (A). This proves (i).

(ii) By Proposition 5.2(i), γ-sint $(A)\leq A$ ----(1).
Again using proposition 6.3(i), $A\leq \gamma$-scl (A).
Then γ-sint $(A) \leq \gamma$-sint $(\gamma$-scl $(A))$ ---(2).
From (1) & (2), we have γ-sint $(A) \leq A\wedge\gamma$-sint $(\gamma$-scl $(A))$. This proves(ii).

(iii) By Proposition 6.4, γ-scl $(A) \leq$ cl (A).
we get int $(\gamma$-scl $(A)) \leq$ int $(cl(A))$.

(iv) By (i), γ-scl $(A) \geq A \vee \gamma$-scl $(\gamma$-sint$(A))$. Then we have
int $(\gamma$-scl$(A) \geq$ int $(A\vee \gamma$-scl $(\gamma$-sint $(A)))$. Since int $(A\vee B) \geq$ int $(A) \vee$ int (B),
int $(\gamma$-scl $(A) \geq$ int $(A) \vee$ int$(\gamma$-scl$(\gamma$-sint $(A))) \geq$ int $(\gamma$-scl $(\gamma$-sint $(A)))$.

The family of all fuzzy semi open (fuzzy semi closed, fuzzy strongly semi open, fuzzy strongly semi closed, fuzzy γ-semi open, fuzzy γ-semi closed, fuzzy γ-open, fuzzy γ-closed) sets of an fuzzy topological space(X, τ) will be denoted by $Fso(\tau)$ $(Fscl(\tau), Fsso(\tau), Fsscl(\tau), F\gamma so(\tau), F\gamma scl(\tau), F\gamma o(\tau), F\gamma cl(\tau))$.

Proposition 6.8: Let (X, τ) be a fuzzy topological space. Then

1) $Fsscl(\tau) \wedge Fscl(\tau) \leq F\gamma scl(\tau)$.

2) $Fsso(\tau) \wedge Fso(\tau) \leq F\gamma so(\tau)$.

3) $F\gamma o(\tau) \wedge Fso(\tau) \leq F\gamma so(\tau)$.

4) $F\gamma cl(\tau) \wedge Fscl(\tau)\leq F\gamma s(\tau)$.

Proof:

Let A be a fuzzy subset of $Fsscl(\tau) \wedge Fscl(\tau)$. Then $A \in Fsscl(\tau)$ and $A \in Fscl(\tau)$. By the Definition of fuzzy strongly semi closed, $A \geq cl(int(cl(A))) \geq int(cl(A)) \geq int(\gamma\text{-}cl(A))$. By the Definition of fuzzy semi closed, $A \geq int(cl(A)) \geq int(\gamma\text{-}cl(A))$. Therefore $A \geq int(\gamma\text{-}cl(A))$. That is A is fuzzy γ-semi closed. This proves (1).

Let $A\in Fsso(\tau) \wedge Fso(\tau)$. Then $A \in Fsso(\tau)$ and $A \in Fso(\tau)$. By the Definition of fuzzy strongly semi open, $A\leq int(cl(int(A))) \leq cl(int (A)) \leq cl(\gamma\text{-}int (A))$. Again using the Definition of fuzzy semi open, $A\leq cl(int(A)) \leq cl(\gamma\text{-}int(A))$. Therefore A is fuzzy γ-semi open. This proves (2).

Let $A\in F\gamma o(\tau) \wedge Fso(\tau)$. Then $A\in F\gamma o(\tau)$ and $A\in Fso(\tau)$. By the Definition of fuzzy γ-open, $A \leq cl(int (A) \vee int(cl (A)$. That is $A \leq cl(int A) \leq cl(\gamma\text{-}int (A))$. Again using the Definition of fuzzy semi open, $A \leq cl(int A)$. This implies that $A\leq cl(\gamma\text{-}int (A))$. Therefore A is fuzzy γ-semi open. Hence proved (3).

Let $A\in F\gamma cl(\tau) \wedge Fscl(\tau)$. Then $A\in F\gamma cl(\tau)$ and $A\in Fscl(\tau)$. By the Definition of fuzzy γ-closed, $A \geq int(cl (A)) \wedge cl(int(A))$ that is $A \geq int(cl(A) \geq int(\gamma\text{-}cl(A))$. Again by Theorem 4.3, $A \geq int (cl (A))$. This implies that $A \geq int(\gamma\text{-}cl(A))$. Therefore A is fuzzy γ-semi closed. Hence proved (4).

Definition 6.9: An fuzzy topological space (X, τ) is fuzzy γ-SO-extremely disconnected if and only if γ-scl(A) is a fuzzy γ-semi open set, for each fuzzy γ-semi open set A of (X, τ).

Theorem 6.10: Let (X, τ) be an fuzzy topological space. Then the following statements are equivalent:

(i) X is γ-SO-extremely disconnected.

(ii) γ-sint(A) is a fuzzy γ-semi closed set, for each fuzzy γ-semi closed set A of X.

(iii) γ-scl(γ-scl $(A))^c$ = (γ-scl $(A))^c$, for each fuzzy γ-semi open set A of X.

(iv) B = (γ-scl $(A))^c$ implies γ-scl(B)=(γ-scl $(A))^c$ for each pair of fuzzy γ- semi open sets A, B of X.

Proof:

(i) ⇒ (ii) Let A be a fuzzy γ-semi closed set of X. Then A^c is a fuzzy γ-semi open set. According to the assumption, γ-scl (A^c) is fuzzy γ-semi open set. So γ-int (A) is a fuzzy γ-semi closed set of X.

(ii) ⇒ (iii) Suppose that A is a fuzzy γ-semi open set of X. Then γ-scl(γ-scl $(A))^c$ = γ-scl (γ-sint($A)^c$). According to the assumption, γ-int (A^c) is a fuzzy γ-semi closed set. So γ-scl (γ-sint (A^c)) = γ-sint (A^c) = (γ-scl $(A))^c$.

(iii)⇒ (iv) Let A and B be a fuzzy γ-semi open set of X such that B =(γ-scl $(A))^c$. From the assumption we have , γ-scl B = γ-scl(γ-scl $(A))^c$ =(γ-scl $(A))^c$.

(iv) ⇒ (i) Let A be a fuzzy γ-semi open set of X. We put B =(γ-scl $(A))^c$. From the assumption, we obtain that γ-scl (B) = (γ-scl $(A))^c$,

so (γ-scl $(B))^c$ = γ-scl(A). Hence γ-sint (B^c) = γ-scl(A). Thus γ-scl(A) is fuzzy γ-semi open set of X.

Definition 6.11: A fuzzy set A of fuzzy topological space (X, τ) is said to fuzzy γ-t-set if int (A)=int(γ-cl(A)).

Theorem 6.12: Let(X, τ) be a fuzzy topological space. Then a fuzzy subset A is fuzzy γ-t-set if and only if A is fuzzy γ-semi closed.

Proof:

Let A be a fuzzy γ-t-set. Then by using Definition 6.10, int(A) = int(γ-cl(A)). Therefore int(γ-cl(A)) = Int(A) ≤ A. Hence A is fuzzy γ-semi closed. Conversely, A is fuzzy γ-semi closed. Then by using Definition 2.1, int(γ-cl(A)) ≤ int(A). Also A ≤ γ-cl(A). This implies that int A ≤ int(γ-cl(A)). Hence int (A)= int(γ-cl (A). Thus A is fuzzy γ-t-set.

Theorem 6.13: Let (X, τ) be an fuzzy topological space. If A is fuzzy γ-closed, then it is fuzzy γ-t-set.

Proof:

Let A be fuzzy γ-closed. Then by Proposition 2.10,A=γ-cl(A) and int(A)=int(γ-cl(A)). Therefore A is fuzzy γ-t-set.

Theorem 6.14: Let (X, τ) be an fuzzy topological space. Then the intersection of any two fuzzy γ-t-set is fuzzy γ-t-set.

Proof:

Let A and B be fuzzy γ-t-set. Then by Definition 6.10, int(A)=int(γ-cl(A)) and int(B)=int(γ-cl(B)).Therefore int(A)∧int(B)=int(γ-cl(A)) ∧ int(γ-cl(B)) = int(γ-cl(A) ∧γ-cl(B)). By Remark 2.6, int(A∧B) = int(γ-cl(A ∧ B)).

The following example shows that union of two fuzzy γ-t-set need not be fuzzy γ-t-set.

Example 6.15: Let X = {a, b} and τ = {0, 1, {$a._1$, $b._3$}, {$a._9$, $b._7$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a._9$, $b._7$}, {$a._1$, $b._3$}}. Consider A = {$a._4$, $b._7$} and B = {$a._8$, $b._6$}. Then int(A)= {$a._1$, $b._3$} and int(B)= {$a._1$, $b._3$}. It follows that int(γ-cl(A))= {$a._1$, $b._3$}and int(γ-cl(B))= {$a._1$, $b._3$}. Therefore by Definition 6.10, A and B are fuzzy γ-t-set. Now A∨B={$a._8$, $b._7$} and int (A∨B)= {$a._1$, $b._3$} but int(γ-cl(A∨B))= {$a._9$, $b._7$}. It shows that A∨B is not an fuzzy γ- t-set.

References

[1] M.Athar and B.Ahmad, Fuzzy Boundary and Fuzzy Semiboundary, *Advances in Fuzzy systems*, vol. 2008, 586893, 9 pages.

[2] K.K.Azad, On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity, *J.Math.Anal.Appl*. 82(1)(1981), 14-32.

[3] Biljana Krsteska , Fuzzy Strongly preopen sets and fuzzy strongly precontinuity, *Mat. Vesnik* 50(1998),111-123.

[4] C.L.Chang, Fuzzy Topological Spaces, J.Math. Anal, Appl.24(1968), 182-190.

[5] Luay A.Al.Swidi, Amed S.A.Oon, Fuzzy γ-open sets and fuzzy γ-closed sets, *Americal Journal of scientific research*, 27(2011), 62-67.

[6] T.Noiri and O.R.Sayed, Fuzzy γ-open sets and fuzzy γ-continuity in fuzzitying topology, *Sci. Math. Jpn*. 55(2002).

[7] A.K. Katsaras and D.B. Liu, Fuzzy vector spaces and fuzzy topological vector spaces , *.Math.Anal.Appl*., vol. 58,no 1, pp. 135-146, 1977.

[8] L.A.Zadch, *Fuzzy Sets*, Information and control, vol.8, Pp 338-353, 1965.