IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728,p-ISSN: 2319-765X, Volume 6, Issue 4 (May. - Jun. 2013), PP 13-17
www.iosrjournals.org

Some forms of N-closed Maps in supra Topological spaces

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Abstract: *In this paper, we introduce the concept of N-closed maps and we obtain the basic properties and their relationships with other forms of N-closed maps in supra topological spaces.*
Keywords: *supra N-closed map, almost supra N-closed map, strongly supra N-closed map.*

I. Introduction:

In 1983, A.S.Mashhour et al [4] introduced the supra topological spaces and studied, continuous functions and s^* continuous functions. T.Noiri and O.R.Syed[5] introduced supra b-open sets and b-continuity on topological spaces.

In this paper, we introduce the concept of supra N-closed maps and study its basic properties. Also we introduce the concept of almost supra N-closed maps and strongly supra N-closed maps and investigate their properties in supra topological spaces.

II. Preliminaries:

Definition 2.1[4]
A subfamily μ of X is said to be supra topology on X if
i) $X, \phi \in \mu$
ii)If $A_i \in \mu \ \forall i \in j$ then $\cup A_i \in \mu$. (X,μ) is called supra topological space.
The element of μ are called supra open sets in (X, μ) and the complement of supra open set is called supra closed sets and it is denoted by μ^c.

Definition 2.2[4]
The supra closure of a set A is denoted by $cl^{\mu}(A)$, and is defined as supra cl(A) = ∩{B : B is supra closed and A ⊆ B}.
The supra interior of a set A is denoted by $int^{\mu}(A)$, and is defined as supra int(A) = ∪{B : B is supra open and A ⊇ B}.

Definition 2.3[4]
Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ, if τ ⊆ μ.

Definition 2.4[3]
Let (X, μ) be a supra topological space. A set A of X is called supra semi- open set, if A ⊆ $cl^{\mu}(int^{\mu}(A))$. The complement of supra semi-open set is supra semi-closed set.

Definition 2.5[1]
Let (X, μ) be a supra topological space. A set A of X is called supra α -open set, if A ⊆ $int^{\mu}(cl^{\mu}(int^{\mu}(A)))$. The complement of supra α -open set is supra α-closed set.

Definition 2.6[5]
Let (X, μ) be a supra topological space. A set A of X is called supra Ω closed set, if $scl^{\mu}(A) \subseteq int^{\mu}(U)$,whenever A ⊆ U, U is supra open set. The complement of the supra Ω closed set is supra Ω open set.

Definition 2.7[5]
The supra Ω closure of a set A is denoted by $\Omega cl^{\mu}(A)$, and defined as $\Omega cl^{\mu}(A)$ =∩{B : B is supra Ω closed and A ⊆ B}.
The supra Ω interior of a set A is denoted by $\Omega int^{\mu}(A)$, and defined as $\Omega int^{\mu}(A)$ =∪{B : B is supra Ω open and A ⊇ B}.

Definition 2.8[6]
Let (X, μ) be a supra topological space . A set A of X is called supra regular open if A= $int^{\mu}(cl^{\mu}(A))$ and supra regular closed if A= $cl^{\mu}(int^{\mu}(A))$.

Definition 2.9[7]

Let (X, μ) be a supra topological space . A set A of X is called supra N-closed set if $\Omega cl^{\mu}(A) \subseteq U$, whenever $A \subseteq U$, U is supra α open set. The complement of supra N-closed set is supra N-open set.

Definition 2.10[7]

The supra N closure of a set A is denoted by $Ncl^{\mu}(A)$, and defined as $Ncl^{\mu}(A) = \cap\{B : B$ is supra N closed and $A \subseteq B\}$.

The supra N interior of a set A is denoted by $Nint^{\mu}(A)$, and defined as $Nint^{\mu}(A) = \cup\{B : B$ is supra N open and $A \supseteq B\}$.

Definition 2.11[7]

Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X, τ) → (Y, σ) is called supra N-continuous function if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra closed set V of (Y, σ).

Definition 2.12[7]

Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X, τ) → (Y, σ) is called supra N-irresolute if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra N-closed set V of (Y, σ).

Notations: Throughout this paper $O^{\mu}(\tau)$ represents supra open set of (X, τ) and $N^{\mu}O(\tau)$ represents supra N-open set of (X, τ).

III. Supra N-Closed Maps

Definition 3.1

A map f:(X, τ) → (Y, σ) is called supra N-closed map(resp. supra N-open) if for every supra closed(resp. supra open) F of X, f(F) is supra N-closed(resp. supra N-open) in Y.

Theorem 3.2

Every supra closed map is supra N-closed map.

Proof

Let f:(X, τ) → (Y, σ) be supra closed map. Let V be supra closed set in X, Since f is supra closed map then f(V) is supra closed set in Y. We know that every supra closed set is supra N-closed, then f(V) is supra N-closed in Y. Therefore f is supra N-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.3

Let X=Y={a, b, c} and τ = {X, φ, {a},{b, c}} , σ = {Y, φ, {a}}.
f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is supra N-closed map but not supra closed map, since V={b,c} is closed in X but f ({b,c}) = {a,c} is supra N-closed set but not supra closed in Y.

Theorem 3.4

A map f:(X, τ) → (Y, σ) is supra N-closed iff $f(cl^{\mu}(V))=Ncl^{\mu}(f(V))$

Proof

Suppose f is supra N-closed map. Let V be supra closed set in (X, τ). Since V is supra closed , $cl^{\mu}(V)=V$. f(V) is supra N-closed in (Y, σ). Since f is supra N-closed map, then $f(cl^{\mu}(V))=f(V)$. Since f(V) is supra N-closed, we have $Ncl^{\mu}(f(V))=f(V)$. Hence $f(cl^{\mu}(V))=Ncl^{\mu}(f(V))$

Conversly, suppose $f(cl^{\mu}(V))=Ncl^{\mu}(f(V))$. Let V be supra closed set in (X, τ), then $cl^{\mu}(V)=V$. since f is a mapping, f(V) is in (Y, σ) and we have $f(cl^{\mu}(V))=f(V)$. Since $f(cl^{\mu}(V))=Ncl^{\mu}(f(V))$, we have $f(V)= Ncl^{\mu}(f(V))$, implies f(V) is supra N-closed in (Y, σ). Therefore f is supra N-closed map.

Theorem 3.5

A map f:(X, τ) → (Y, σ) is supra N-open iff $f(int^{\mu}(V))=Nint^{\mu}(f(V))$

Proof

Suppose f is supra N-open map. Let V be supra open set in (X, τ). Since V is supra open, $int^{\mu}(V)=V$, f(V) is supra N-open in (Y, σ). Since f is supra N-open map, Therefore $f(int^{\mu}(V))=f(V)$. Since f(V) is supra N-open, we have $Nint^{\mu}(f(V))=f(V)$. Hence $f(int^{\mu}(V))= Nint^{\mu}(f(V))$

Conversly, suppose $f(int^{\mu}(V))=Nint^{\mu}(f(V))$. Let V be a supra open set in (X, τ), then $int^{\mu}(V)=V$. Since f is a mapping, f(V) is in (Y, σ) and we have $f(int^{\mu}(V))=f(V)$. Since $f(int^{\mu}(V))=Nint^{\mu}(f(V))$, we have $f(V)= Nint^{\mu}(f(V))$, implies f(V) is supra N-open in (Y, σ). Therefore f is supra N-open map.

Remark:3.6

If f:(X, τ) → (Y, σ) is supra N-closed map and g: (Y, σ) → (Z, υ) is supra N-closed map then its composite need not be supra N-closed map in general and this is shown by the following example.

Example 3.7

Let X=Y=Z={a, b, c} and τ = {X, φ, {a} {b, c}} , σ = {Y, φ, {a}}. υ={Z, φ {a},{b},{a,b},{b,c}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. and g: (Y, σ) → (Z, υ) be the function defined by g(a)=b,g(b)=c,g(c)=a. Here f and g is supra N-closed map, but its composition is not N-closed map, since gof{b, c} ={a,b} is not N-closed in (Z,υ).

Theorem:3.8

If f:(X, τ) → (Y, σ) is supra closed map and g: (Y, σ) → (Z, υ) is supra N-closed map then the composition gof is supra N-closed map.

Proof

Let V be supra closed set in X. Since f is a supra closed map, f(V) is supra closed set in Y. Since g is supra N-closed map, g(f(V)) is supra N-closed in Z. This implies gof is supra N-closed map.

IV. Almost supra N-closed map and strongly supra N-closed map .

Definition 4.1

A map f:(X, τ) → (Y, σ) is said to be almost supra N-closed map if for every supra regular closed set F of X, f(F) is supra N-closed in Y.

Definition 4.2

A map f:(X, τ) → (Y, σ) is said to be strongly supra N-closed map if for every supra N-closed set F of X, f(F) is supra N-closed in Y.

Theorem 4.3

Every strongly supra N-closed map is supra N-closed map.

Proof

Let V be supra closed set in X. Since every supra closed set is supra N-closed set, then V is supra N-closed in X. Since f is strongly supra N-closed map, f(V) is supra N-closed set in Y. Therefore f is supra N-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.4

Let X=Y={a, b, c} and τ = {X, φ, {a}} , σ = {Y, φ, {b},{a,b},{b,c}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is supra N-closed map but not strongly supra N-closed map, since V={a,b} is supra N-closed set in X, but f ({a,b}) = {b,c} is not a supra N-closed set in Y.

Theorem 4.5

Every supra N-closed map is almost supra N-closed map.

Proof

Let V be a supra regular closed set in X. We know that every supra regular closed set is supra closed set. Therefore V is supra closed set in X. Since f is supra N-closed map, f(V) is supra N-closed set in Y. Therefore f is almost supra N-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.6

Let X=Y={a, b, c} and τ = {X, φ, {a}, {b}, {a,b},{b, c}}, σ = {Y, φ, {a},{c},{a,c}}. f:(X, τ) → (Y, σ) be a function defined by f(a)=c, f(b)=b, f(c)=a. Here f is almost supra N- closed map but it is not supra N-closed map, since V={a, c} is supra closed set in X but f({a, c}) = {a, c} is not supra N-closed set in Y.

Theorem 4.7

Every strongly supra N-closed map is almost supra N-closed map.

Proof

Let V be supra regular closed set in X. We know that every supra regular closed set is supra closed set and every supra closed set is supra N-closed set. Therefore V is supra N-closed set in X. Since f is strongly supra N-closed map, f(V) is supra N-closed set in Y. Therefore f is almost supra N-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.8

Let X=Y={a, b, c} and τ = {X, φ, {a}, {c}, {a,c}} , σ = {Y, φ,{a}, {b},{a,b},{b,c}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is almost supra N- closed map but it is not strongly supra N-closed map, since V={a} is supra N-closed in X but f({a}) = { b} is not supra N-closed set in Y.

Theorem:4.9

If f:(X, τ) → (Y, σ) is strongly supra N-closed map and g: (Y, σ) → (Z, υ) is strongly supra N-closed map then its composition gof is strongly supra N-closed map.

Proof

Let V be supra N-closed set in X. Since f is strongly supra N-closed, then f(V) is supra N-closed in Y. Since g is strongly supra N-closed, then g(f(V)) is supra N-closed in Z. Therefore gof is strongly supra N-closed map.

Theorem 4.10

If f:(X, τ) → (Y, σ) is almost supra N-closed map and g: (Y, σ) → (Z, υ) is strongly supra N-closed map then its composite gof is almost supra N-closed map.

Proof

Let V be supra regular closed set in X. Since f is almost supra N-closed, then f(V) is supra N-closed set in Y. Since g is strongly supra N-closed,then g(f(V)) is supra N-closed in Z. Therefore gof is almost supra N-closed map.

Theorem 4.11

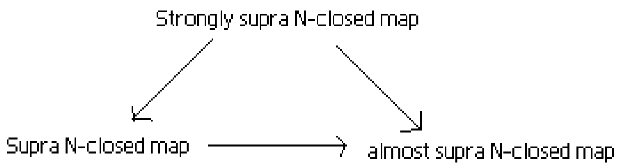
Let f:(X, τ) → (Y, σ) and g: (Y, σ) → (Z, υ) be two mappings such that their composition gof:(X, τ) → (Z, υ) be a supra N-closed mapping then the following statements are true:

(i) If f is supra continuous and surjective then g is supra N-closed map
(ii) If g is supra N-irresolute and injective then f is supra N-closed map.

Proof

i) Let V be a supra closed set of (Y, σ). Since f is supra continuous $f^{-1}(V)$ is supra closed set in (X, τ). Since gof is supra N-closed map, We have (gof)(f^{-1}(V)) is supra N-closed in (Z, υ). Therefore g(V) is supra N-closed in (Z, υ), since f is surjective. Hence g is supra N-closed map.

ii) Let V be supra closed set of (X, τ). Since gof is supra N-closed, we have gof(V) is supra N-closed in (Z, υ). Since g is injective and supra N-irresolute g^{-1}(gof(V) is supra N-closed in (Y, σ).Therefore f(V) is supra N-closed in (Y, σ). Hence f is supra N-closed map.



V. Applications

Definition:5.1

A supra topological space (X, τ) is $T_N^\mu - space$ if every supra N-closed set is supra closed in (X, τ).

Theorem:5.2

Let (X, τ) be a supra topological space then

(i) $O^\mu(\tau) \subseteq N^\mu O(\tau)$
(ii) A space (X, τ) is $T_N^\mu - space$ iff $O^\mu(\tau) = N^\mu O(\tau)$

Proof

(i) Let A be supra open set, then X-A is supra closed set. We know that every closed set is N-closed. Therefore X-A is N-closed, implies A is N-open. Hence $O^\mu(\tau) \subseteq N^\mu O(\tau)$

(ii) Let (X, τ) be $T_N^\mu - space$. Let A∈ $N^\mu O(\tau)$, then X-A is N-closed , by hypothesis X-A is closed and therefore A∈ $O^\mu(\tau)$. Hence we have $O^\mu(\tau) = N^\mu O(\tau)$. Conversely the proof is obvious

Theorem:5.3

If (X, τ) is $T_N^\mu - space$, then every singleton set of (X, τ) is either supra α-closed set or supra open set.

Proof

Suppose that for some x∈X, the set {x} is not supra α-closed set of (X, τ), then {x} is not supra N-closed set in (X, τ), Since we know that every α-closed set is supra N-closed set. So trivially $\{x\}^c$ is N-closed set. From the hypothesis $\{x\}^c$ is supra closed set in (X, τ). Therefore {x} is supra open set

Theorem:5.4

Let f:(X, τ) → (Y, σ) be supra N-closed map and g: (Y, σ) → (Z, υ) be supra N-closed map then their composition gof: (X, τ) → (Z, υ) is a supra N-closed map if (Y, σ) is $T_N^{\mu} - space$.

Proof

Let V be a supra closed set in X. Since f is supra N-closed map, then f(V) is supra N-closed set in Y. Since Y is $T_N^{\mu} - space$, f(V) is supra closed set in Y. Since g is supra N-closed map, we have g(f(V)) is supra N-closed in Z. Hence gof is a N-closed map.

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