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Non- Newtonian behavior of blood in very narrow vessels

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Abstarct: *The purpose of the study is to get some qualitative and quantitative insight into the problem of flow in vessels under consideration where the concentration of lubrication film of plasma is present between each red cells and tube wall. This film is potentially important in region to mass transfer and to hydraulic resistance, as well as to the relative resistance times of red cells and plasma in the vessels network.*

I. Introduction

Prothero and Burton [10,11]. It is expected that the cells are deformed elastically to enable them to pass through the tube which also suffer some small elastic distension.. Prothero and Buston [10] pointed out that the bolus of viscous plasma between two cells must perform, relative to their motion, a toroidal circulation, towards on the tube axis and backward near the walls. Prothero and Buston estimated from their model experimental, the pressure drop in the bolus of moving plasma between two red cells and deduced a contribution to over all vessels resistance less than that given by Poiseuille law, mainly because plasma with depleted RBC has a viscosity considerably lower than typical values measured for whole blood.

A large number of theoretical and experimental efforts have been made in the literature to explain the blood flow behavior when it flows through the vessels of circulatory system of the living things. To account for the numerous relevant and important contribution of Bayliss[2], Womeresly[18,19], Whitemore [20], Attingn [1], Fung[5] and many others, mathematical modeling of blood flow has been subject to constant changes and modification. Above listed investigators have been used single – phase homogeneous Newtonian viscous fluid, a classical approach that does not account for the presence of red cells in blood while flowing the circulatory system. Although, this approach provides satisfactory tools to describe certain aspects of blood flow in aorta and large arteries, it fails to give an adequate representation of flow field, especially in the vessels of small diameter Srivastva and Srivastva[12-15] and Vann and Fitz-Gerald [17]. Several researcher Carmand Kurland[3],Gupta et al.[6], Chaturmani and Mahajan[4], Jean and Peddison[7], Jung and Cowokers[8,9] have pointed out that blood being a suspension of corpuscles, behave like a non-Newtonian fluid at low shear rates. Thurston [16] has developed a mathematical model for the flow of closely fitting incompressible elastic sphere in a tube under zero drag condition.

II. Mathematical Analysis

We consider the flow of elastic incompressible sphere in a rigid tube of uniform radius. Single file flow of RBC surrounded by an annulus of plasma is considered. In the case of movable buoyant particle treated in the present study, the condition of zero drag on the particle must be satisfied in addition to the Reynolds equation. It can be used to eliminate leak-back (which is equal to the discharge of the fluid observed relative to a reference frame fixed to the particle) leaving only pressure drop as an unknown.

The single RBC of biconcave –disk shape is deformed during the flow passage in very narrow vessels, as shown in figure.2.1.

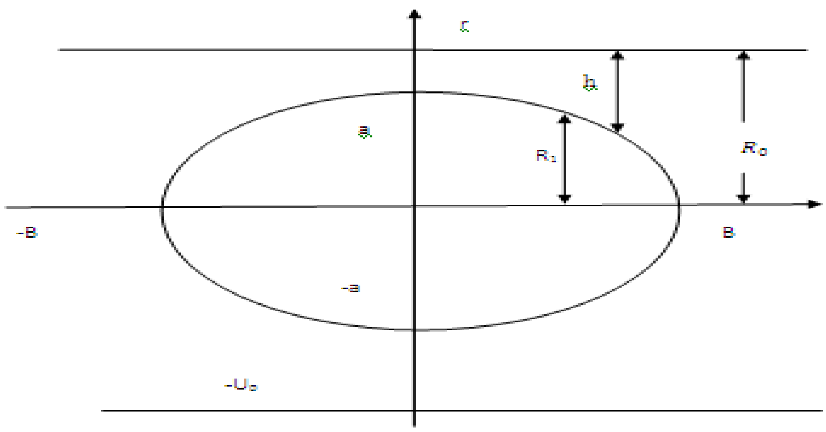

Figure.2.1

It is assumed that the inertial terms are negligible, the equation of motion in cylindrical polar co-ordinate about the axis of symmetry is

$$\frac{\partial p}{\partial z} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) \qquad (2.2.1)$$

The equation of continuity is

$$\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial z}(rv_z) = 0 \qquad (2.2.2)$$

where v_r and v_z are the radial and axial velocity respectively, μ the dynamic viscosity of fluid , p the pressure not varying with r.

Boundary conditions are

$$r = R_0 ; \quad v_z = - v_0 ; \quad v_r = 0 \qquad (2.2.3)$$

$$r = R_1 ; \quad v_z = 0 ; \quad v_r = 0 \qquad (2.2.4)$$

$$P(-b) - P(b) = \Delta P_0 ; \quad h = R_0 - R_1 \qquad (2.2.5)$$

For incompressible, continuity equation (2.2.2), we have

$$-\int \frac{\partial}{\partial r}(rv_r)dr = \int \frac{\partial}{\partial z}(rv_z)dr$$

Or $$\frac{\partial}{\partial z}\int_{R_1}^{R_0} rv_z dr = -\int_{R_1}^{R_0}\frac{\partial}{\partial r}(rv_r)dr = 0$$

Further, $$\int_{R_1}^{R_0} rv_z dr = C = -Q_0 \qquad (2.2.6)$$

where Q_0 is the leak- back , given by

$$2\pi R_0 Q_0 = \pi R_0^2 U_0 - \pi R_0^2 V_0 \qquad (2.2.7)$$

V_0 is the average velocity of the fluid in lubrication zone.

From equation (2.2.7), we have

$$Q_0 = \frac{R_0}{2}(U_0 - V_0) \qquad (2.2.8)$$

Integrating equation (2.2.1), we get

$$v_z = \frac{1}{4\mu}\frac{dp}{dz}r^2 + A\log r + B \qquad (2.2.9)$$

where A and B are constant to be determine, by boundary condition (2.2.3) and (2.2.4) as

$$A = \frac{-\left[U_0 + \frac{1}{4\mu}\frac{dp}{dz}(R_0^2 - R_1^2)\right]}{\log\frac{R_0}{R_1}}$$

and $$B = \frac{\frac{1}{4\mu}\frac{dp}{dz}R_1^2 + \left[U_0 + \frac{1}{4\mu}\frac{dp}{dz}(R_0^2 - R_1^2)\right]\log R_1}{\log\frac{R_0}{R_1}}$$

Substituting the value of A and B in equation (2.2.9), we get

$$v_z = \frac{1}{4\mu}\frac{dp}{dz}\left[r^2 - R_1^2 + \frac{(R_0^2 - R_1^2)}{\log\frac{R_1}{R_0}}\log\frac{r}{R_1} + U_0\frac{\log\frac{r}{R_1}}{\log\frac{R_1}{R_0}}\right] \qquad (2.2.10)$$

Put $R_0 = R_1 + h$ in equation (2.2.10) becomes

$$v_z = \frac{1}{4\mu}\frac{dp}{dz}\left[r^2 - R_1^2 + \frac{(2R_1 h + h^2)}{\log\left(1+\frac{h}{R_1}\right)}\log\frac{r}{R_1} + U_0\frac{\log\frac{r}{R_1}}{\log\left(1+\frac{h}{R_1}\right)}\right] \qquad (2.2.11)$$

Integrating equation (2.2.11) and using (2.2.6), we have

$$\frac{dp}{dz} = \frac{16\mu\left[R_0 Q_0 - U_0\left(\frac{R_0^2}{2} + \frac{(R_0^2 - R_1^2)}{4\log\frac{R_1}{R_0}}\right)\right]}{(R_0^2 - R_1^2)\left[\left(R_0^2 + R_1^2 + \frac{(R_0^2 - R_1^2)}{\log\frac{R_1}{R_0}}\right)\right]} \qquad (2.2.12)$$

Under zero – drag condition, we have

Pressure force acting on the particle + viscous stresses experienced by the particle = 0.

i.e. $$\pi\int_{-b}^{b} R_1^2\frac{dp}{dz}dz - 2\pi\mu\int_{-b}^{b} R_1\left(\frac{\partial v_z}{\partial r}\right)_{r=R_1} dz = 0 \qquad (2.2.13)$$

If Ω is fluid volume, then from equation (2.2.1), we have

$$\int_\Omega \frac{dp}{dz}.d\Omega = \mu\int_\Omega \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)d\Omega \qquad (2.2.14)$$

$$\frac{1}{\mu}\left\{\pi R_0^2[P(b) - P(-b)] - \pi\int_{-b}^{b} R_1^2\frac{dp}{dz}dz\right\}$$

$$= 2\pi R_0\int_{-b}^{b}\left(\frac{\partial v_z}{\partial r}\right)_{r=R_0} dz - 2\pi\int_{-b}^{b} R_1\left(\frac{\partial v_z}{\partial r}\right)_{r=R_1} dz \qquad (2.2.15)$$

From equation (2.2.13) and equation (2.2.15) , we have

$$\pi R_0^2\,\Delta P_0 = -2\pi\mu R_0\int_{-b}^{b}\left(\frac{\partial v_z}{\partial r}\right)_{r=R_0} dz \qquad (2.2.16)$$

Using the quantity $\mu U/a$, in pressure and stress terms, the non dimensional quantities becomes,

$b' = \frac{pa}{\mu U}$, $\tau' = \frac{\tau a}{\mu U}$, $U_0{}' = \frac{U_0}{U}$, $v_z{}' = \frac{v_z}{U}$, $z' = \frac{z}{R_0}$, $r' = \frac{r}{a}$,

$R_1{}' = \frac{R_1}{R_0}$, H= $\frac{h}{R_0}$, $\alpha = \frac{a}{b}$, $\beta = \frac{a}{R_0}$, $C_0 = \frac{2Q_0}{UR_0}$, (2.2.17)

Velocity field v_z and pressure gradient $\frac{dp}{dz}$ given by equation (2.2.11) and (2.2.12) take the form,

$$v_z'(r',z') = \frac{1}{4\beta}\frac{dp'}{dz'}\left[\beta^2 r'^2 - R_1'^2 + \frac{1-R_1'^2}{\log R'_1}\log\frac{\beta r'}{R'_1}\right] + U'_0\frac{\log\frac{\beta r'}{R'_1}}{\log R'_1} \qquad (2.2.18)$$

$$\frac{dp'}{dz} = 8\beta\frac{\left[C_0 - U_0\left(1+\frac{1-R_1^2}{2\log R'_1}\right)\right]}{(1-R_1^2)\left[\left(1+R_1^2+\frac{(1-R_1^2)}{\log R_1}\right)\right]} \qquad (2.2.19)$$

For the shake of convenience, we omit '`' in proceeding expression.

Equation (2.2.18) gives

$$\frac{\partial V_z}{\partial r} = \frac{1}{4\beta}\frac{dp}{dz}\left[2\beta^2 r + \frac{1-R_1^2}{\log R_1}\right] + \frac{U_0}{2\log R_1} \qquad (2.2.20)$$

With the help of equation (2.2.20) and (2.2.16) , we obtain

$$\Delta P_0 = 4\beta\int_{-\beta/\alpha}^{\beta/\alpha}\left[\frac{\left\{2+\frac{1-R_1^2}{\log R_1}\right\}\left\{C_0 - U_0\left(1+\frac{1-R_1^2}{2\log R_1}\right)\right\}}{(1-R_1^2)\left\{1+R_1^2+\frac{(1-R_1^2)}{\log R_1}\right\}} + \frac{U_0}{2\log R_1}\right]dz \qquad (2.2.21)$$

As $\Delta P_0 = P\left(-\frac{\beta}{\alpha}\right) - P\left(\frac{\beta}{\alpha}\right)$ (2.2.22)

Then

$$\Delta P_0 = 8\beta\int_{-\beta/\alpha}^{\beta/\alpha}\frac{\left\{C_0 - U_0\left(1+\frac{1-R_1^2}{2\log R_1}\right)\right\}}{(1-R_1^2)\left\{1+R_1^2+\frac{(1-R_1^2)}{\log R_1}\right\}}dz \qquad (2.2.23)$$

If we put

$$D_{11} = 4\beta\int_{-\beta/\alpha}^{\beta/\alpha}\left[\frac{\left\{2+\frac{1-R_1^2}{\log R_1}\right\}}{(1-R_1^2)\left\{1+R_1^2+\frac{(1-R_1^2)}{\log R_1}\right\}}\right]dz \qquad (2.2.24)$$

$$D_{12} = 4\beta\int_{-\beta/\alpha}^{\beta/\alpha}\left[\frac{\left\{2+\frac{1-R_1^2}{\log R_1}\right\}\left\{1+\frac{(1-R_1^2)}{2\log R_1}\right\}}{(1-R_1^2)\left\{1+R_1^2+\frac{(1-R_1^2)}{\log R_1}\right\}} - \frac{1}{2\log R_1}\right]dz \qquad (2.2.25)$$

$$D_{21} = 8\beta\int_{-\beta/\alpha}^{\beta/\alpha}\frac{dz}{(1-R_1^2)\left\{1+R_1^2+\frac{(1-R_1^2)}{\log R_1}\right\}} \qquad (2.2.26)$$

$$D_{21} = 8\beta\int_{-\beta/\alpha}^{\beta/\alpha}\frac{\left\{1+\frac{(1-R_1^2)}{2\log R_1}\right\}}{(1-R_1^2)\left\{1+R_1^2+\frac{(1-R_1^2)}{\log R_1}\right\}}dz \qquad (2.2.27)$$

Then equation (2.2.21) takes the form

$$D_{11}C_0 + \Delta P_0 = D_{12}U_0 \qquad (2.2.28)$$

$$D_{21}C_0 + \Delta P_0 = D_{22}U_0 \qquad (2.2.29)$$

For U_0 = 1, above equation gives,

$$C_0 = \frac{D_{12}-D_{22}}{D_{11}-D_{21}}\ , \qquad \Delta P_0 = D_{12} - D_{11}C_0 \qquad (2.2.30)$$

$$\frac{U_0}{V_0} = (1-C_0)^{-1}\ ;$$

Effective viscosity

$$\eta = \frac{\alpha U_0 \Delta P_0}{16 V_0} \qquad (2.2.31)$$

III. Result and Discussion

We have calculated the value of $\frac{U_0}{V_0}$ and η and compared the calculated value from the results obtained by other authors. The results are given in the form of tables.

Variation of velocity field v_z can be obtained from equation (2.2.18). For different values of α (= 1.5, 1.0, 0.5) and β (= 0.90, 0.95,0.99), the variation of v_z with respect to gap thickness H have been shown in the table s 2.1 and 2.2.

Table – 2.1

S. No.	Shape of the particle	β	α	Value of η obtained by other authors		Value of η obtained in the present analysis
				[17]	[20]	
1	Sphere	0.70	1.0	1.196	1.196	0.853
2	Sphere	0.80	1.0	1.455	1.455	1.177
3	Sphere	0.90	1.0	2.201	2.200	1.971
4	Sphere	0.93	1.0	2.813	2.611	2.540
5	Sphere	0.95	1.0	3.485	3.259	3.207
6	Sphere	0.98	1.0	6.207	5.870	5.853
7	Sphere	0.99	1.0	9.274	9.013	8.964
8	Ellipsoid	0.90	4.0	1.503	1.268	1.971
9	Ellipsoid	0.90	0.5	1.795	1.714	1.971

Table – 2.3.2

S. No.	Shape of the particle	β	α	Value of $\frac{U_0}{V_0}$ obtained by other authors		Value of $\frac{U_0}{V_0}$ obtained in the present analysis
				[17]	[20]	
1	Sphere	0.70	1.0	1.416	1.414	1.403
2	Sphere	0.80	1.0	1.272	1.274	1.268
3	Sphere	0.90	1.0	1.136	1.132	1.133
4	Sphere	0.93	1.0	1.092	1.092	1.092
5	Sphere	0.95	1.0	1.068	1.066	1.066
6	Sphere	0.98	1.0	1.029	1.026	1.027
7	Sphere	0.99	1.0	1.019	1.103	1.013
8	Ellipsoid	0.90	4.0	1.138	1.132	1.133
9	Ellipsoid	0.90	0.5	1.136	1.132	1.133

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