IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728,p-ISSN: 2319-765X, Volume 6, Issue 3 (May. - Jun. 2013), PP 25-29
www.iosrjournals.org

Notions via β*-open sets in topological spaces

J.Antony Rex Rodgio , Jessie Theodore And Hanaselvi Jansi.J
Department of mathematics,V.O.Chidambaram College,Tuticorin-62800Tamil Nadu,INDIA
Ph.D. Scholar ,Department of mathematics,Sarah Tucker college,Tirunelveli- 627007,Tamil Nadu,INDIA .
Ph.D. Scholar ,Department of mathematics,Sarah Tucker college,Tirunelveli- 627007,Tamil Nadu,INDIA .

Abstract: *In this paper, first we define β*-open sets and β*-interior in topological spaces.J.Antony Rex Rodrigo[3] has studied the topological properties of η̂* -derived, η̂* -border, η̂* -frontier and η̂* exterior of a set using the concept of η̂* -open following M.Caldas,S.Jafari and T.Noiri[5]. By the same technique the concept of β*-derived, β*-border, β*-frontier and β*exterior of a set using the concept of β*-open sets are introduced.Some interesting results that shows the relationships between these concepts are brought about.*
***Key words**: η̂* -border, η̂* -frontier and η̂* exterior, β*-derived, β*-border, β*-frontier and β*exterior*

I. Introduction:

For the first time the concept of generalized closed sets was considered by Levine in 1970 [7]. After the works of Levine on semi-open sets, various mathematicians turned their attention to the generalizations of topology by considering semi open sets instead of open sets. In 2002, M. Sheik John [8] introduced a class of sets namely ω-closed set which is properly placed between the class of semi closed sets and the class of generalized closed sets. The complement of an ω-closed set is called an ω-open set.The concept of semi pre open sets was defined by Andrijevic[2] in 1986 and are also known under the name β sets.

We have already introduced a class of generalized closed sets called β*-closed sets using semipreopen sets and ω-open sets. The complement of a β*-closed set is called β*-open set.In this paper the concept of of β*-kernel, β*-derived, β*-border, β*-frontier and β*exterior of a set using the concept of β*-open sets are introduced.

II. Preliminaries:

Throughout the paper (X, τ), (Y, σ) and (Z, η) or simply X, Y and Z denote topological spaces on which no separation axioms are assumed unless otherwise mentioned explicitly.
We recall some of the definitions and results which are used in the sequel.

Definition 2.1

A subset A of a topological space (X, τ) is called

(i) A semi-open set [7] if A⊂cl(int(A)) and a semi-closed set if int(cl(A))⊂A,
(ii) A semipre open set [6] (= β-open set [1]) if A⊂cl(int(cl(A))) and a semi-pre closed set(=β closed) if int(cl(int(A)))⊂A
(iii) ω-open [8] if cl(A)⊂ U whenever A⊂ U and U is semi open.
(iv) A β*-closed set [4] if spcl(A) ⊂int(U) whenever A⊂U and U is ω-open

Theorem 2.2:[4] Every closed(resp.open) set is β*- closed(resp.β*open).

3.1. β*-Open sets

Definition 3.1.1: A subset A in X is called β*-open in X if A^c is β*-closed in X. We denote the family of all β*-open sets in X by β*O(τ).

Definition 3.1.2: For every set E⊂ X, we define the β*-closure of E to be the intersection of all β*-closed sets containing E. In symbols, β*cl(E) = ∩ {A:E⊂ A, A∈ β*c(τ)}.

Lemma 3.1.3: For any E⊂ X, E⊂ β*cl(E)⊂ cl(E).

Proof: Follows from Theorem 2.2.

Proposition 3.1.4: Let A be a subset of a topological space X.For any x∈ X, x∈ β*cl(A) if and only if U∩A≠∅ for every β*-open set U containing x.

Proof: Necessity: Suppose that x∈ β*cl(A). Let U be a β*-open set containing x such that A∩ U = ∅ and so A⊂U^c. But U^c is a β* closed set and hence β*cl(A) ⊆U^c. Since x∉U^c we obtain x∉ β*cl(A) which is contrary to the hypothesis.

Sufficiency:

Suppose that every β*-open set of X containing x intersects A. If x∉ β*cl(A), then there exist a β* closed set F of X such that A⊂ F and x∉ F. Therefore x∈ F^c and F^c is and β*-open set containing x. But F^c∩A= ∅. This is contrary to the hypothesis.

Definition 3.1.5: For any A⊂ X, β*int(A) is defined as the union of all β*-open set contained in A. That is β*int(A) =∪ {U:U⊂Aand U∈ β*O(τ)}.
Proposition 3.1.6: For any A⊂ X, int(A)⊂β*int(A).
Proof: Follows from Theorem 2..2.
Proposition 3.1.7: For any two subsets A_1 and A_2 of X.
(i) If A_1⊂ A_2, then β*int(A_i)⊂ β*int(A_2).
(ii) β*int(A_1∪ A_2)⊃β*int(A_1)∪ β*int(A_2).
Proposition 3.1.8: If A is β*-open then A=β*int(A).
Remark 3.1.9: Converse of Proposition 3.1.8 is not true. It can be seen by the following example.
Example 3.1.10: Let X={a, b, c} and τ = {∅, {a}, X} then for the set A={b, c}, β*int(A) = A but {b, c} is not β* closed.
Proposition 3.1.11: Let A be a subset of a space X. Then the following are true
(i) (β*int(A))^c = β*cl(A^c)
(ii) (β*int(A)) = (β*cl(A^c))^c
(iii) β*cl(A)= (β*int(A^c))^c
Proof:
(i) Let x∈ (β*int(A))^c. Then x∉ β*int(A). That is every β* open set U containing x is such that U⊄A. Thus every β*-open set U containing x is such that U∩A^c≠∅. By proposition 3.1.4, x∈ β*cl(A^c) and therefore (β*int(A))^c⊂ β*cl(A^c). Conversely, let x∈ β*cl(A^c). Then by proposition 3.1.4, every β* open set U containing x is such that U∩A^c ≠ ∅. By definition 3.1.5, x∉ β*int(A). Hence x∈ (β*int(A))^c and so β*cl(A^c)⊂ (β*int(A))^c. Hence (β*int(A))^c = β*cl(A^c).
(ii) Follows by taking complements in (i).
(iii) Follows by replacing A by A^c in (i).
Proposition 3.1.12: For a subset A of a topological space X, the following conditions are equivalent.
(i) β*O(τ) is closed under any union.
(ii) A is β* closed if and only if β*cl(A) = A.
(iii) A is β* open if and only if β*int(A) = A.
Proof: (i) ⇒ (ii): Let A be a β* closed set. Then by the definition of β*-closure we get β*cl(A) = A.
Conversely, assume β*cl(A) = A. For each x∈ A^c, x∉ β*cl(A), by proposition 3.1.4, there exists a β* open set $G_x$containing x such that G_x∩A=∅ and hence x∈G_x⊂ A^c. Therefore we obtain $A^c = \bigcup_{x\in A^c} G_x$. By (i) A^c is β*-open and hence A is β* closed.
(ii)⇒ (iii): Follows by (ii) and proposition 3.1.11.
(iii)⇒ (i): Let {U_α /α ∈∧ } be a family of β*-open sets of X. Put $U = \bigcup_\alpha U_\alpha$. For each x∈ U, there exists α(x)∈∨ such that x∈ $U_{\alpha(x)}$⊂ U. Since $U_{\alpha(x)}$ is β*-open, x∈ β*int(U) and so U=β*int(U). By (iii), U is β*-open. Thus β*O(τ) is closed under any union.
Proposition 3.1.13: In a topological space X, assume that β*O(τ) is closed under any union. Then β*cl(A) is a β*closed set for every subset A of X.
Proof: Since β*cl(A) = β*cl(β*cl(A)) and by proposition 3.1.12, we get β*cl(A) is a β*closed set.

3.2. β-*Kernel

Definition 3.2.1: For any A⊂ X, β*ker(A) is defined as the intersection of all β*-open sets containing A. In notation, β*ker(A) = ∩{U/A⊂ U, U∈ β*O(τ)}.
Example 3.2.2: Let X= {a, b, c}, τ = {∅, {a}, {a, c}, X}. Here β*O(τ) = P(X) –{{b}, {b, c}}. Let A= {b, c}then kerA=X and B={a}, then kerB={a}.
Definition 3.2.3: A subset A of a topological space X is a U-set if A=β*ker(A).
Example 3.2.4: Let X={a, b, c} and τ ={∅, {a}, {a, c}, X}. Here {a}, {c}, {a, b}, {a, c} are U-sets. The set {b, c} is not a U-set.
Lemma 3.2.5: For subsets A, B and A_α(α∈∧) of a topological space X, the following hold,
(i) A⊂ β*ker(A).
(ii) If A⊂ B, then β*ker(A)⊂ β*ker(B).
(iii) β*ker(β*ker(A)) = β*ker(A).
(iv) If A is β*-open then A=β*ker(A).
(v) β*ker(∪ {A_α/α ∈∧ })⊂∪ {β*ker(A_α)/α ∈∧ }
(vi) β *ker(∩ {A_α /α ∈∧ })⊂ ∩ {β*ker(A_α)/α ∈∧ }.
Proof:
(i) Clearly follows from Definition 3.2.1.
(ii) Suppose x∉ β*ker(B), then there exists a subset U∈ β*O(τ) such that U⊃B with x∉ U. since A⊂ B, x∉ β*ker(A). Thus β*ker(A)⊂ β*ker(B).

(iii) Follows from (i) and Definition 3.2.1.
(iv) By definition 3.2.1 and A∈ β*O(τ), we have β*ker(A)⊂ A. By (i) we get A = β*ker(A).
(v) For each α ∈∧ , β*ker(A_α)⊂ β*ker($\bigcup_{\alpha\in\wedge} A_\alpha$). Therefore we $\bigcup_{\alpha\in\wedge}$ β * ker(A_α)⊂β*ker($\bigcup_{\alpha\in\vee} A_\alpha$).
(vi) Suppose that x∉∩{β*ker(A_α)/α ∈} then there exists an α_0∈∧ , such that x∉β*ker($A\alpha_0$) and there exists a β*-open set U such that x∉ U and $A\alpha_0$⊂ U. We have $\bigcap_{\alpha\in\wedge} A_\alpha$⊂$A\alpha_0$⊂ U and x∉ U. Therefore x∉ β*ker{∩ A_α /α ∈∧ }. Hence ∩{β*ker(A_α)/α ∈∧ }⊃β*ker(∩ {A_α/α ∈∧ }.

Remark 3.2.6: In (v) and (vi) of Lemma 3.2.5, the equality does not necessarily hold as shown by the following example.

Example 3.2.7: Let X={a, b, c, d} and τ = {∅, {a}, X}. Let A = {b} and B = {c, d}. Hereβ*kerA = {b} and β*ker(B)={c, d}. β*ker(A)∪ β*kerB = {b}∪ {b}∪ {c, d} = {b, c, d}. β*ker(A∪ B) = β*ker({b, c, d}) = X.

Let X= {a, b, c} and τ= {∅, {a}, X}. Let P= {a, b} and Q= {b, c}. Here β*ker(P∩Q) = β*ker({b}) = {b}. But β*ker(P)∩ β*ker(Q) = {a, b}∩ X = {a, b}.

Remark 3.2.8: From (iii) of Lemma 3.2.5 it is clear that β*ker(A) is a U-set and every open set is a U-set.

Lemma 3.2.9: Let A_α(α ∈Λ) be a subset of a topological space X. If A_α is a U-set then ($\bigcap_{\alpha\in\wedge} A_\alpha$) is a U-set.

Proof: β*ker($\bigcap_{\alpha\in\wedge} A_\alpha$) ⊂$\bigcap_{\alpha\in\wedge}$ β*ker(A_α), by lemma 3.2.5. Since A_α is a U-set, we getβ*ker($\bigcap_{\alpha\in\wedge} A_\alpha$)⊂($\bigcap_{\alpha\in\wedge} A_\alpha$). Again by (i) of lemma 2.4.28, ($\bigcap_{\alpha\in\wedge} A_\alpha$)⊂β*ker($\bigcap_{\alpha\in\wedge} A_\alpha$). Thus β*ker($\bigcap_{\alpha\in\wedge} A_\alpha$)= ($\bigcap_{\alpha\in\wedge} A_\alpha$) which implies ($\bigcap_{\alpha\in\wedge} A_\alpha$) is U-set.

Definition 3.2.10: A subset A of a topological space X is said to be U-closed if A= L∩ F where L is an U-set and F is a closed set of X.

Remark 3.2.11: It is clear that every U-set and closed sets are U-closed.

Theorem 3.2.12: For a subset A of a topological space X, the following conditions are equivalent.
(i) A is U-closed
(ii) A = L∩cl(A) where L is a U-set.
(iii) A = β*ker(A)∩ cl(A).

Proof:
(i)⇒(ii): Let A = L∩F where L is a U-set and F is a closed set. Since A⊂ F, we have cl(A)⊂ F and A⊂ L∩ cl(A)⊂ L∩ F = A. Therefore, we obtain L∩cl(A) = A.
(ii)⇒ (iii): Let A = L∩cl(A) where L is a U-set. Since A⊂ L, we have β*ker(A)⊂β*ker(L) = L. Therefore β*ker(A)∩ cl(A)⊂ L∩ cl(A) = A. Hence A = β*ker(A)∩ cl(A).
(iii)⇒ (i): Since β*ker(A) is a U-set, the proof follows.

3.3. β*-Derived set

Definition 3.3.1: Let A be a subset of a space X. A point x∈ X is said to be a β* limit point of A, if for each β*-open set U containing x, U∩ (A-{x}) ≠∅. The set of all β* limit point of A is called a β*-derived set of A and is denoted by $D_{\beta*}$(A).

Theorem 3.3.2.: For subsets A, B of a space X, the following statements hold
(i) $D_{\beta*}$(A) ⊂D(A) where D(A) is the derived set of A.
(ii) If A⊂ B, then $D_{\beta*}$(A)⊂ $D_{\beta*}$(B).
(iii) $D_{\beta*}$(A)∪ $D_{\beta*}$(B)⊂ $D_{\beta*}$(A∪B) and $D_{\beta*}$(A∩B)⊂ $D_{\beta*}$(A)∩ $D_{\beta*}$(B).
(iv) $D_{\beta*}$($D_{\beta*}$(A))-A⊂ $D_{\beta*}$(A).
(v) $D_{\beta*}$(A∪$D_{\beta*}$(A))⊂ A∪$D_{\beta*}$(A).

Proof:
(i) Since every open set is β*-open, the proof follows.
(ii) Follows from definition 3.3.1.
(iii) Follows by (i).
(iv) If x∈ $D_{\beta*}$($D_{\beta*}$(A))-A and U is a β*-open set containing x, then U∩($D_{\beta*}$(A)-{x})≠∅. Let y∈ U∩($D_{\beta*}$(A)-{x}). Then since y∈$D_{\beta*}$(A) and y∈ U, U∩ (A-{y})≠∅. Let z∈ U∩{A-{y}}. Then z≠x for z∈A and x∉ A. Hence U∩(A-{x})≠∅. Therefore, x∈$D_{\beta*}$(A).
(v) Let x∈$D_{\beta*}$(A∪$D_{\beta*}$(A)). If x∈ A, the result is obvious. So let x∈$D_{\beta*}$(A∪$D_{\beta*}$(A))-A, then for an β*-open set U containing x. U∩((A∪$D_{\beta*}$(A))-{x})≠∅. Thus U∩(A-{x})≠∅ or U∩($D_{\beta*}$(A)-{x})≠∅. By the same argument in (iv), it follows that U∩ (A-{x}) ≠∅. Hence x∈$D_{\beta*}$(A). Therefore in either case$D_{\beta*}$(A∪$D_{\beta*}$(A))⊂ A∪$D_{\beta*}$(A).

Remark 3.3.3: In general, the converse of (i) is not true. For example, Let X={a, b, c} and τ = {∅, {a, b}, X}. Then β*O(τ)={∅, {a}, {b}, {a, b}, X}. Let A = {a, b} then D(A) = X and $D_{\beta*}$(A) = c. Therefore D(A) ⊄ $D_{\beta*}$(A).

Proposition 3.3.4: $D_{\beta*}$(A∪B) ≠$D_{\beta*}$(A)∪$D_{\beta*}$(B).

Example 3.3.5: Let X = {a, b, c. d} and τ = {∅, {a, b}, X}. Then β*O(τ) = P(X) – [a}, {b}, {a, b}. Let A = {a, b, d} and B ={c}. Then $D_{\beta*}$(A∪ B) = {a, b} and $D_{\beta*}$(A) = ∅, $D_{\beta*}$(B)=∅.

Theorem 3.3.6: For any subset A of a space X, β*cl(A)= A∪$D_{\beta*}$(A).

Proof: Since $D_{\beta*}(A)\subset \beta^*cl(A)$, $A\cup D_{\beta*}(A)\subset \beta^*cl(A)$. On the other hand, let $x\in \beta^*cl(A)$. If $x\in A$, then the proof is complete. If $x\notin A$, each β*-open set U containing x intersects A at a point distinct from x, so $x\in D_{\beta*}(A)$. Thus $\beta^*cl(A)\subset D_{\beta*}(A)\cup A$ and hence the theorem.

3. 4. β-*Border

Definition 3.4.1: Let A be a subset of a space X. Then the β* border of A is defined as $b_{\beta*}(A) = A-\beta^*int(A)$.

Theorem.3.4.2: For a subset A of a space X, the following statements hold.

(i) $b_{\beta*}(A)\subset b(A)$ where b(A) denote the border of A.

(ii) $A = \beta^*int(A) \cup b_{\beta*}(A)$.

(iii) $\beta^*int(A)\cap b_{\beta*}(A)=\emptyset$.

(iv) If A is β*-open then $b_{\beta*}(A) =\emptyset$.

(v) $\beta^*int(b_{\beta*}(A))=\emptyset$.

(vi) $b_{\beta*}(b_{\beta*}(A))=b_{\beta*}(A)$.

(vii) $b_{\beta*}(A)=A\cap \beta^*cl(A^c)$.

Proof: (i),(ii) and (iii) are obvious from the definitions of β*interior of A and β*-border of A where A is any subset of X.

vi) If A is β*-open, then $A=\beta^*int(A)$. Hence the result follows.

v) If $x\in \beta^*int(b_{\beta*}(A))$, then $x\in b_{\beta*}(A)$. Now $b_{\beta*}(A)\subset A$ implies $\beta^*int(b_{\beta*}(A))\subset \beta^*int(A)$. Hence $x\in \beta^*int(A)$ which is a contradiction to $x\in b_{\beta*}(A)$. Thus $\beta^*int(b_{\beta*}(A)) = \emptyset$.

vi) $b_{\beta*}(b_{\beta*}(A)) = b_{\beta*}(A-\beta^*int(A)) = (A-\beta^*int(A))-\beta^*int(A-\beta^*int(A))$ which is $b_{\beta*}(A)-\emptyset$, by (iv). Hence $b_{\beta*}(b_{\beta*}(A)) = b_{\beta*}(A)$.

vii) $b_{\beta*}(A) = A-\beta^*int(A) = A-(\beta^*cl(A^c))^c = A\cap \beta^*cl(A^c)$.

Remark 3.4.3.: In general, the converse of (i) is not true. For example, let X = {a, b, c} and τ = {∅, {a}, {a, b}, X}. Then β*O(τ) = {∅, {a}, {b}, {a, b}, {a, c}, X}. Let A = {a, c}, then $b_{\beta*}(A)$ = {a, c} – {a, c} = ∅ and b(A) = {a, c}-{a} = {c}. Therefore $b(A)\not\subset b_{\beta*}(A)$.

3.5 β-*Frontier

Definition 3.5.1: Let A be a subset of a space X. Then β*-frontier of A is defined as $Fr_{\beta*}(A)-\beta^*int(A)$.

Theorem 3.5.2: For a subset A of a space X, the following statements hold

i) $Fr_{\beta*}(A) \subset Fr(A)$, where Fr(A) denotes the frontier of A.

ii) $\beta^*cl(A)= \beta^*int(A)\cup Fr_{\beta*}(A)$

iii) $\beta^*int(A)\cap Fr_{\beta*}(A) = \emptyset$.

iv) $b_{\beta*}(A)\subset Fr_{\beta*}(A)$

v) $Fr_{\beta*}(A) = b_{\beta*}(A)\cup D_{\beta*}(A)$

vi) If A is β*-open, then $Fr_{\beta*}(A) =D_{\beta*}(A)$

vii) $Fr_{\beta*}(A) =\beta^*cl(A)\cap \beta^*cl(A^c)$

viii) $Fr_{\beta*}(A) =Fr_{\beta*}(A^c)$

ix) $Fr_{\beta*}(\beta^*int(A))\subset Fr_{\beta*}(A)$.

x) $Fr_{\beta*}(\beta^*cl(A)) \subset Fr_{\beta*}(A)$.

Proof:

i) Since every open set is β*-open we get the proof.

ii) $\beta^*int(A)\cup Fr_{\beta*}(A)= \beta^*int(A)\cup (\beta^*cl(A)-\beta^*int(A)) = \beta^*cl(A)$.

iii) $\beta^*int(A)\cap Fr_{\beta*}(A)=\beta^*int(A)\cap(\beta^*cl(A)-\beta^*int(A))=\emptyset$.

iv) Obvious from the definition.

v) $\beta^*int(A)\cup Fr_{\beta*}(A)=\beta^*int(A)\cup b_{\beta*}(A)\cup D_{\beta*}(A)$, is obvious from the definition. Therefore we get $Fr_{\beta*}(A) = b_{\beta*}(A)\cup D_{\beta*}(A)$.

vi) If A is β*-open, then $b_{\beta*}(A) = \emptyset$, then by (V)$Fr_{\beta*}(A) =D_{\beta*}(A)$.

vii) $Fr_{\beta*}(A) =\beta^*cl(A)-\beta^*int(A) = \beta^*cl(A)-(\beta^*cl(A^c))^c = \beta^*cl(A)\cap \beta^*cl(A^c)$.

viii) Follows from (vii).

ix) Obvious.

x) $Fr_{\beta*}(\beta^*cl(A))=\beta^*cl(\beta^*cl(A))-\beta^*int(\beta^*cl(A))= \beta^*cl(A)-\beta^*int(A) =Fr_{\beta*}(A)$.

In general the converse of (i) of theorem 3.5.2 is not true.

Example 3.5.3: Let X={a, b, c} and τ = {∅, {a}, {a, b}, X}. Then β*cl(τ) = {∅, {b}, {c}, {b, c}, {a, c}, X}. Let A = {a, b}. Then $\beta^*cl(A)-\beta^*int(A) = Fr_{\beta*}(A)$=X-{a, b} = {c}. But cl(A)-int(A) = Fr(A) = X-{a} = {b,c}. Therefore $Fr(A)\not\subset Fr_{\beta*}(A)$.

3.6.β-*Exterior

Definition 3.6.1: $\beta^*Ext(A) = \beta^*int(A^c)$ is said to be the β* exterior of A.

Theorem 3.6.2: For a subset A of a space X, the following statements hold

(i) $Ext(A) \subset \beta^*Ext(A)$ where Ext(A) denote the exterior of A.
(ii) $\beta^*Ext(As) = \beta^*int(A^c) = (\beta^*cl(A))^c$.
(iii) $\beta^*Ext(\beta^*Ext(A)) = \beta^*int(\beta^*cl(A))$
(iv) If $A \subset B$, then $\beta^*Ext(A) \supset \beta^*Ext(B)$.
(v) $\beta^*Ext(A \cup B) \subset \beta^*Ext(A) \cup \beta^*Ext(B)$.
(vi) $\beta^*Ext(A \cap B) \supset \beta^*Ext(A) \cap \beta^*Ext(B)$.
(vii) $\beta^*Ext(X) = \emptyset$.
(viii) $\beta^*Ext(\emptyset) = X$.
(ix) $\beta^*int(A) \subset \beta^*Ext(\beta^*Ext(A))$.

Proof: (i) & (ii) follows from definition 3.6.1.
iii) $\beta^*Ext(\beta^*Ext(A)) = \beta^*Ext(\beta^*int(A^c)) = \beta^*Ext(\beta^*cl(A)^c) = \beta^*int(\beta^*cl(A))$.
iv)If $A \subset B$, then $A^c \supset B^c$. Hence $\beta^*int(A^c) \supset \beta^*int(B^c)$ and so $\beta^*Ext(A) \supset \beta^*Ext(B)$.
v) and (vi) follows from (iv).
(vii) and (viii) follows from 3.6.1.
ix) $\beta^*int(A) \subset \beta^*int(\beta^*cl(A)) = \beta^*int(\beta^*int(A^c)) = \beta^*int(\beta^*Ext(A))^c = \beta^*Ext(\beta^*Ext(A))$.

Proposition 3.6.3: In general equality does not hold in (i), (v) and (vi) of Theorem 2.4.49.

Example 3.6.4: Let X= {a, b, c} and τ = {∅, {a, b}, X}. Then β*O(τ) = {∅, {a}, {b},{a, b}, X}. If A= {a}, B={b} and C={c} then β*Ext(A) ={b},β*Ext(B) ={a} and Ext(A) = ∅. Therefore $\beta^*Ext(A) \not\subset Ext(A)$, $\beta^*Ext(A) \cup \beta^*Ext(B) \not\subset \beta^*Ext(A \cup B)$ and $\beta^*Ext(A \cap B) \not\subset \beta^*Ext(A) \cap \beta^*Ext(B)$.

References:

[1]. Abd EI-Mobsef M.E,EI-Deeb S.N and Mahmoud R.A., β-open sets and β-continuous mappings, Bull.Fac.Sci.Assiut Univ., 12(1983), 77-90.
[2]. Andrijevic D., Semi-preopen sets, Mat. Vesnik, 38(1986), 24-32.
[3]. Antony Rex Rodrigo. J. Some Characterizations of η̂* closed sets and η̂* -continuous maps in Topological and bitopological spaces, Ph.D., Thesis, Alagappa university, Karaikudi(2007)
[4]. Antony Rex Rodrigo J,Jessie Theodore and Hana selvi Jansi,β^*-Closed sets in topological spaces,International journal of mathematical Archive-3(3),2012,1065-1070
[5]. Caldas M Jafari S., and Noiri,T., Notions via g-open sets,Kochi J. of maths (accepted)
[6]. Dontchev. J, On generalizing semi-preopen sets. Mem. Fac. Sci. Kochi Uni. Ser. A Math., (1995), 35-48.
[7]. . Levine N., Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963)
[8]. .Sheik John M., A Study on generalizations of closed sets and continuous maps in topological and bitopological spaces, Ph.D., Thesis, Bharathiar University, Coimbatore(2002).

.