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Invention of the plane geometrical formulae - Part II

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Abstract: *In this paper, I have invented the formulae for finding the area of an Isosceles triangle. My finding is based on pythagoras theorem.*

I. Introduction

A mathematician called Heron invented the formula for finding the area of a triangle, when all the three sides are known. Similarly, when the base and the height are given, then we can find out the area of a triangle. When one angle of a triangle is a right angle, then we can also find out the area of a right angled triangle. Hence forth, We can find out the area of an equilateral triangle by using the formula of an equilateral triangle. These some formulae for finding the areas of a triangles are not exist only but including in educational curriculum also.

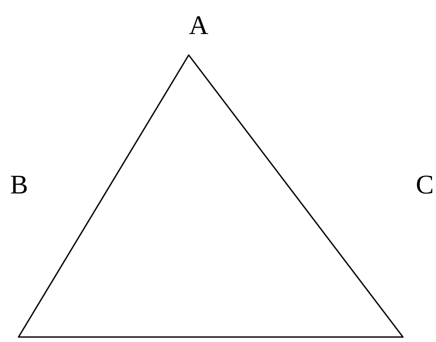
But, In educational curriculum. I don't appeared the formula for finding the area of an isosceles triangle with doing teaching – learning process . Hence, I have invented the new formula for finding the area of an isosceles triangle by using Pythagoras theorem.

I used pythagoras theorem with geometrical figures and algebric equations for the invention of the new formula of the area of an isosceles triangle. I Proved it by using geometrical formulae & figures, 20 examples and 20 verifications (proofs).

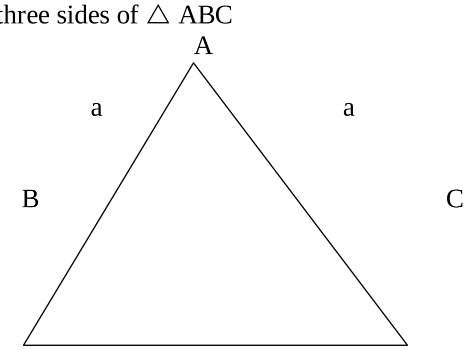
Here myself is giving you the summary of the research of the plane geometrical formulae- Part II

II. Method

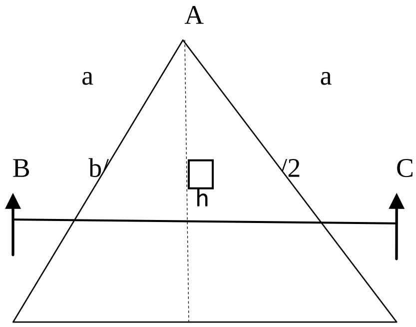
First taking an isosceles triangle ABC



Now taking a, a & b for the lengths of three sides of $\triangle$ ABC



Draw perpendicular AD on BC.



△ABC is an isosceles triangle and it is an acute angle also.
In △ABC,
Let us represent the lengths of the sides of a triangle with the letters a,a,b. Side AB and side AC are congruent side. Third side BC is the base. AD is perpendicular to BC.
Hence, BC is the base and AD is the height.
Here, taking AB=AC = a
Base , BC = b Height, AD = h
In △ ABC, two congruent right angled triangle are also formed by the length of perpendicular AD drawn on the side BC from the vertex A. By the length of perpendicular AD drawn on the side BC, Side BC is divided into two equal parts of segment. Therefore, these two equal segments are seg DB and seg DC. Similarly, two a right angled triangles are also formed, namely, △ ADB and △ADC which are congruent.
Thus,

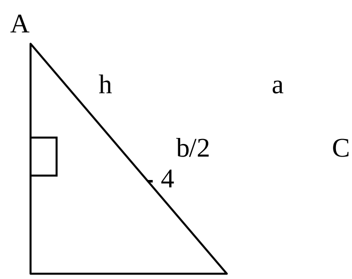
$DB = DC = 1/2 \times BC$

$DB = DC = 1/2 \times b = b/2$

△ ADB and △ ADC are two congruent right angled triangle.

Taking first right angled △ADC,
In △ADC, Seg AD and Seg DC are both sides forming the right angle. Seg AC is the hypotenuse.

Here, AC =a
Height , AD = h
DC = b/2 and $m \angle ADC = 90^0$



According to Pythagoras Theorem,
$(\text{hypotenuse})^2 = (\text{one side forming the right angle})^2 + (\text{second side forming the right angle})^2$
In short,

$(\text{Hypotenuse})^2 = (\text{one side})^2 + (\text{second side})^2$

$$AC^2 = AD^2 + DC^2$$
$$AD^2 + DC^2 = AC^2$$
$$h^2 + (b/2)^2 = a^2$$
$$h^2 = a^2 - (b/2)^2$$
$$h^2 = a^2 - \frac{b^2}{4}$$
$$h^2 = \frac{a^2 \times 4}{4} - \frac{b^2}{4}$$
$$h^2 = \frac{4a^2}{4} - \frac{b^2}{4}$$
$$h^2 = \frac{4a^2 - b^2}{4}$$

Taking the square root on both side,

$$\sqrt{h^2} = \sqrt{\frac{4a^2 - b^2}{4}}$$

$$\sqrt{h^2} = \sqrt{\frac{1}{4} \times (4a^2 - b^2)}$$

$$\sqrt{h^2} = \sqrt{\frac{1}{4}} \times \sqrt{4a^2 - b^2}$$

The square root of h^2 is h and the square root of ¼ is ½

$$\therefore h = \tfrac{1}{2} \times \sqrt{4a^2 - b^2}$$

$$\therefore \text{Height, } h = \tfrac{1}{2}\sqrt{4a^2 - b^2}$$

$$\therefore AD = h = \tfrac{1}{2}\sqrt{4a^2 - b^2}$$

Thus,

Area of $\triangle ABC$ = ½ × Base × Height

= ½ × BC × AD

=½ × b × h

$$\text{But Height, } h = \tfrac{1}{2}\sqrt{4a^2 - b^2}$$

$$\therefore \text{Area of } \triangle ABC = \tfrac{1}{2} \times b \times \tfrac{1}{2}\sqrt{4a^2 - b^2}$$

$$\therefore \text{Area of } \triangle ABC = \frac{b}{2} \times \frac{1}{2}\sqrt{4a^2 - b^2}$$

$$= \frac{b \times 1}{2 \times 2} \times \sqrt{4a^2 - b^2}$$

$$= \frac{b}{4}\sqrt{4a^2 - b^2}$$

$$\therefore \text{Area of an isosceles } \triangle ABC = \frac{b}{4}\sqrt{4a^2 - b^2}$$

For example- Now consider the following examples:-

Ex. (1) If the sides of an isosceles triangle are 10 cm, 10 cm and 16 cm.

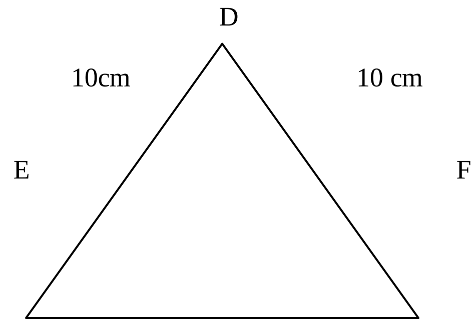
Find it's area

△DEF is an isosceles triangle.

In △DEF given alongside,

l (DE) = 10 cm.

l l (DF) = 10 cm. l (EF) = 16 cm



Let,

a = 10 cm

Base, b = 16 cm.

By using The New Formula of an isosceles triangle,

∴ Area of an isosceles △DEF = A (△DEF)

$$= \frac{b}{4}\sqrt{4a^2 - b^2}$$

$$= \frac{16}{4} \times \sqrt{4(10)^2 - (16)^2}$$

$$= 4 \times \sqrt{4 \times 100 - 256}$$

$$= 4 \times \sqrt{400 - 256}$$

$$= 4 \times \sqrt{144}$$

The square root of 144 is 12

= 4 × 12 = 48 sq.cm.

∴ Area of an isosceles △DEF = 48 sq.cm.

Verification :-

☞ Here,

l (DE) = a = 10 cm.

l (EF) = b = 16 cm.

l (DF) = c = 10 cm.

By using the formula of Heron's

Perimeter of △DEF = a + b + c

= 10 + 16 + 10 = 36 cm

Semiperimeter of △DEF,

$$S = \frac{a + b + c}{2}$$

$$S = \frac{36}{2}$$

S = 18 cm.

$$\therefore \text{Area of an isosceles } \triangle \text{ DEF} = \sqrt{s\,(s - a)\,(s - b)\,(s - c)}$$

$$= \sqrt{18 \times (18 - 10) \times (18 - 16) \times (18 - 10)}$$

$$= \sqrt{18 \times 8 \times 2 \times 8}$$

$$= \sqrt{(18 \times 2) \times (8 \times 8)}$$

$$= \sqrt{36 \times 64}$$

$$= \sqrt{36} \times \sqrt{64}$$

The square root of 36 is 6 and the square root of 64 is 8

= 6 × 8 = 48 sq.cm

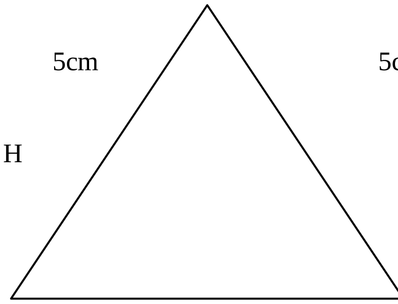
∴ Area of △DEF = 48 sq.cm

Ex. (2) In $\triangle$GHI, l (GH) = 5 cm, l (HI) = 6 cm and l (GI) = 5 cm.
Find the area of $\triangle$ GHI.

☞ $\triangle$GHI is an isosceles triangle.

In $\triangle$GHI given alongside,

l (GH) = 5 cm.
l (HI) = 6 cm.
l (GI) = 5 cm



Let,
a = 5 cm
Base, b = 6 cm.

By using The New Formula of area of an isosceles triangle,

$\therefore$ Area of an isosceles $\triangle$GHI = $\frac{b}{4}\sqrt{4a^2 - b^2}$

$= \frac{6}{4} \times \sqrt{4 \times (5)^2 - (6)^2}$

The simplest form of $\frac{6}{4}$ is $\frac{3}{2}$

$= \frac{3}{2} \times \sqrt{(4 \times 25) - 36}$

$= \frac{3}{2} \times \sqrt{100 - 36}$

$= \frac{3}{2} \times \sqrt{64}$

The square root of 64 is 8

$= \frac{3}{2} \times 8 = \frac{3 \times 8}{2} = \frac{24}{2}$

= 12 sq.cm.

$\therefore$ Area of an isosceles $\triangle$GHI = 12 sq.cm.

Verification :-

Here,

l (GH) = a = 5 cm.
l (HI) = b = 6 cm.
l (GI) = c = 5 cm.

By using the formula of Heron's

Perimeter of $\triangle$GHI = a + b + c
= 5 + 6 + 5
= 16 cm

Semiperimeter of $\triangle$GHI,

$S = \frac{a + b + c}{2}$

$S = \frac{16}{2}$

S = 8 cm.

$\therefore$Area of an isosceles $\triangle$ GHI = $\sqrt{s\,(s-a)\,(s-b)\,(s-c)}$

$$= \sqrt{8 \times (8-5) \times (8-6) \times (8-5)}$$
$$= \sqrt{8 \times 3 \times 2 \times 3}$$
$$= \sqrt{(8 \times 2) \times (3 \times 3)}$$
$$= \sqrt{16 \times 9}$$
$$= \sqrt{144}$$

The square root of 144 is 12

= 12 sq.cm

∴ Area of an isosceles △GHI = 12 sq.cm.

Explanation:-

We observe the above solved examples and their verifications, it is seen that the values of solved examples by using the new formula of an isosceles triangle and the values of their verifications are equal.

Hence, The new formula of the area of an isosceles triangle is proved.

III. Conclusions:-

Area of an isosceles triangle = $\dfrac{b}{4} \times \sqrt{4a^2 - b^2}$

From the above new formula , we can find out the area of an isosceles triangle. This new formula is useful in educational curriculum, building and bridge construction and department of land records. This new formula is also useful to find the area of an isosceles triangular plots of lands, fields, farms, forests, etc. by drawing their maps.

References:-

[1] Geometry concepts and Pythagoras theorem.