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Splitting of Bigraphs

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Abstract: *In this paper we are going to define splitting on bigraphs. The purpose of this paper is that however investigation of created graph by splitting is bipartite graph (Bigraph) or not? We show that after of operation of splitting on the one of the vertices of bigraph, the graph is also bigraph. Another rule is proved in this paper, is that if the graph is connected then the resulting bi graph is also connected.*

Keywords: *bipartite graph, bigraph, splitting, connected graph, matrix, incident matrix, adjacency matrix*

I. Introduction

Definition of Splitting Graph:

In graph theory - a breach of mathematics - a split graph is a graph in which the vertices can be partitioned in to a clique and independent set. Split graphs were first studied by Folders and Hummer (1997, 1977) and independently introduced by Tyshkevich and Chernyak (1979).

Definition of Bigraph (Bipartite graph):

A graph $G= (V, E)$ is called bipartite if V can be partitioned into two sets X and one vertex in Y.

Definition of vertex *v* on bigraph *G*:

Suppose G be an arbitrary connected bigraph such that $G=XUY$ (where X and Y are partition of G and suppose v can be an arbitrary vertex of G (v can be of X or Y).
Let $v\in X$ and $d(v)=n$ and T is the set of all edges of bigraphs G that there are on the v.
$T= \{e_1, e_2, \dots, en\}$
So, $|T|=n$ and $d(v)=|T|$

Now, we define the splitting on a vertex v we add the vertex v' to a set of X and put a copy of edges on top of the vertex v of graph G on the v' that the set of this edge is called $T'=\{e_1', e_2',\dots, e_n'\}$ then we have $T=\{e_1, e_2,\dots, e_n\}$ and $T'=\{e_1', e_2',\dots, e_n'\}$

So, if the vertex $v\epsilon X$ with the vertex $y_j\epsilon Y$ are adjacent then $v'\epsilon X$ in the new graph will be adjacent with the graph $y_j\epsilon Y$. Thus we introduce a new graph G_T and $\begin{cases} G = XUY \\ G_T = (XU\{v'\})UY \end{cases}$ and $X\,U\{v'\}=X'$

Sentences and the main theorems are proved in this paper are in clued:

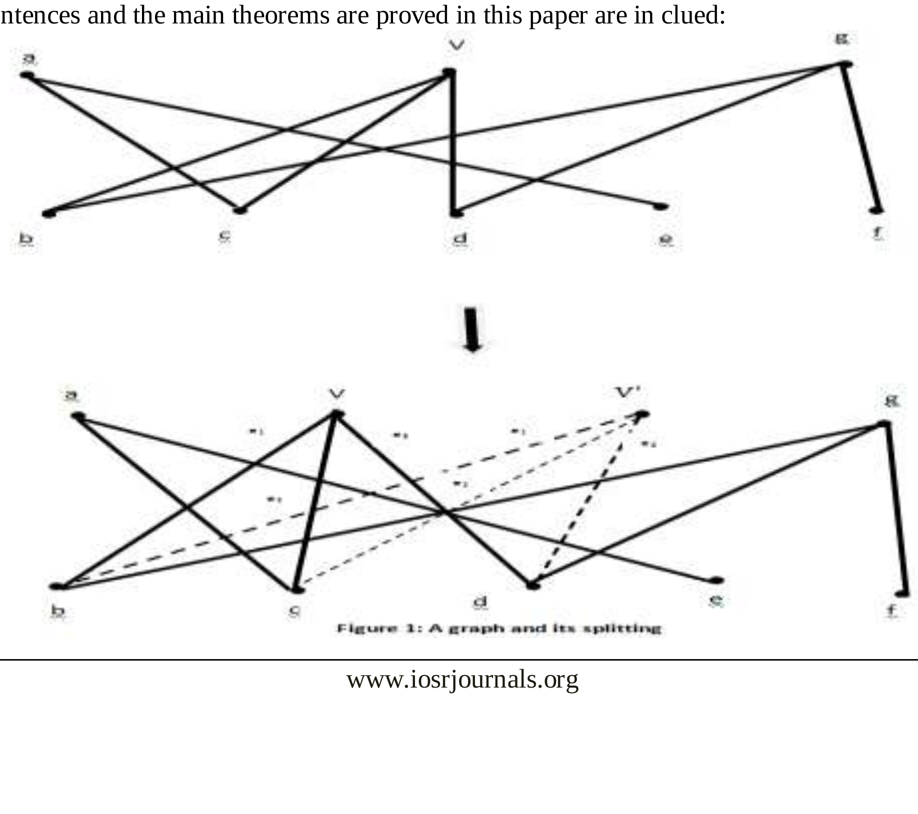


Figure 1: A graph and its splitting

Theorem: If G is a complete bigraph $K_{m,n}$ then G_T is also a complete bigraph as follows:

$$K_{m,n} \rightarrow \begin{cases} k'_{m+1,n} \\ k'_{n,m+1} \end{cases}$$

Proposition: prove that G_T is connective if G is connective.

In the first section we define the operation of splitting of bigraph and then review the cycles of this graph and then in this paper we proved the two main ruling. In the next section we introduce the incident matrix and the splitting of adjacency matrix bigraph .

II. The Cycles of Graph G_T:

Proposition: suppose C is the cycle of the Bigraph G, then the cycles of graph $G_{T'}$ would be equivalent to the following:

1. The same as cycles of graph G
2. If C consists of two elements of set T, then $(C\text{-}\{e_1,e_2\})\ U\ \{e'_1 , e'_2\}$ such that $\{e'_1 ,e'_2\} \epsilon\ T'$ and $\{e_1 ,e_2\} \epsilon\ T$ are the cycles of G_T.
3. Two elements of T and two elements of T' results in a cycle of length 4 in G_T.

Proof: since the set T' is the copy of the set T, then

1. In the graph G_T without considering the vertex v' and the edges are reached on the same graph G, so the cycles of graph G are entirely passed in to the new graph G_T.
2. Assuming that ve_1bgde_3v be the arbitrary cycle of the graph G where $\{e_1,e_3\} \epsilon\ T$, with replaced $\{e'_1,e'_3\} \epsilon\ T'$ instead of $\{e_1,e_2\}$ to cycle $v'e_1'bgde'_3v'$so, we receive the new graph G_T.
3. If $d(v)=1$ then the edge e of graph G will not be on any cycle of graph G, and since e' is also a copy of e on v' then e' is also will not be the cycle of graph G_T.

Let $d(v)>1$ and let's start of graph G_T of the vertex $v\epsilon X'$ we move in the graph G_T consider the edge from vertex v. this edge connects the vertex v of the vertex set Y, same like edge vy_i.

According to definition of graph G_T, a copy of this edge is also on the vertex v' like $v'y_i$, therefore get out of the set Y with this edge and we reach the set X' on the vertex v'. Since $d(v)>1$ then $d(v')>1$ (according the definition of splitting on vertex v) thus, there is an output edge of v', which will be to vertex y_j of y like $v'y_j$.

With the edge we are going to the set Y. this edge is also a copy of edges where the edges are on the vertex v.

So, from the vertex y_j we will have the outgoing edge to the vertex v like vy_j then vy_i, $v'y_i$, $v'y_j$, vy_j are the cycles of graph G_T.

Proof of main results

Theorem: If G is a complete bigraph $K_{m,n}$ then G_T is also a complete bigraph as follows:

$$K_{m,n} \rightarrow \begin{cases} k'_{m+1,n} \\ k'_{n,m+1} \end{cases}$$

Proof: we assume G is a complete bigraph, according to the definition above for G_T, G_T is bigraph.

$G=XUY$ and $G_T=X'U\,Y$

Now, if the bigraph G is complete, we show that bigraph G_T is also complete $G=K_{m,n}$. let v' be the vertex that is created by split of the vertex $v\epsilon X$. according to previous explanation about G_T ,all the edges on vertex $v\epsilon X$ are copied the new vertex v' in G_T . since the bigraph G is complete , then the vertex $v\epsilon X$ is adjacent with all vertices of set Y and the vertex $v'\epsilon X'$ is also adjacent with all vertices of set Y, thus the bigraph G_T is also complete and $G_T= K'_{m+1,n}$.

Proposition: prove that G_T is connective if G is connective.

Proof: Let G_T is not connective that means (a subset of the vertex set y is not adjacent with any vertex subset of the set Y). We remove the vertex $v'\epsilon X'$ and the edges on this vertex, and then we get the bigraph G. By removing the vertex and edges, does not change as disconnect. Sub graph, even though it may also increase the number of blocks. This means that the bigraph G is also disconnected and this is a contradiction. Therefore if the graph G is connective, G_T is connective.

III. Incident Matrix and adjacency matrix G:

Definition of adjacency matrix graph G:

Suppose $G= (V, E)$ is a graph without loop where V is the set of vertices of the graph G and E. the set of all edges of graph G.

Let $V(G)=\{v_1, v_2, \ldots, v_n\}$ and $E(G)=\{e_1, e_2, \ldots, e_n\}$ displayed by $A(G)$ that $n_x n$ matrix and where a_{ij} is denote the number of edges in vertexes v_i and v_j then we have $A(G)=(a_{ij})_{n,n}$. $A(G)$ is a symmetric square matrix where the entries of the main diagonal are equal of zero. Because the graph G is a bigraph and does not have a loop.

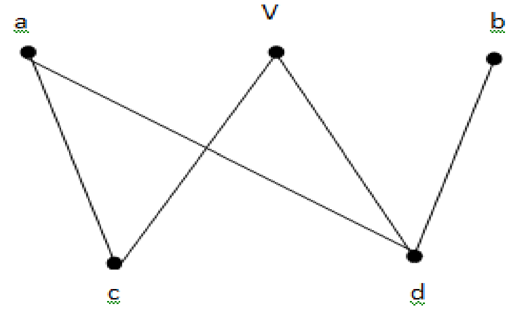
The adjacency matrix bigraph *G*:

Adjacency matrix of bigraph is a matrix of the form:

$$A(G) = \begin{bmatrix} O_{n\times n} & A_{n\times m} \\ (A_{n\times m})^T & O_{m\times m} \end{bmatrix}_{(n+m)(n+m)}$$

$|X|=n$ and $|Y|=m$, $G=XUY$ and $(A_{n,m})$ and $(A_{n,m})^T=A^T_{m,n}$

For example if G is the bigraph as the fig below:

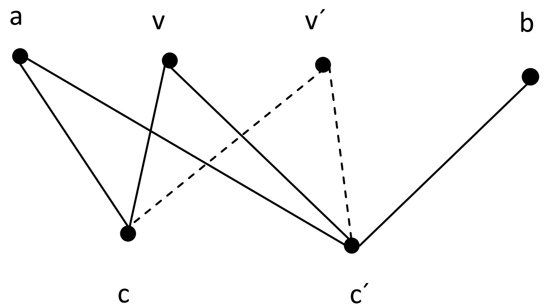


$$\begin{array}{c} \\ a \\ v \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} a & v & b & c & D \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

On the basis of bigraph G the bigraph $G_{T,}$ since $X'=XU\{v'\}$ then $|X'|=n+1$

$$A(G_T) = \begin{bmatrix} O_{(n+1)(n+1)} & A_{(n+1)m} \\ A_{(n+1)m} & O_{(m,m)} \end{bmatrix}_{((n+1)+m)\,\square\,((n+1)+m)}$$

For example, in the previous example if G_T construct:



$$\begin{array}{c} \\ a \\ v \\ v' \\ b \\ c \\ c' \end{array} \begin{array}{c} \begin{array}{cccccc} a & v & v' & b & c & c' \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

So, with an adjacency matrix bigraph G be the adjacency matrix bigraph G_T is also created. Thus the sub matrix $O_{n,n}$ in $A(G)$ to $O_{(n+1)(n+1)}$

To convert and in the sub matrix A_{nxm} a copy of the entries in the top vertex v in front vertex v' the following matrix is reached.

Incident matrix *G*:

An occurrence matrix G on a graph n□m is M (G) = (mij)n□m will be represented by

$$M_{ij} = \begin{cases} 1, & \text{if vertex } v_i \text{ on an edge } e_i \text{ is located} \\ 0, & \text{Otherwise} \end{cases}$$

N: the vertex of graph

M: the edges of graph

Incident matrix G_T:

We add a row matrix M (G) to the vertex v´

We add the members of the set T´ (this mean $|T'|$ of the matrix M (G) column

We will add all entries in the row v´ to zero apart from the entries that, we will take one the other.

The elements that are the column set T (apart from the entries in the row v) are located on the members of the same element T´ also put.

IV. Conclusion

We prove that if G is a complete bigraph $K_{m,n}$ then G_T is also a complete bigraph as follows:

$$K_{m,n} \rightarrow \begin{cases} k'_{m+1,n} \\ k'_{n,m+1} \end{cases}$$

And G_T is connective if G is connective.

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