IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728.Volume 5, Issue 1 (Jan. - Feb. 2013), PP 91-94
www.iosrjournals.org

Linear Algebra and Quantum Mechanics: Unveiling the Hidden Connections

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ABSTRACT
It is possible to think of algebra as the language of mathematics, and it plays a significant part in determining whether or not pupils are able to follow a diverse range of educational paths in today's culture. Because of this, it could seem to be common sense that algebra should play a significant part in mathematics. However, investigations based on data from a number of large-scale international studies have indicated that there are significant discrepancies across nations in terms of algebra. Algebra plays a significant role in certain countries, whereas this is not the case in other countries. It has been shown that these disparities remain constant across time and at various levels inside. This highlights and examines the ways in which these inequalities may impede upon individual rights and chances to pursue the education that the person desires, as well as the ways in which this may impede upon societies' needs to recruit individuals to a variety of professions.
Keywords: *Mathematical Usage, Algebra*

I. INTRODUCTION

One way to think about algebra is as the mathematical equivalent of a language. It is a truth universally acknowledged that one's level of fluency in the native tongue of a nation is directly correlated to the possibilities available to them in that country. One may make the same point regarding algebra. The ability to solve algebraic equations is necessary for individuals to possess in all fields of study and work that include the usage of this language. It takes time to acquire the language of a nation, and that language develops over the course of time via intense instruction through hearing and through training to use it yourself. In general, it is simpler for young children to begin learning a language than it is for adults. The same can be said, to a certain degree, about algebra. The only difference is that, in mathematics, you start with arithmetic since arithmetic is the foundation for algebra. Therefore, it appears acceptable to begin with algebra after achieving some level of fluency in arithmetic.

Everyone in a contemporary society spends a significant amount of time in school, which helps to prepare them for becoming responsible citizens who are able to take care of their own day-to-day lives and also have jobs that allow them to contribute to society while also providing for their own needs. The question of what kind of competency should be emphasized in schools in contemporary cultures is one that has to be addressed. Is it sufficient to teach them basic arithmetic and statistics in mathematics to prepare them for their everyday lives, or do we need to put in more effort to teach them algebra as a mathematical language? A large number of individuals who have an extensive education in many fields of technology, such as engineering and computer science, are required for a contemporary civilization. A contemporary society is confronted with challenges that are connected to the economy and the environment. In each of these areas, a solid understanding of the mathematical language known as algebra is absolutely necessary. In today's world, there are a wide variety of fields in which an understanding of algebra is necessary to pursue a career. In addition to this, it is essential for all forms of education in the natural sciences, such as physics, biology, and chemistry, as well as education in mathematics itself. It is necessary to have a strong command of the algebraic language if you want to enroll in a university program that focuses on geometry.

Algebra is included in the curricula of schools all over the globe for the same reasons: there is a legitimate purpose for it to be there, and the school is responsible for teaching it to the pupils. Despite this, a number of studies have shown that the importance placed on children studying algebra differs greatly from one region of the globe to another. This report summarizes the findings of a number of similar analyses carried out over the course of the last two decades, drawing on data collected from a variety of research conducted at various educational levels. Using these findings as a jumping off point, certain ramifications will be pointed out and explored for both individual students and societies that do not place a strong emphasis on the study of algebra in their schools.

This includes discussions of students' equal rights to pursue all types of education and, as a result, have the opportunity for a number of different positions in society, possible reasons for the low emphasis on algebra in some countries, the relation between pure and applied mathematics, as well as some reflections about teaching and learning algebra from the perspective that algebra is a language.

when I was first introduced to abstract algebra, I was mesmerized by the power and beauty of the subject. When working with algebraic structures, you do not need to be concerned with what the objects are; rather, it is how they act that is crucial. This is one of the most enlightening notions that the topic presents, and it is also one of the most freeing. The behavior of mathematical objects may be wonderfully articulated via the use of the language of algebra. My interest in algebra eventually led me to the field of algebraic geometry, which at the time was one of the most abstract subfields of pure mathematics. At the time, I never in a million years would have guessed that twenty-five years later I would be authoring papers alongside computer scientists, where we employ algebraic geometry and commutative algebra to address issues in geometric modeling. I never would have believed it. The purely theoretical and conceptual aspects of algebra that I first learnt have turned out to have important real-world applications. What does the fact that these applications exist, along with others, say about the connection between algebra and applied mathematics? This essay's goal is to investigate some elements of this interaction in the aim of generating helpful debates between those working in pure mathematics and those working in applied mathematics. In this context, the term "applied mathematics" refers to not only the mathematical concepts that students study in mathematics and applied mathematics departments, but also the mathematical concepts that students study in computer science, engineering, and operations research departments. In order to show the many potential applications of algebra, I will first use examples from the fields of geometric modeling, economics, and splines. After that, I will talk about computer algebra, and then I will wrap up with some thoughts on the place of algebra in the curriculum for applied mathematics.

Matrix Operations

It is conventional to create matrices in a format that utilizes rectangular patterns, as the reader most certainly is already aware. The first index will always give you the number of the row, and the second index will always tell you the number of the column. This is how the convention works. Consequently, a regular two-by-three matrix of the fact that addition and scalar multiplication are both defined on an entry-by-entry basis, each of these identities follows logically from the identities of the corresponding members of F. The operation of multiplying matrices is described in such a manner that the kind of system of linear equations that was covered in the prior section may be expressed in the form of a matrix equation.

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ & \ddots & \\ a_{k1} & \cdots & a_{kn} \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \quad \text{and} \quad B = \begin{pmatrix} b_1 \\ \vdots \\ b_k \end{pmatrix}$$

To be more specific, if matrix A is a k-by-m matrix and matrix B is an m-by-n matrix, then the k-by-n matrix described by C = AB is the product of the two matrices.

$$C_{ij} = \sum_{l=1}^{m} A_{il} B_{lj}.$$

Let us underline that the need for defining a product AB is that the number of columns of A should be equal to the number of rows of B. This is the condition that must be met. According to this interpretation, the equations in the system that were discussed before have the form AX = B.

In point of fact, there is a further inquiry to be made. In the process of applying row reduction, there is one more option, as stated in Proposition and that is that the left side contains a row of 0. In this scenario, is it possible to reason that A1 does not exist? Or, to put it another way, are we able to be certain that there is at least one row in the reduced row-echelon form that has all zeroes on the left side of the vertical line and anything that is not zero on the right side? These questions can all be answered in the affirmative, and we will demonstrate why in just a minute. First, we have to establish that multiplications of matrices are the source of all row operations, including the most fundamental ones.

While Cramer's Rule should be easily recognizable, free resolutions (whatever they are) could come out as fairly vague. In the next section, we shall show that there are issues in geometric modeling in which these themes naturally occur. The study that Tom Sederberg and Falai Chen did on parametric curves in the plane is where my portion of the narrative gets started. In the event that we are provided with relatively prime polynomials a(t), b(t), and c(t) of degree n, then the parametric equations will be as follows:

$$x = \frac{a(t)}{c(t)} \quad \text{and} \quad y = \frac{b(t)}{c(t)}$$

Describe a curve that is contained inside the plane. In order to approach this problem from a geometric standpoint, Sederberg and Chen took into consideration lines defined by.

$$A(t)x + B(t)y + C(t) = 0,$$

where A(t), B(t), and C(t) are polynomials that change based on the value of the variable t. Because the line shifts in response to changes in t, it is referred to as a moving line. A parametrization is said to be followed by a moving line if, for each possible value of the parameter, the point can be found on the line that corresponds to that value. To put it another way, is a solution of for each and every t. After making these changes to the numerators and denominators, we are able to get the equation.

$$A(t)a(t) + B(t)b(t) + C(t)c(t) \equiv 0,$$

But then Tom surprised me by asking me what the word "module" meant. I was caught off guard. At first, I was taken aback by the fact that he was unfamiliar with such a fundamental phrase; but, then I recalled that civil engineers are not required to attend classes in abstract algebra. (Tom has a degree in civil engineering, but he is now working in the field of computer science.) Tom's work involves the use of vectors of polynomials rather often, and modules over polynomial rings are the language that is most suited to the discussion of such things. Despite this, Tom was completely unaware of the word "module" until I brought it to his attention. After considering this, I came to the conclusion that our understanding of "applied algebra" probably needs some expansion. I will elaborate more on this topic in the next paragraphs, but first allow me to conclude the tale of shifting lines. Both Sederberg and Chen came to the realization independently that when two moving lines obey the parametrization, the curve may be described by the point of intersection between the lines. When they investigated the matter further, they found that there are always two moving lines.

$$p := A_1(t)x + B_1(t)y + C_1(t) = 0,$$
$$q := A_2(t)x + B_2(t)y + C_2(t) = 0$$

OBJEACTIVES

1. The Study Mathematical Usage Of Algebra And It's Importance.
2. The Study Algebra Can Be Viewed As A Language Of Mathematics.

Symbolic Linear Algebra

Not so long ago, a coworker and I were having a discussion on the importance of Cramer's Rule in linear algebra. Cramer's Rule may have played a significant part in the development of linear algebra in the past, but it appears to have less of a place in the field now, particularly given the focus that is placed on solving equations via the use of Gaussian elimination. In addition, Cramer's Rule is rendered worthless whenever a system of equations contains a coefficient matrix that is not in a well-conditioned state. Because of these factors, one of my coworkers was considering whether or not the subject should be skipped; after all, why should students be required to memorize a superfluous formula? Cramer's Rule has long been one of my favorites because of the inherent beauty it has, however this method is not always successful for students who want to perceive mathematics through the lens of its applications. After giving it some consideration, I came to the conclusion that Cramer's Rule, despite the fact that it is not useful for numerical linear algebra, does have a use in the broader field of applied linear algebra. Note that the application of Cramer's Rule in is nonnumerical and part of geometric modeling. This is the key to understanding the distinction between the two. Consider the following straightforward example from the field of economics as yet another non-mathematical use of Cramer's Rule.

Algebra and Applied Algebra

Over the course of the last forty years, we have seen the development of significant computing capacity as well as the discovery and, in some instances, rediscovery of basic algorithms for symbolic computation. Because of the interconnected nature of these recent advancements, there has been an explosion of study, both fundamental and applied. There are many different kinds of publications in this field, ranging from textbooks for undergraduate students to specialized monographs. Some of these works are geared for scholars in algebraic geometry and commutative algebra, while others are written for a wider variety of readers. The field of computer algebra covers a broad range of topics, some of which include coset enumeration, the theory of Galois, modular arithmetic, symbolic integration, symbolic summation, difference equations, power series, and special functions. Polynomials, as one would anticipate, play a significant part in the field of computer algebra, which is where one may find algorithms for greatest common divisors, factorization, Grobner bases, resultants, characteristic sets, quantifier elimination, and cylindrical algebraic decomposition. The survey is often included in introductions to computer algebra. Problems in areas such as robotics, splines, differential equations, statistics, coding theory, computational chemistry, computer-aided geometric design, geometric theorem proving, and systems of polynomial equations may be solved with the help of computer algebra. These uses, along with others, are detailed in the. computer algebra that were mentioned before might provide the answer for those branches of the applied mathematics community that place an extreme premium on their access to computational tools. (This is not yet the ideal answer since the textbooks do not always make full use of the language and algebra, and even the ones that do sometimes make too many assumptions or need a high learning curve.) However, in many areas of practical research, computational methods are not the primary concern; rather, the emphasis is placed on gaining a knowledge of the overarching structure of the issues that are being investigated. In situations like these, the language of algebra may be of great use. Due to this fact, the field of applied mathematics need to consider how it may provide its students with improved access to algebra.

II. CONCLUSION

The discussion of algebra instruction that has taken place up to this point has prompted a few thoughts, and these are the thoughts with which we will conclude this chapter. To begin, we need to think about the technologies that were there throughout each era. These tools have undeniably had a significant impact in the evolution of algebra instruction over the ages. Because keeping written records and replicating them was so difficult, oral teaching was prevalent for a significant portion of human history. The development of print made it possible to make extensive use of written content, which was later seen as a more effective alternative to oral techniques and presented in this light. In later times, the contradiction between oral and written education evolved into a blend of oral and written instruction, with changes to a greater or lesser degree depending on the local customs. The current epoch, in which we find ourselves, is one of fast growth of new digital technologies (the internet, web interactive boards), which will unquestionably have a substantial impact on the teaching of mathematics, and in particular the teaching of algebra.

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