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Static Deformation of a Uniform Half-Space with Rigid Boundary Due To a Vertical Dip-Slip Line Source

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Abstract: *The Airy stress function for a vertical dip-slip line source buried in a homogeneous, isotropic, perfectly elastic half-space with rigid boundary is obtained. This Airy stress function is used to derive closed-form analytical expressions for the stresses and displacements at an arbitrary point of the half-space caused by vertical dip-slip line source. The variation of the displacements and stress fields with distance from the fault and depth from the fault is studied numerically.*

Keywords – *Dip-slip faulting, Half-space, Rigid boundary, Static deformation*

I. Introduction

Strains and stresses within the Earth constitute important precursors of earthquakes. Therefore, the determination of the static deformation of an Earth model around surface faults is important for any scheme for prediction of earthquakes. Static dislocation models are used to analyze the static deformation of the medium caused by earthquake faults. Steketee (1958a, b) applied the elasticity theory of dislocations in the field of seismology. For the sake of simplicity, Steketee ignored the curvature of the Earth, its gravity, anisotropy and non-homogeneity and dealt with a semi-infinite, non-gravitating, isotropic and homogeneous medium. Homogeneity means that the medium is uniform throughout, whereas isotropy specifies that the elastic properties of the medium are independent of direction. Maruyama (1964) calculated all the sets of Green's functions required for the displacement and stress fields around faults in a half-space. Jungels and Frazier (1973) described a finite element variational method applied to plain strain analysis. This technique presents a suitable tool for the analysis of permanent displacements, tilts and strains caused by seismic events. The accuracy of technique was demonstrated by comparing the numerical results for the static field due to long dislocation in a homogeneous half-space from closed form analytical solution with those obtained from the finite element method. Sato (1971) and Sato and Yamashita (1975) derived the expressions for the static surface deformations due to two-dimensional strike slip and dip-slip faults located along the dipping boundary between the two different media. Freund and Barnett (1976) gave a two-dimensional analysis of surface deformation due to dip-slip faulting in a uniform half-space, using the theory of analytic functions of a complex variable.

Singh and Garg (1986) obtained the integral expressions for the Airy stress function in an unbounded medium due to various two- dimensional seismic sources. Singh et al. (1992) followed a similar procedure to obtain closed-form analytical expression for the displacements and stresses at any point of either of two homogeneous, isotropic, perfectly elastic half-spaces in welded contact due to two-dimensional sources. Singh and Rani (1991) obtained closed-form analytical expressions for the displacements and stresses at any point of a two-phase medium consisting of a homogeneous, isotropic, perfectly elastic half-space in welded contact with a homogeneous, orthotropic, perfectly elastic half-space caused by two-dimensional seismic sources located in the isotropic half-space. Bonafede and Rivalta (1999a) obtained analytical solutions for the elementary tensile dislocation problem in a layered elastic medium composed of two welded, semi-infinite half-spaces. A plain - strain configuration was considered and different rigidities and Poisson ratios were assumed for the two half-spaces. The elementary dislocation problem refers to a dislocation surface over which a jump discontinuity with constant amplitude (Burgers vector) is prescribed for the displacement field. Similar dislocation models in homogeneous half-spaces (e.g. Okada, 1992) are often employed to model dyke injection within the crust (e.g. Bonaccorso and Davis 1993), although a constant-displacement discontinuity, in general, is not the most realistic description of dyke opening. Bonafede and Rivalta (1999b) obtained the solutions for the displacement and stress fields produced by a vertical tensile crack, opening under the effect of an assigned overpressure within it, in the proximity of the welded boundary between two media characterized by different elastic parameters. Singh *et al.*(2011) obtained analytical expressions for stresses at an arbitrary point of homogeneous, isotropic, perfectly elastic half-space with rigid boundary caused by a long tensile fault of finite width.

Beginning with the expressions obtained by Singh and Garg (1986), we have obtained the integral expressions for the Airy stress function, displacements and stresses in a homogeneous, isotropic, perfectly elastic half-space by applying the boundary conditions of rigid boundary at the surface of the half-space. The integrals were then evaluated analytically, obtaining closed-form expressions for the Airy stress function, the displacements and the stresses at any point of the half-space caused by two-dimensional buried sources. The expressions for a vertical dip-slip dislocation follow immediately.

II. Theory

Let the Cartesian co-ordinates be denoted by $(x, y, z) \equiv (x_1, x_2, x_3)$ with z - axis vertical. Consider a two-dimensional approximation in which the displacement components u_1, u_2 and u_3 are independent of x so that $\partial / \partial x \equiv 0$. Under this assumption, the plane strain problem $(u_1 \equiv 0)$ can be solved in terms of the Airy stress function U such that

$$p_{22} = \frac{\partial^2 U}{\partial z^2}, \quad p_{23} = -\frac{\partial^2 U}{\partial y \partial z}, \quad p_{33} = \frac{\partial^2 U}{\partial y^2} \tag{1}$$

$$\nabla^2 \nabla^2 U = 0. \tag{2}$$

where p_{ij} are the components of stress. As shown by Singh and Garg (1986), the Airy stress function U_0 for a line source parallel to the x-axis passing through the point (0, 0, h) in an infinite medium can be expressed in the form

$$U_0 = \int_0^\infty \left[\left(L_0 + M_0 k |z-h| \right) \sin ky + \left(P_0 + Q_0 k |z-h| \right) \cos ky \right] k^{-1} e^{-k|z-h|} dk \tag{3}$$

where the source coefficients L_0, M_0, P_0 and Q_0 are independent of k. Singh and Garg (1986) have obtained these source coefficients for various seismic sources.

For a line source parallel to the x-axis acting at the point (0, 0, h) of the half-space $z \geq 0$, a suitable solution of the biharmonic equation (2) is of the form

$$U = U_0 + \int_0^\infty \left[\left(L + Mkz \right) \sin ky + \left(P + Qkz \right) \cos ky \right] k^{-1} e^{-kz} dk \tag{4}$$

where U_0 is given by the equation (3) and L, M, P and Q are unknowns to be determined from the boundary conditions. From the equations (1) and (4), the stresses and the displacements are found to be

$$\begin{aligned} p_{22} = & \int_0^\infty \left[\left(L_0 - 2M_0 + M_0 k |z-h| \right) e^{-k|z-h|} + \left(L - 2M + Mkz \right) e^{-kz} \right] \sin ky \; k \; dk \\ & + \int_0^\infty \left[\left(P_0 - 2Q_0 + Q_0 k |z-h| \right) e^{-k|z-h|} + \left(P - 2Q + Qkz \right) e^{-kz} \right] \cos ky \; k \; dk \end{aligned} \tag{5}$$

$$\begin{aligned} p_{23} = & \int_0^\infty \left[\pm \left(L_0 - M_0 + M_0 k |z-h| \right) e^{-k|z-h|} + \left(L - M + Mkz \right) e^{-kz} \right] \cos ky \; k \; dk \\ & + \int_0^\infty \left[\mp \left(P_0 - Q_0 + Q_0 k |z-h| \right) e^{-k|z-h|} + \left(P - Q + Qkz \right) e^{-kz} \right] \sin ky \; k \; dk \end{aligned} \tag{6}$$

$$\begin{aligned} p_{33} = & -\int_0^\infty \left[\left(L_0 + M_0 k |z-h| \right) e^{-k|z-h|} + \left(L + Mkz \right) e^{-kz} \right] \sin ky \; k \; dk \\ & - \int_0^\infty \left[\left(P_0 + Q_0 k |z-h| \right) e^{-k|z-h|} + \left(P + Qkz \right) e^{-kz} \right] \cos ky \; k \; dk \end{aligned} \tag{7}$$

$$2\mu u_2 = -\int_0^\infty \left[\left(L_0 - \frac{M_0}{\alpha} + M_0 k|z-h| \right) e^{-k|z-h|} + \left(L - \frac{M}{\alpha} + Mkz \right) e^{-kz} \right] \cos ky\, dk$$
$$+\int_0^\infty \left[\left(P_0 - \frac{Q_0}{\alpha} + Q_0 k|z-h| \right) e^{-k|z-h|} + \left(P - \frac{Q}{\alpha} + Qkz \right) e^{-kz} \right] \sin ky\, dk \quad (8)$$

$$2\mu u_3 = \int_0^\infty \left[\pm \left(L_0 - M_0 + \frac{M_0}{\alpha} + M_0 k|z-h| \right) e^{-k|z-h|} + \left(L - M + \frac{M}{\alpha} + Mkz \right) e^{-kz} \right] \sin ky\, dk$$
$$+\int_0^\infty \left[\pm \left(P_0 - Q_0 + \frac{Q_0}{\alpha} + Q_0 k|z-h| \right) e^{-k|z-h|} + \left(P - Q + \frac{Q}{\alpha} + Qkz \right) e^{-kz} \right] \cos ky\, dk \quad (9)$$

where the upper sign is for $z > h$ and the lower sign for $z < h$ and $\alpha = \dfrac{\lambda+\mu}{\lambda+2\mu}$.

We assume that the surface of the half-space $z \geq 0$ is with rigid boundary. Therefore, the boundary conditions are

$$u_2 = u_3 = 0 \text{ at } z = 0 \quad (10)$$

It is noticed that L_0, M_0, P_0 and Q_0 have different values for $z > h$ and $z < h$. Let L^-, M^-, P^- and Q^- be, respectively, the values of L_0, M_0, P_0 and Q_0 for $z < h$.

Equations (8) and (9) using boundary conditions of equation (10) yield

$$L = \frac{\alpha}{2-\alpha}\left[L^- - \frac{2}{\alpha}\left(1 - \frac{1}{\alpha}\right) M^- + M^- kh \right] e^{-kh}$$

$$M = \frac{\alpha}{2-\alpha}\left[2L^- - M^- + 2M^- kh \right] e^{-kh} \quad (11)$$

$$P = \frac{\alpha}{2-\alpha}\left[P^- - \frac{2}{\alpha}\left(1 - \frac{1}{\alpha}\right) Q^- + Q^- kh \right] e^{-kh}$$

$$Q = \frac{\alpha}{2-\alpha}\left[2P^- - Q^- + 2Q^- kh \right] e^{-kh}$$

Putting the values of L, M, P and Q in equations (4) to (9), we get the Airy stress function, the stresses and the displacements at any point of the half-space in the form of integrals. These integrals can be evaluated by using standard integral transforms given in Appendix. The final results are given below where we have used the notations

$$R_1^2 = y^2 + (z-h)^2,\; R_2^2 = y^2 + (z+h)^2,\; z \neq h \quad (12)$$

$$U = L_0\left[\tan^{-1}\left(\frac{y}{h-z}\right) \right] + L^-\left[\left(\frac{\alpha}{2-\alpha}\right)\tan^{-1}\left(\frac{y}{h+z}\right) + \left(\frac{\alpha}{2-\alpha}\right)\frac{2yz}{R_2^2} \right] + M_0\left[\frac{y(h-z)}{R_1^2} \right] + M^-\left[\left(\frac{2(1-\alpha)}{\alpha(2-\alpha)} \right) \right.$$
$$\times \tan^{-1}\left(\frac{y}{h+z}\right) + \left(\frac{\alpha}{2-\alpha}\right)\frac{y(h-z)}{R_2^2} + \left(\frac{\alpha}{2-\alpha}\right)\frac{4hyz(h+z)}{R_2^4} \Big] - P_o \log R_1 - P^-\left[\left(\frac{\alpha}{2-\alpha}\right)\log R_2 \right.$$
$$-\left(\frac{\alpha}{2-\alpha}\right)\frac{2z(h+z)}{R_2^2} \Big] + Q_o\left[\frac{(h-z)^2}{R_1^2} \right] + Q^-\left[\left(\frac{2}{2-\alpha}\right)\left(1 - \frac{1}{\alpha}\right)\log R_2 + \left(\frac{\alpha}{2-\alpha}\right)\frac{(h^2-z^2)}{R_2^2} \right.$$
$$\left. + \left(\frac{\alpha}{2-\alpha}\right)\frac{2hz}{R_2^2}\left(\frac{2(h+z)^2}{R_2^2} - 1 \right) \right] \quad (13)$$

$$p_{22} = L_0\left[\frac{2y(h-z)}{R_1^4} \right] + L^-\left[-\left(\frac{\alpha}{2-\alpha}\right)\frac{6y(h+z)}{R_2^4} + \left(\frac{\alpha}{2-\alpha}\right)\frac{4yz}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2} - 1 \right) \right] + M_0\left[-\frac{4y(h-z)}{R_1^4} \right.$$

$$+\frac{2y(h-z)}{R_1^4}\left(\frac{4(h-z)^2}{R_1^2}-1\right)\bigg]+M^-\left[\left(\frac{4}{2-\alpha}\right)\left(\alpha-1+\frac{1}{\alpha}\right)\frac{y(h+z)}{R_2^4}-\left(\frac{\alpha}{2-\alpha}\right)\frac{2y(3h+z)}{R_2^4}\right.$$

$$\times\left(\frac{4(h+z)^2}{R_2^2}-1\right)+\left(\frac{\alpha}{2-\alpha}\right)\frac{48hyz(h+z)}{R_2^6}\left(\frac{2(h+z)^2}{R_2^2}-1\right)\bigg]+P_o\left[\frac{1}{R_1^2}\left(\frac{2(h-z)^2}{R_1^2}-1\right)\right]$$

$$+P^-\left[\left(\frac{\alpha}{2-\alpha}\right)\frac{3}{R_2^2}\left(1-\frac{2(h+z)^2}{R_2^2}\right)+\left(\frac{\alpha}{2-\alpha}\right)\frac{4z(h+z)}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-3\right)\right]+Q_o\left[\frac{-2}{R_1^2}\right.$$

$$\times\left(\frac{2(h-z)^2}{R_1^2}-1\right)+\frac{2(h-z)^2}{R_1^4}\left(\frac{4(h-z)^2}{R_1^2}-3\right)\bigg]+Q^-\left[\left(\frac{2}{2-\alpha}\right)\left(\alpha-1+\frac{1}{\alpha}\right)\frac{1}{R_2^2}\right.$$

$$\left(\frac{2(h+z)^2}{R_2^2}-1\right)-\left(\frac{\alpha}{2-\alpha}\right)\frac{2(h+z)(3h+z)}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-3\right)+\left(\frac{\alpha}{2-\alpha}\right)\frac{12hz}{R_2^4}$$

$$\times\left(\frac{8(h+z)^4}{R_2^4}-\frac{8(h+z)^2}{R_2^2}+1\right)\bigg] \tag{14}$$

$$p_{23}=L_0\left[\frac{1}{R_1^2}\left(1-\frac{2(h-z)^2}{R_1^2}\right)\right]+L^-\left[\left(\frac{\alpha}{2-\alpha}\right)\frac{1}{R_2^2}\left(1-\frac{2(h+z)^2}{R_2^2}\right)+\left(\frac{\alpha}{2-\alpha}\right)\frac{4z(h+z)}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-3\right)\right]$$

$$+M_0\left[\frac{1}{R_2^2}\left(\frac{2(h-z)^2}{R_1^2}-1\right)-\frac{2(h-z)^2}{R_1^4}\left(\frac{4(h-z)^2}{R_1^2}-3\right)\right]+M^-\left[\left(\frac{\alpha}{2-\alpha}\right)\left(1-\frac{2}{\alpha}\left(1-\frac{1}{\alpha}\right)\right)\right.$$

$$\times\frac{1}{R_2^2}\left(\frac{2(h+z)^2}{R_2^2}-1\right)-\left(\frac{\alpha}{2-\alpha}\right)\frac{2(h+z)^2}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-3\right)+\left(\frac{\alpha}{2-\alpha}\right)\frac{12hz}{R_2^4}\left(\frac{8(h+z)^4}{R_2^4}-\frac{8(h+z)^2}{R_2^2}+1\right)\bigg]$$

$$+P_o\left[\frac{2y(h-z)}{R_1^4}\right]+P^-\left[\left(\frac{\alpha}{2-\alpha}\right)\frac{2y(h+z)}{R_2^4}-\left(\frac{\alpha}{2-\alpha}\right)\frac{4yz}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-1\right)\right]+Q_o\left[\frac{-2y(h-z)}{R_1^4}\right.$$

$$-\frac{2y(z-h)}{R_1^4}\left(\frac{4(h-z)^2}{R_1^2}-1\right)+Q^-\left[\left(\frac{\alpha}{2-\alpha}\right)\left(\frac{2}{\alpha}\left(1-\frac{1}{\alpha}\right)-1\right)\frac{2y(h+z)}{R_2^4}+\left(\frac{\alpha}{2-\alpha}\right)\frac{2y(h+z)}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-1\right)\right.$$

$$-\left(\frac{\alpha}{2-\alpha}\right)\frac{48hyz(h+z)}{R_2^6}\left(\frac{2(h+z)^2}{R_2^2}-1\right)\bigg] \tag{15}$$

$$p_{33}=L_0\left[\frac{2y(z-h)}{R_1^4}\right]-L^-\left[\left(\frac{\alpha}{2-\alpha}\right)\frac{2y(h+z)}{R_2^4}+\left(\frac{\alpha}{2-\alpha}\right)\frac{4yz}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-1\right)\right]+M_0\left[\frac{2y(z-h)}{R_1^4}\right.$$

$$\times\left(\frac{4(h-z)^2}{R_1^2}-1\right)\bigg]+M^-\left[\left(\frac{4}{2-\alpha}\right)\left(1-\frac{1}{\alpha}\right)\frac{y(h+z)}{R_2^4}-\left(\frac{\alpha}{2-\alpha}\right)\frac{2y(h-z)}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-1\right)\right.$$

$$-\left(\frac{\alpha}{2-\alpha}\right)\frac{48hyz(h+z)}{R_2^6}\left(\frac{2(h+z)^2}{R_2^2}-1\right)\bigg]+P_o\left[\frac{1}{R_1^2}\left(1-\frac{2(h-z)^2}{R_1^2}\right)\right]-P^-\left[\left(\frac{\alpha}{2-\alpha}\right)\right.$$

$$\times\frac{1}{R_2^2}\left(\frac{2(h+z)^2}{R_2^2}-1\right)+\left(\frac{\alpha}{2-\alpha}\right)\frac{4z(h+z)}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-3\right)\bigg]+Q_o\left[\frac{2(z-h)^2}{R_1^4}\left(3-\frac{4(h-z)^2}{R_1^2}\right)\right]$$

$$+Q^-\left[\left(\frac{2}{2-\alpha}\right)\left(1-\frac{1}{\alpha}\right)\frac{1}{R_2^2}\left(\frac{2(h+z)^2}{R_2^2}-1\right)+\left(\frac{\alpha}{2-\alpha}\right)\frac{2(h^2-z^2)}{R_2^4}\left(3-\frac{4(h+z)^2}{R_2^2}\right)\right.$$

$$-\left(\frac{\alpha}{2-\alpha}\right)\frac{12hz}{R_2^4}\left(\frac{8(h+z)^4}{R_2^4}-\frac{8(h+z)^2}{R_2^2}+1\right)\bigg] \tag{16}$$

$$2\mu u_2 = L_0\left[\frac{z-h}{R_1^2}\right] + L^-\left[\left(\frac{2}{\alpha}-1\right)\left(\frac{\alpha}{2-\alpha}\right)\frac{(h+z)}{R_2^2} + \left(\frac{\alpha}{2-\alpha}\right)\frac{2z}{R_2^2}\left(1-\frac{2(h+z)^2}{R_2^2}\right)\right] + M_0\left[\frac{1}{\alpha}\frac{(h-z)}{R_1^2} + \frac{(z-h)}{R_1^2}\left(\frac{2(h-z)^2}{R_2^2}-1\right)\right]$$
$$+M^-\left[\left(1-\frac{2}{\alpha}\right)\left(\frac{1}{2-\alpha}\right)\frac{(h+z)}{R_2^2} + \left(\frac{\alpha}{2-\alpha}\right)\left(z+\frac{2h}{\alpha}-h\right)\frac{1}{R_2^2}\left(\frac{2(h+z)^2}{R_2^2}-1\right) - \left(\frac{\alpha}{2-\alpha}\right)\frac{4hz(h+z)}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-3\right)\right]$$
$$+P_0\left[\frac{y}{R_1^2}\right] + P^-\left[\left(1-\frac{2}{\alpha}\right)\left(\frac{\alpha}{2-\alpha}\right)\frac{y}{R_2^2} + \left(\frac{\alpha}{2-\alpha}\right)\frac{4yz(h+z)}{R_2^4}\right] + Q_0\left[\frac{-y}{\alpha R_1^2} + \frac{2y(z-h)^2}{R_1^2}\right]$$
$$+Q^-\left[\left(\frac{2}{\alpha}-1\right)\left(\frac{1}{2-\alpha}\right)\frac{y}{R_2^2} + \left(\frac{\alpha}{2-\alpha}\right)\left(h-\frac{2h}{\alpha}-z\right)\frac{2y(h+z)}{R_2^2} + \left(\frac{\alpha}{2-\alpha}\right)\frac{4hyz}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-1\right)\right] \tag{17}$$

$$2\mu u_3 = L_0\left[\frac{-y}{R_1^2}\right] + L^-\left[\frac{y}{R_2^2} + \left(\frac{\alpha}{2-\alpha}\right)\frac{4yz(h+z)}{R_2^4}\right] + M_0\left[\left(1-\frac{1}{\alpha}\right)\frac{y}{R_1^2} - \frac{2y(z-h)^2}{R_1^4}\right] + M^-\left[\left(\frac{1}{\alpha}-1\right)\frac{y}{R_2^2}\right.$$
$$\left. +\left(\frac{\alpha}{2-\alpha}\right)\left(\frac{2h}{\alpha}-h-z\right)\frac{2y(h+z)}{R_2^4} + \left(\frac{\alpha}{2-\alpha}\right)\frac{4hyz}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-1\right)\right] + P_0\left[\frac{z-h}{R_1^2}\right] + P^-\left[\frac{z+h}{R_2^2}\right.$$
$$\left. +\left(\frac{\alpha}{2-\alpha}\right)\frac{2z}{R_2^2}\left(\frac{2(h+z)^2}{R_2^2}-1\right)\right] + Q_0\left[\left(1-\frac{1}{\alpha}\right)\frac{(h-z)}{R_1^2} + \frac{(z-h)}{R_1^2}\left(\frac{2(h-z)^2}{R_1^2}-1\right)\right] + Q^-\left[\left(\frac{1}{\alpha}-1\right)\right.$$
$$\left. \times\frac{(h+z)}{R_2^2} + \left(\frac{\alpha}{2-\alpha}\right)\left(\frac{2h}{\alpha}-h-z\right)\frac{1}{R_2^2}\left(\frac{2(h+z)^2}{R_2^2}-1\right) + \left(\frac{\alpha}{2-\alpha}\right)\frac{4hz(h+z)}{R_2^4}\left(\frac{4(h+z)^2}{R_2^2}-3\right)\right] \tag{18}$$

III. Dip-Slip Dislocation

The field due to a line dip-slip fault of arbitrary dip can be expressed in terms of the fields due to a vertical dip-slip fault and a dip-slip on a 45^0 dipping fault:

$$U = \mu bds\left[U_{(23)+(32)}\cos 2\delta + U_{(33)-(22)}\sin 2\delta\right] \tag{19}$$

IV. Vertical Dip-Slip Dislocation

From equation (19), the double couple (23) + (32) of moment D_{23} is equivalent to a vertical dip-slip line source such that

$$D_{23} = \mu bds \tag{20}$$

where b is the slip. Therefore, from Appendix II, the source coefficients for a vertical dip-slip line source are given by

$$\begin{aligned} &L_0 = P_0 = Q_0 = 0, \quad M_0 = \pm\frac{\alpha\mu bds}{\pi} \\ &L^- = P^- = Q^- = 0, \quad M^- = -\frac{\alpha\mu bds}{\pi} \end{aligned} \tag{21}$$

On putting the values of source coefficients from equation (21) into equations (13) - (18), the results for the Airy stress function, the stresses and the displacements for a vertical dip-slip are found to be:

$$U = \frac{\alpha\mu bds}{\pi}\left[\frac{y(z-h)}{R_1^2} + \frac{2(\alpha-1)}{\alpha(2-\alpha)}\tan^{-1}\left(\frac{y}{h+z}\right) + \left(\frac{\alpha}{2-\alpha}\right)\frac{y(z-h)}{R_2^2} - \left(\frac{\alpha}{2-\alpha}\right)\frac{4hyz(z+h)}{R_2^4}\right] \tag{22}$$

$$p_{22} = \frac{\alpha\mu bds}{\pi}\left[\frac{6y(h-z)}{R_1^4} - \frac{8y(h-z)^3}{R_1^6} - \left(\frac{1}{2-\alpha}\right)\left(\alpha-1+\frac{1}{\alpha}\right)\frac{4y(h+z)}{R_2^4} - \left(\frac{\alpha}{2-\alpha}\right)\frac{2y(3h+z)}{R_2^4}\right.$$
$$\left. +\left(\frac{\alpha}{2-\alpha}\right)\frac{8y(3h+z)(h+z)^2}{R_2^6} + \left(\frac{\alpha}{2-\alpha}\right)\frac{48hyz(h+z)}{R_2^6} - \left(\frac{\alpha}{2-\alpha}\right)\frac{96hyz(h+z)^3}{R_2^8}\right] \tag{23}$$

$$p_{23}=\frac{\alpha\mu bds}{\pi}\left[\frac{1}{R_1^2}-\frac{8(h-z)^2}{R_1^4}+\frac{8(h-z)^4}{R_1^6}-\left(\frac{\alpha}{2-\alpha}\right)\left(1-\frac{2}{\alpha}\left(1-\frac{1}{\alpha}\right)\right)\frac{2(h+z)^2}{R_2^4}\right.$$
$$+\left(\frac{\alpha}{2-\alpha}\right)\left(1-\frac{2}{\alpha}\left(1-\frac{1}{\alpha}\right)\right)\frac{1}{R_2^4}-\left(\frac{\alpha}{2-\alpha}\right)\frac{6(h+z)^2}{R_2^4}-\left(\frac{\alpha}{2-\alpha}\right)\frac{12hz}{R_2^4}+\left(\frac{\alpha}{2-\alpha}\right)\frac{8(h+z)^4}{R_2^6} \tag{24}$$
$$\left.+\left(\frac{\alpha}{2-\alpha}\right)\frac{96hz(h+z)^2}{R_2^6}-\left(\frac{\alpha}{2-\alpha}\right)\frac{96hz(h+z)^4}{R_2^8}\right]$$

$$p_{33}=\frac{\alpha\mu bds}{\pi}\left[\frac{2y(z-h)}{R_1^4}-\frac{8y(z-h)^3}{R_1^6}-\left(\frac{1}{2-\alpha}\right)\left(1-\frac{1}{\alpha}\right)\frac{4y(h+z)}{R_2^4}-\left(\frac{\alpha}{2-\alpha}\right)\frac{2y(h-z)}{R_2^4}\right.$$
$$\left.+\left(\frac{\alpha}{2-\alpha}\right)\frac{8y(h-z)(h+z)^2}{R_2^6}-\left(\frac{\alpha}{2-\alpha}\right)\frac{48hyz(h+z)}{R_2^6}+\left(\frac{\alpha}{2-\alpha}\right)\frac{96hyz(h+z)^3}{R_2^8}\right] \tag{25}$$

$$2\mu u_2=\frac{\alpha\mu bds}{\pi}\left[\left(1+\frac{1}{\alpha}\right)\frac{(z-h)}{R_1^2}-\frac{2(z-h)^3}{R_1^4}-\left(\frac{1}{2-\alpha}\right)\left(1-\frac{2}{\alpha}\right)\frac{(h+z)}{R_2^2}+\left(\frac{\alpha}{2-\alpha}\right)\left(z+\frac{2h}{\alpha}-h\right)\frac{1}{R_2^2}\right.$$
$$\left.-\left(\frac{\alpha}{2-\alpha}\right)\left(z+\frac{2h}{\alpha}-h\right)\frac{2(z+h)^2}{R_2^4}-\left(\frac{\alpha}{2-\alpha}\right)\frac{12hz(h+z)}{R_2^4}+\left(\frac{\alpha}{2-\alpha}\right)\frac{16hz(h+z)^3}{R_2^6}\right] \tag{26}$$

$$2\mu u_3=\frac{\alpha\mu bds}{\pi}\left[\left(\frac{1}{\alpha}-1\right)\frac{y}{R_1^2}+\frac{2y(z-h)^2}{R_1^4}+\left(1-\frac{1}{\alpha}\right)\frac{y}{R_2^2}-\left(\frac{\alpha}{2-\alpha}\right)\left(\frac{2h}{\alpha}-h-z\right)\frac{2y(h+z)}{R_2^4}\right.$$
$$\left.+\left(\frac{\alpha}{2-\alpha}\right)\frac{4hyz}{R_2^4}-\left(\frac{\alpha}{2-\alpha}\right)\frac{16hyz(h+z)^2}{R_2^6}\right] \tag{27}$$

V. Numerical Results

We study numerically the stress and the displacement field at any point of the uniform isotropic perfectly elastic half-space caused by a vertical dip-slip line source. We define the following dimensionless quantities

$$Y=\frac{y}{h}, \qquad Z=\frac{z}{h} \tag{28}$$

where h is the distance of the line source from the interface. The displacements are calculated in units of $\frac{bds}{\pi h}$ and $\frac{\mu bds}{\pi h^2}$, where b is the slip and ds is the width of the fault. Let the dimensionless stresses and displacements be denoted by U_i and P_{ij}. Then,

$$u_i=\frac{bds}{\pi h}U_i\ , \qquad p_{ij}=\frac{\mu bds}{\pi h^2}P_{ij} \tag{29}$$

From equations (22) - (27) and (28) and (29), we get the following expressions for the dimensionless stresses and displacements for a vertical dip-slip line source:

$$P_{22}=\frac{2}{3}\left[\frac{6Y(1-Z)}{A^4}-\frac{8Y(1-Z)^3}{A^6}-\frac{7Y(1+Z)}{2B^4}-\frac{Y(3+Z)}{B^4}+\frac{4Y(3+Z)(1+Z)^2}{B^6}+\frac{24YZ(1+Z)}{B^6}-\frac{48YZ(1+Z)^3}{B^8}\right] \tag{30}$$

$$P_{23}=\frac{2}{3}\left[\frac{1}{A^2}-\frac{8(1-Z)^2}{A^4}+\frac{8(1-Z)^4}{A^6}-\frac{5(1+Z)^2}{2B^4}+\frac{5}{4B^2}-\frac{3(1+Z)^2}{B^4}-\frac{6Z}{B^4}+\frac{4(1+Z)^4}{B^6}+\frac{48Z(1+Z)^2}{B^6}-\frac{48Z(1+Z)^4}{B^8}\right] \tag{31}$$

$$P_{33} = \frac{2}{3}\left[\frac{2Y(Z-1)}{A^4} - \frac{8Y(Z-1)^3}{A^6} + \frac{3(1+Z)}{2B^4} - \frac{Y(1-Z)}{B^4} + \frac{4Y(1-Z)(1+Z)^2}{B^6} - \frac{24YZ(1+Z)}{B^6} + \frac{48YZ(1+Z)^3}{B^8}\right] \tag{32}$$

$$U_2 = \frac{1}{3}\left[\frac{5(Z-1)}{2A^2} - \frac{2(Z-1)^3}{A^4} + \frac{3(Z+1)}{2B^2} + \frac{(Z+2)}{2B^2} - \frac{(Z+2)(Z+1)^2}{B^4} - \frac{6Z(Z+1)}{B^4} + \frac{8Z(Z+1)^3}{B^6}\right] \tag{33}$$

$$U_3 = \frac{1}{3}\left[\frac{Y}{2}\left(\frac{1}{A^2} - \frac{1}{B^2}\right) + \frac{2Y(Z-1)^2}{A^4} + \frac{Y(Z-2)(Z+1)}{B^4} + \frac{2YZ}{B^4} - \frac{8YZ(Z+1)^2}{B^6}\right] \tag{34}$$

where $A^2 = Y^2 + (Z-1)^2, \qquad B^2 = Y^2 + (Z+1)^2$

VI. Discussion

Figures 1.1 - 1.3 show the variation of dimensionless stresses P_{22} , P_{23} and P_{33} at the interface with the horizontal distance from the fault. Figure 1.1 shows the variation of normal stress P_{22} with distance from the fault at z = 2h, 2.5h and 3h respectively. Moreover, P_{22} tends to zero as y approaches to infinity. Figure 1.2 shows the variation of the dimensionless shear stress P_{23} with the horizontal distance from the fault at z = 2h, 2.5h and 3h respectively. At y = 0, P_{23} attains its maximum value for z = 2h and minimum value at z = 3h. P_{23} approaches to zero as y approaches to infinity. Figure 1.3 shows the variation of the dimensionless normal stress P_{33} with y at z = 2h, 2.5h and 3h. It is observed that P_{33} is zero at y = 0 and also tends to zero as y approaches to infinity. Figure 1.4 – 1.5 shows the variation of dimensionless displacements U_2 and U_3 at the interface with the horizontal distance from the fault. The variation of U_2 and U_3 for z = 3h is smooth, but for z = 2h, has sharp maxima and minima. It is noticed that the displacements U_2 and U_3 approaches to zero as y approaches to infinity.

Figure 1.6 shows the variation of dimensionless stresses P_{22} at the interface with the depth at two epicentral locations at y = 2h, 2.5h and 3h respectively. It is observed that for y = 3h , the variation is smooth but for y = 2h , P_{22} varies strongly in the range $0 < z < 2h$. Moreover it tends to zero as z approaches to infinity. Figure 1.7 shows the variation of the dimensionless shear stress P_{23} with the depth at y = 2h, 2.5h and 3h respectively. The variation of P_{23} for y =2h depends strongly on z whereas for y = 2.5h and y = 3h, the variation of stress component P_{23} is smooth. P_{23} tends to zero as z approaches to infinity. Figure 1.8 shows the variation of the dimensionless normal stress P_{33} with z at y = 2h, 2.5h and 3h. For y = 2h, P_{33} attains the maximum value. The variation is significant in the range o < z < 2h. P_{33} tends to zero as z approaches to infinity. Figure 1.9 and Figure 1.10 shows the variation of dimensionless displacements U_2 and U_3 with the depth at y = 2h, 2.5h, 3h from the fault. The variation of U_2 and U_3 for y = 3h is smooth, but for y = 2h, has sharp maxima and minima. U_2 and U_3 approache to zero as z approaches to infinity.

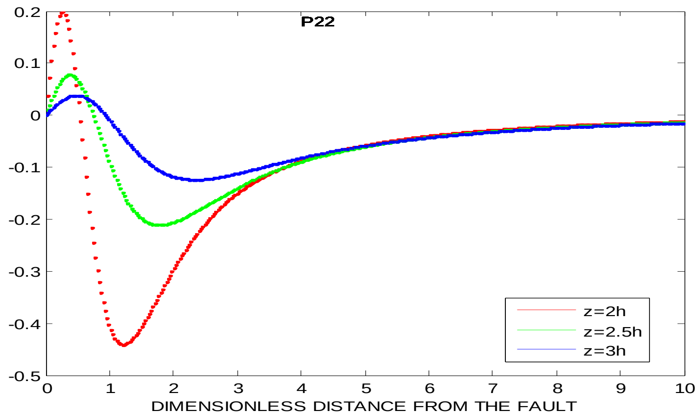

Fig. 1.1 Variation of dimensionless normal stress P_{22} with the distance from the fault

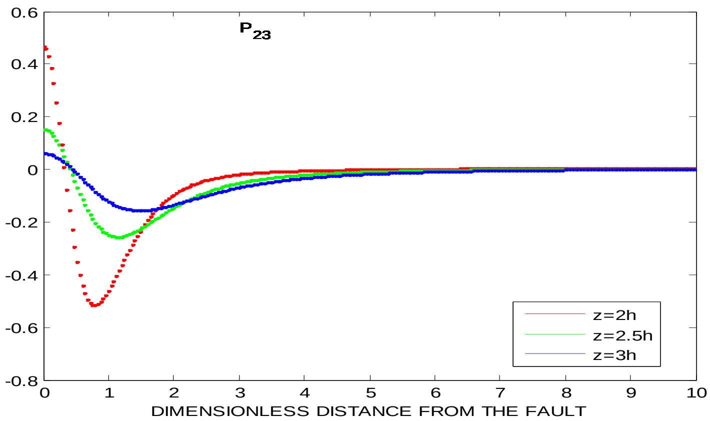

Fig. 1.2 Variation of dimensionless shear stress P_{23} with the distance from the fault

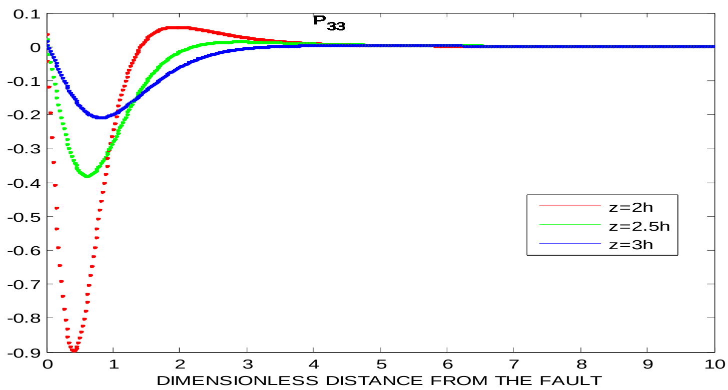

Fig. 1.3 Variation of dimensionless normal stress P_{33} with the distance from the fault

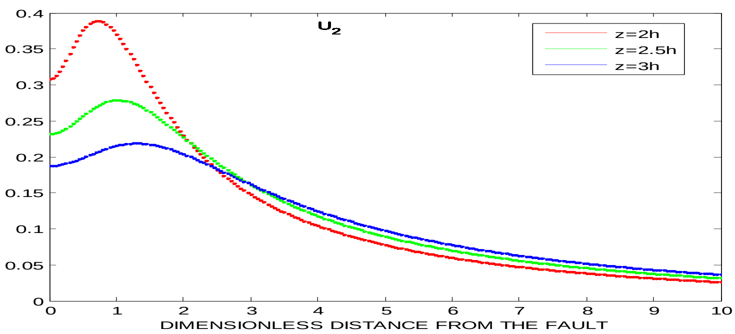

Fig. 1.4 Variation of dimensionless displacement U_2 with the distance from the fault

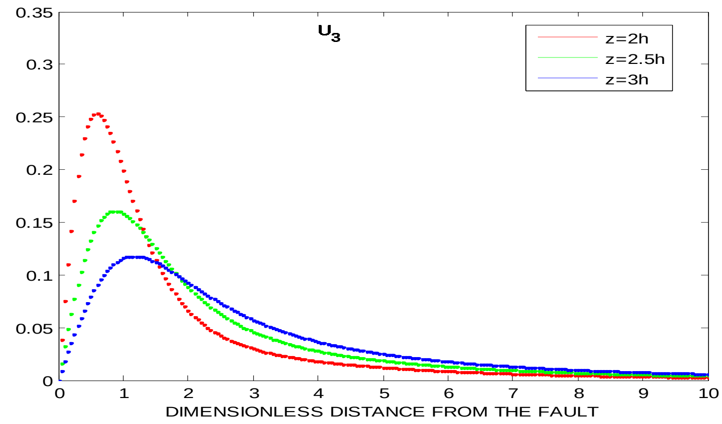

Fig. 1.5 Variation of dimensionless displacement U_3 with the distance from the fault

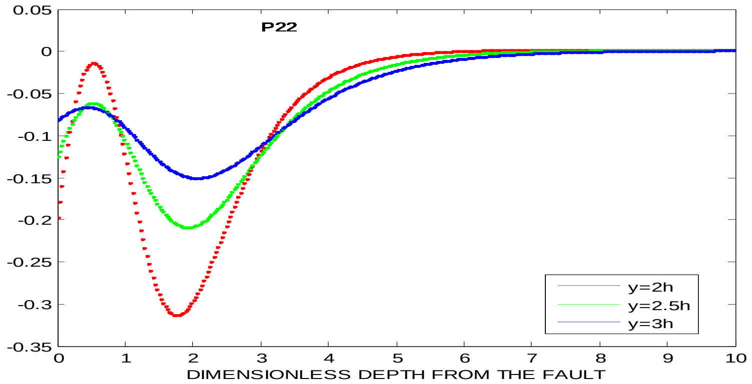

Fig. 1.6 Variation of dimensionless normal stress P_{22} with the depth from the fault

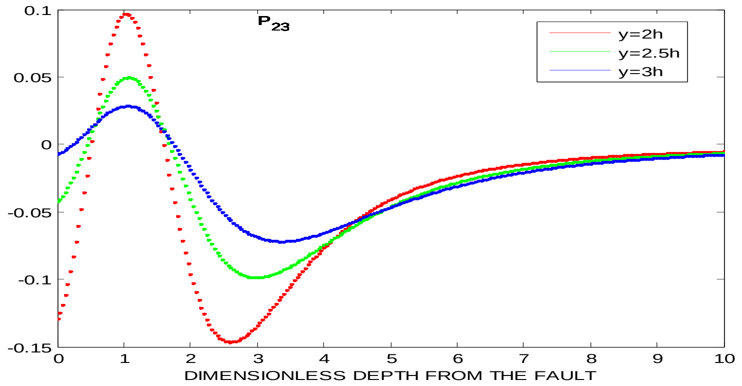

Fig. 1.7 Variation of dimensionless shear stress P_{23} with the depth from the fault

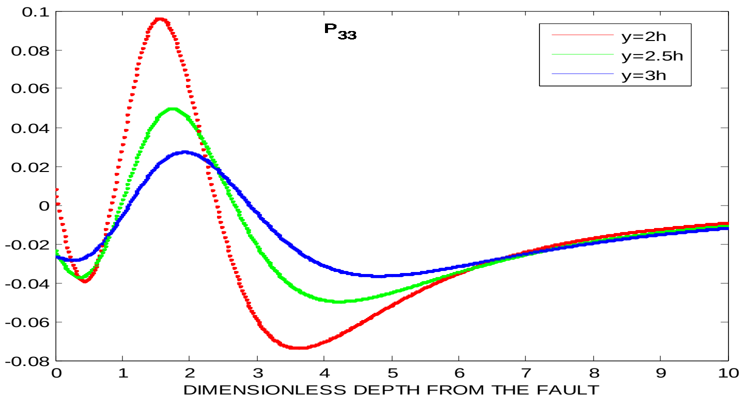

Fig. 1.8 Variation of dimensionless normal stress P_{33} with the depth from the fault

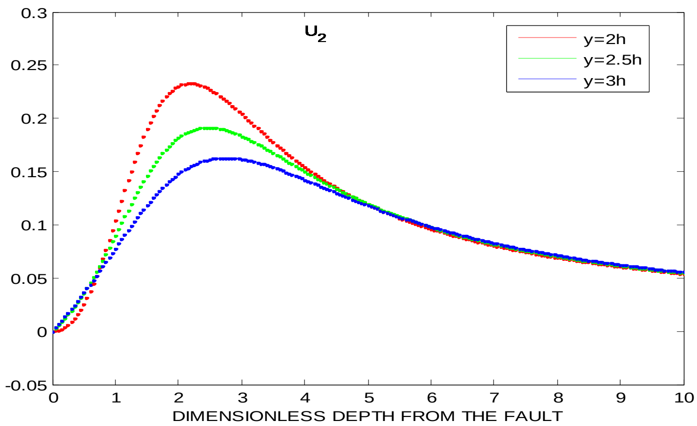

Fig. 1.9 Variation of dimensionless displacement U_2 with the depth from the fault

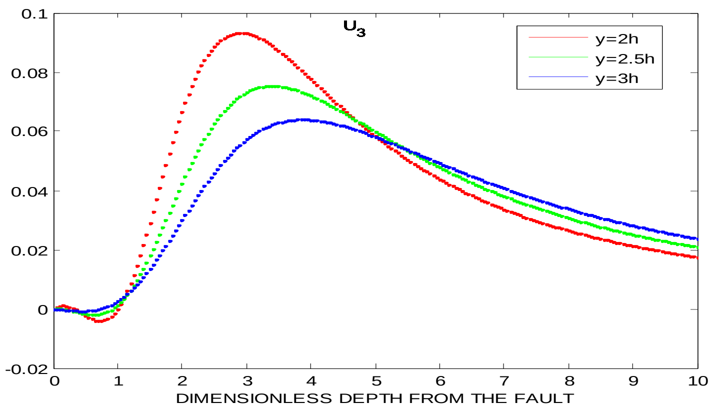

Fig. 1.10 Variation of dimensionless displacement U_3 with the depth from the fault

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Appendix I

$$\left[z>0,\ \ y^2+z^2=R^2\right]$$

i. $$\int_0^\infty e^{-kz}\frac{\sin ky}{k}dk=\tan^{-1}\left(\frac{y}{z}\right)$$

ii. $$\int_0^\infty e^{-kz}\frac{\cos ky}{k}dk=-\log R$$

iii. $$\int_0^\infty e^{-kz}\sin kydk=\left(\frac{y}{R^2}\right)$$

iv. $$\int_0^\infty e^{-kz}\cos kydk=\left(\frac{y}{R^2}\right)$$

v. $$\int_0^\infty e^{-kz}\sin kykdk=\frac{2yz}{R^4}$$

vi $$\int_0^\infty e^{-kz}\cos kykdk=\frac{1}{R^2}\left(\frac{2z^2}{R^2}-1\right)$$

vii $$\int_0^\infty e^{-kz}\sin kyk^2dk=\frac{2y}{R^4}\left(\frac{4z^2}{R^2}-1\right)$$

viii $$\int_0^\infty e^{-kz}\cos kyk^2dk=\frac{2z}{R^4}\left(\frac{4z^2}{R^2}-3\right)$$

ix $$\int_0^\infty e^{-kz}\sin kyk^3dk=\frac{24yz}{R^6}\left(\frac{2z^2}{R^2}-1\right)$$

x $$\int_0^\infty e^{-kz}\cos kyk^3dk=\frac{6}{R^4}\left(\frac{8z^4}{R^4}-\frac{8z^2}{R^2}+1\right)$$

Appendix II

Source coefficients for various sources. The upper sign is for z > h and the lower sign for z < h. $\left[\alpha=(\lambda+\mu)/(\lambda+2\mu)\right]$

Source	**L_0**	**M_0**	**P_0**	**Q_0**
Single Couple (23)	$\mp\frac{F_{23}}{2\pi}$	$\pm\alpha\frac{F_{23}}{2\pi}$	0	0
Single Couple (32)	$\pm\frac{F_{32}}{2\pi}$	$\pm\alpha\frac{F_{32}}{2\pi}$	0	0
Double Couple (23) + (32)	0	$\pm\frac{\alpha}{\pi}D_{23}$	0	0
$F_{23}=F_{32}=D_{23}$				
Centre of rotation (32) - (23)	$\pm\frac{R_{23}}{\pi}$	0	0	0
$F_{23}=F_{32}=R_{23}$				

Dipole (22)	0	0	$(1-\alpha)\frac{F_{22}}{2\pi}$	$-\frac{\alpha}{2\pi}F_{22}$
Dipole (33)	0	0	$(1-\alpha)\frac{F_{33}}{2\pi}$	$\frac{\alpha}{2\pi}F_{33}$
Centre of dilatation (22) + (33) $F_{22} = F_{33} = C_0$	0	0	$(1-\alpha)\frac{C_0}{\pi}$	0
Double Couple (33) - (22) $F_{22} = F_{33} = D'_{23}$	0	0	0	$\frac{\alpha}{\pi}D'_{23}$