Homogeneous Dusty Fluid Turbulence In A First Order Reactant For The Case Of Multi Point And Multi Time Prior To The Final Period Of Decay

M.S.Alam.Sarker[1], M.A.Bkar PK[2], M.A.K. Azad[3]

[1](Professor, Dept. of Applied Mathematics, Rajshahi University, Rajshahi- 6205, Bangladesh)
[2](Assist. Prof., Dept. of Applied Mathematics, Rajshahi University, Rajshahi- 6205, Bangladesh)
[3](Assoct. Prof., Dept. of Applied Mathematics, Rajshahi University, Rajshahi- 6205, Bangladesh)

Abstract : *Using Deissler's approach, the decay for the concentration of a dilute contaminant undergoing a first-order chemical reaction in dusty fluid homogeneous turbulence at times prior to the ultimate phase for the case of multi-point and multi-time is studied. Here two and three point correlations between fluctuating quantities have been considered and the quadruple correlations are ignored in comparison to the second and third order correlations. Taking Fourier transform the correlation equations are converted to spectral form. Finally, integrating the energy spectrum over all wave numbers we obtained the decay law for the concentration fluctuations in a homogeneous turbulence prior to the final period in presence of dust particle for the case of multi-point and multi-time.*

Keywords: *Deissler's method, Dust particle, First order reactant, Navier-Stock's equation, Turbulent flow.*

I. INTRODUCTION

Chemical kinetics deals with the rates of chemical reactions and with how the rates depend on factors such as concentration and temperature. Such studies are important in providing essential evidence as to the mechanisms of chemical processes. The essential characteristic of turbulent flows is that turbulent fluctuations are random in nature. Chemical reactions occur in the gas phase, in solution in a variety of solvents, at gas-solid and other interfaces, in the liquid state, and in the solid state. It is sometimes convenient to work with amounts of substances instead of with concentrations. Experimental methods, some of them very sophisticated, have been developed for studying the rates of these various types of reaction and even for following very rapid reactions such as explosions. Theoretical treatments also have been worked out for the various types of reaction. Experiments of this kind can be referred to as "bulk" or "bulb" experiments. Chemical reaction as used in chemistry, chemical engineering, physics, fluid mechanics, heat and mass transport. The mathematical models that describe chemical reaction kinetics provide chemists and chemical engineers with tools to better understand and describe chemicals processes such as food decomposition, stratospheric ozone decomposition, and the complex chemistry of biological systems. In recent year; the motion of dusty viscous fluids in a rotating system has developed rapidly. The motion of dusty fluid occurs in the movement of dust –laden air, in problems of fluidization, in the use of dust in a gas cooling system and in the sedimentation problem of tidal rivers. The behavior of dust particles in a turbulent flow depends on the concentrations of the particles and the size of the particles with respect to the scale of turbulent fluid.

Following Deissler's approach [1, 2], the two-point, two-time correlations are obtained by considering the equation for the concentration of a dilute contaminant undergoing a first order chemical reaction. In order to solve the equations for the final period, the triple order correlation terms are neglected in comparison to the second-order ones. Loeffer and Deissler [3] used the theory, developed by Deissler [1, 2] to study the temperature fluctuations in homogeneous turbulence before the final period. In the study of homogeneous fluid turbulence a method is describing theoretically the concentration fluctuations of dilute contaminant a first order reactant prior to the ultimate phage of decay by Kumar and Patel [4]. Kumar and Patel [5] extended their problem [4] for the case of multi-point and multi-time concentration correlation. In [6], Sarker and Kishore studied the decay of MHD turbulence at times before the final period using Chandrasekhar's relation [7].Sarker and Islam [8] discussed the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Aziz *et al.* [9] also extended their previous problem in presence of dust particle. Corrsin [10] obtained on the spectrum of isotropic temperature fluctuations in isotropic turbulence. Azad *et al.* [11] obtained first order reactant in magneto-hydrodynamic turbulence before the final period of decay in presence of dust particles. Azad *et al.* [12] also studied the statistical theory of certain distribution functions in MHD turbulent flow for velocity and concentration undergoing a first order reaction in a rotating system**.**

In thi**s** work, we studied the fluctuation of concentration of a dilute contaminant undergoing a first-order chemical reaction in homogeneous dusty fluid turbulence prior to the final phase of decay for the case of

multi-point and multi-time. Here, we have considered two-point and three-point correlation equations and solved these equations after neglecting fourth-order correlation terms. Finally we obtained the decay law of energy fluctuations of concentration of dilute contaminant undergoing a first order chemical reaction for the case of multi-point and multi-time in homogeneous dusty fluid turbulence comes out to the form

$$\left\langle X^2 \right\rangle = \exp(-2RT_m) \left\{ AT_m^{-3/2} + \exp(fQ)BT_m^{-5} \right\}$$

where $\left\langle X^2 \right\rangle$ denotes the concentration fluctuation energy. It is seen that the demolition of the impurity is more rapid than that in the case of pure mixing. This result has been shown in the figure also.

II. BASIC EQUATION

The differential equation governing the concentration of a dilute contaminant undergoing a first-order chemical reaction in dusty fluid homogeneous turbulence could be written as

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho} \frac{\partial p}{\partial x_k} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} - Ru_i + f(u_i - v_i) \tag{1}$$

The subscripts can take on the values 1, 2, and 3. Here, $u_i(\hat{x})$ is a random function of position and time at a point p, $u_k(\hat{x},t)$=turbulent velocity, R=constant reaction rate, D =diffusivity, t= time, ε_{mki}=alternating tensor, Ω_m =constant angular velocity components, $f = \frac{kN}{\rho}$, dimension of frequency, N=constant number density of dust particle, $m_s = \frac{4}{3}\pi R_s^3 \rho_s$, mass of single spherical dust particle of radius R_s, ρ_s =constant density of the material in dust particle, $p(\hat{x},t)$ =Pressure fluctuation, ρ = Fluid density, ν = Kinematics viscosity, u_k =turbulent velocity component, v_i = dust particle velocity component, x_k = space-coordinate, and repeated subscript in a term indicates a summation of terms, with the subscripts successively taking on the values 1, 2, 3.

III. TWO-POINT, TWO-TIME CORRELATION AND SPECTRAL EQUATIONS

Under the limitations that (i) the turbulence and the concentration fields are homogeneous (ii) the chemical reaction and the local mass transfer have no effect on the velocity field and (iii) the reaction rate and the diffusivity are constant, differential equation governing the concentration of a dilute contaminant undergoing a first-order chemical reaction we take the Navier-Stokes equations at the point P and the concentration equation at P′ and separated by the vector $\hat{r}$ could be written as

$$\frac{\partial X}{\partial t} + u_k \frac{\partial X}{\partial x_k} = D \frac{\partial^2 X}{\partial x_k \partial x_k} - RX \tag{2}$$

$$\frac{\partial X'}{\partial t'} + u_k' \frac{\partial X'}{\partial x_k'} = D \frac{\partial^2 X'}{\partial x_k' \partial x_k'} - RX' \tag{3}$$

where $X(\hat{x},t)$ is a random function of position and time. The other symbols are as usual.

Multiplying equation (2) by X', equation (3) by X, and averaging, we get

$$\frac{\partial \langle XX' \rangle}{\partial t} + \frac{\partial \langle u_k XX' \rangle}{\partial x_k} = D \frac{\partial^2 \langle XX' \rangle}{\partial x_k \partial x_k} - R\langle XX' \rangle \tag{4}$$

$$\frac{\partial \langle XX' \rangle}{\partial t'} + \frac{\partial \langle u_k' XX' \rangle}{\partial x_k'} = D \frac{\partial^2 \langle XX' \rangle}{\partial x_k' \partial x_k'} - R\langle XX' \rangle \tag{5}$$

where the conditions of continuity and the fact that the quantities at a point at a particular time are independent of the positions at the other points have been utilized.

Using the transformations. $\frac{\partial}{\partial x_k} = -\frac{\partial}{\partial r_k}, \frac{\partial}{\partial x_k'} = \frac{\partial}{\partial r_k}, \frac{\partial}{\partial t} = \left(\frac{\partial}{\partial t}\right)_{\Delta t} - \frac{\partial}{\partial \Delta t}, \frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t},$

in to equations (4) and (5), we obtains

$$\frac{\partial \langle XX' \rangle}{\partial t} + \frac{\partial \langle u_k XX' \rangle}{\partial r_k}(-\hat{r}, -\Delta t, t + \Delta t) - \frac{\partial \langle u_k XX' \rangle}{\partial r_k}(\hat{r}, \Delta t, t) = 2D \frac{\partial^2 \langle XX' \rangle}{\partial r_k \partial r_k} - 2R\langle XX' \rangle \tag{6}$$

$$\frac{\partial \langle XX' \rangle}{\partial \Delta t} + \frac{\partial \langle u_k XX' \rangle}{\partial r_k}(-\hat{r}, -\Delta t, t + \Delta t) = D \frac{\partial^2 \langle XX' \rangle}{\partial r_k \partial r_k} - R\langle XX' \rangle \tag{7}$$

In order to reduce Eqs. (6) and (7) to spectral form by using three-dimensional Fourier transform

$$\langle XX'(\hat{r},\Delta t,t)\rangle = \int_{-\infty}^{\infty}\theta(\hat{k},\Delta t,t)\exp(i\hat{k}.\hat{r})d\hat{k} \tag{8}$$

$$\langle XX'(\hat{r},\Delta t,t)\rangle = \int_{-\infty}^{\infty}\phi_k(\hat{k},\Delta t,t)\exp(i\hat{k}.\hat{r})d\hat{k} \cdot \tag{9}$$

We get

$$\frac{\partial\theta}{\partial t} + (2Dk^2 + 2R)\theta = ik_k\phi_k(\hat{k},\Delta t,t) + i(-k_k)\phi_k(-\hat{k}_k,-\Delta t,t+\Delta t) , \tag{10}$$

$$\frac{\partial\theta}{\partial\Delta t} + (Dk^2 + R)\theta = -ik_k\phi_k(-\hat{k},-\Delta t,t+\Delta t) \cdot \tag{11}$$

IV.   Solution for the Ultimate Phase of Decomposing Turbulence

For the ultimate phase of homogeneous turbulence decompose, the third-order correlations can be ignored in comparison to the second-order correlations, with this approximation the solutions of Eqs. (10) and (11) may be obtained as

$$\theta = f_1(\hat{k},\Delta t)\exp[-(2Dk^2 + 2R)(t - t_0)] \tag{12}$$

$$\theta = f_2(\hat{k},t)\exp[-(Dk^2 + R)\Delta t]. \tag{13}$$

For consistent solution of Eqs (12) and (13) we must have

$$G(k) = f(k)\exp[(-2Dk^2 + 2R)(t - t_0 + \frac{\Delta t}{2})] . \tag{14}$$

where $G(k) = 2\pi k^2\theta$ is the concentration spectrum function. We evaluate f (k) by Corrsion [10]

i.e. $f(k) = N_o k^2/\pi$ .where N_0 is a constant depend on initial condition. Thus, we obtain

$$G(k) = \frac{N_0 k^2}{\pi}\exp[(-2Dk^2 + 2R)(t - t_0 + \frac{\Delta t}{2})] \tag{15}$$

By integrating equation (15) with respect to k, we obtain

$$\langle XX'\rangle(\hat{r},t_m) = \frac{N_o D^{1/2}}{4\sqrt{4\pi}(t_m - t_o)^{3/2}}\exp\left\{-\left[\frac{2C(t_m - t_o) + r^2}{8D(t_m - t_o)}\right]\right\} , \quad \text{where } t_m = t + \Delta t/2. \tag{16}$$

V.   Three-point, Three-time Correlation and Spectral Equations

Under the same assumptions as before, we take the Navier-Stokes equation for dusty fluid homogeneous turbulence at the point P and the concentration equations at P' and P'' as

$$\frac{\partial u_i}{\partial t} + u_k\frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_k} + \nu\frac{\partial^2 u_i}{\partial x_k\partial x_k} - Ru_i + f(u_i - v_i) \tag{17}$$

$$\frac{\partial X'}{\partial t'} + u_k'\frac{\partial X'}{\partial x_k'} = D\frac{\partial^2 X'}{\partial x_k'\partial x_k'} - RX' \tag{18}$$

$$\frac{\partial X''}{\partial t''} + u_k''\frac{\partial X''}{\partial x_k''} = D\frac{\partial^2 X''}{\partial x_k''\partial x_k''} - RX'' \tag{19}$$

Multiplying equation (17) by XX'', (18) by $u_i X''$ and (19) by $u_i X'$ and then taking space averages, we obtain.

$$\frac{\partial}{\partial t}\langle u_i X'X''\rangle + \frac{\partial}{\partial x_k}\langle u_i u_k X'X''\rangle = -\frac{1}{\rho}\frac{\partial}{\partial x_k}\langle pX'X''\rangle + \nu\frac{\partial^2}{\partial x_k\partial x_k}\langle u_i X'X''\rangle - R\langle u_i X'X''\rangle + f[\langle u_i X'X''\rangle - \langle v_i X'X''\rangle] \tag{20}$$

$$\frac{\partial}{\partial t'}\langle u_i X'X''\rangle + \frac{\partial}{\partial x_k'}\langle u_i u_k' X'X''\rangle = D\frac{\partial^2}{\partial x_k'\partial x_k'}\langle u_i X'X''\rangle - R\langle u_i X'X''\rangle \tag{21}$$

$$\frac{\partial}{\partial t''}\langle u_i X'X''\rangle + \frac{\partial}{\partial x_k''}\langle u_i u_k'' X'X''\rangle = D\frac{\partial^2}{\partial x_k''\partial x_k''}\langle u_i X'X''\rangle - R\langle u_i X'X''\rangle \tag{22}$$

Using the transformations

$$\frac{\partial}{\partial x_k} = -(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'}), \frac{\partial}{\partial x_k'} = \frac{\partial}{\partial r_k}, \frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'}, \frac{\partial}{\partial t}t',t'' = \left(\frac{\partial}{\partial t}\right)_{\Delta t,\Delta t'} - \frac{\partial}{\partial\Delta t} - \frac{\partial}{\partial\Delta t'}, \frac{\partial}{\partial t'} = \frac{\partial}{\partial\Delta t}, \frac{\partial}{\partial t''} = \frac{\partial}{\partial\Delta t'}$$

Into equations (20)-(22), we get

$$\frac{\partial}{\partial t}\langle u_i X'X''\rangle - (\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'})\langle u_i u_k X'X''\rangle + \frac{\partial}{\partial r_k}\langle u_i u_k' X'X''\rangle + \frac{\partial}{\partial r_k'}\langle u_i u_k'' X'X''\rangle$$

$$= -\frac{1}{\rho}(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'})\langle pX'X''\rangle + \nu(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'})^2\langle u_i X'X''\rangle + D(\frac{\partial^2}{\partial r_k \partial r_k}+\frac{\partial^2}{\partial r_k'\partial r_k'})\langle u_i X'X''\rangle$$

$$-(2R)\langle u_i X'X''\rangle + f[\langle u_i X'X''\rangle - \langle v_i X'X''\rangle] \tag{23}$$

$$\frac{\partial}{\partial \Delta t}\langle u_i X'X''\rangle + \frac{\partial}{\partial r_k}\langle u_i u_k' X'X''\rangle = D\frac{\partial^2}{\partial r_k'\partial r_k'}\langle u_i X'X''\rangle - R\langle X' u_i X''\rangle \tag{24}$$

$$\frac{\partial}{\partial \Delta t'}\langle u_i X'X''\rangle + \frac{\partial}{\partial r_k'}\langle u_i u_k'' X'X''\rangle = D\frac{\partial^2}{\partial r_k'\partial r_k''}\langle u_i X'X''\rangle - R\langle X' u_i X''\rangle \tag{25}$$

Using the six-dimensional Fourier transform of the type

$$\langle XX' u_k''(\hat{r},\hat{r}',\Delta t,\Delta t',t)\rangle = \int_{-\infty}^{\infty}.\int_{-\infty}^{\infty}\psi_k(\hat{k},\hat{k}',\Delta t,\Delta t',t)\exp(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')d\hat{k}d\hat{k}'$$

and with the fact that, $\langle X'X''v_k\rangle = \int_{-\infty}^{\infty}.\int_{-\infty}^{\infty}\psi_k'(\hat{k},\hat{k}',\Delta t,\Delta t',t)\exp(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')d\hat{k}d\hat{k}'$

and the assumption that the quintuple correlations representing the transfer terms in equations (23)-(25) can be neglected as they decay faster than the lower-order correlation terms. Then the equations (23) - (25) in Fourier space can be written as

$$\frac{\partial \psi_i}{\partial t}(\hat{k},\hat{k}',\Delta t,\Delta t',t) + D[(1+N_S)k^2 + 2N_s kk'\cos\{(1+N_s)k'^2 + (2R-fQ)/D\}]\psi_i(\hat{k},\hat{k}',\Delta t,\Delta t',t) = 0 \tag{26}$$

$$\frac{\partial \psi_i}{\partial \Delta t}(\hat{k},\hat{k}',\Delta t,\Delta t',t) + D[(k^2+2R)/D]\psi_i(\hat{k},\hat{k}',\Delta t,\Delta t',t) = 0 \tag{27}$$

$$\frac{\partial \psi_i}{\partial \Delta t'}(\hat{k},\hat{k}',\Delta t,\Delta t',t) + D[(k'^2+2R)/D]\psi_i(\hat{k},\hat{k}',\Delta t,\Delta t',t) = 0 \tag{28}$$

where $N_s = \nu/D$, the Schmidt number and $\langle X'X''v_l\rangle = L\psi_i'\langle X'X''v_l\rangle$, $1-L=Q$.

As the pressure force terms are related to higher-order correlations, therefore, these along with the quadruple correlations are also neglected.

Integrating equations (26)-(28) between t_o and t, we obtain

$$\psi_i = f_i \exp\{-D[(1+N_s)k^2 + 2N_s kk'\cos\theta + (1+N_s)k'^2 + (2R-fQ)/D](t-t_0)\} \ \psi_i = g_i \exp\{[-D(k^2+R/D)]\Delta t\}$$

$$\psi_i = h_i \exp\{[-D(k'^2+R/D)]\Delta t'\}.$$

For these relations to be consistent, we have

$$k_i\psi_i = k_i(\psi_i)_0 \exp[-D\{(1+N_s)(k^2+k'^2)(t-t_0) + k^2\Delta t + k'^2\Delta t' + 2N_s kk'\cos\theta(t-t_0)$$

$$+(2R/D)[t-t_0+(\Delta t+\Delta t')/2]+[(-fQ)/D](t-t_0)\}] \tag{29}$$

where the subscript 0 refers to the value of ψ_i at $t=t_o$, $\Delta t=\Delta t'=0$ and θ is the angle between k

and k'. The relation between ϕ_i and ψ_i is given by

$$k_i\phi_i(\hat{k},\Delta t,t) = \int_{-\infty}^{\infty}k_i\psi_i(\hat{k},\Delta t,\hat{k}',0,t)dk' \tag{30}$$

Substituting equations (30) and (29) into equation (10), we obtain

$$\frac{\partial G}{\partial t} + (2k^2 D + 2R)G = W \tag{31}$$

where $G = 2\pi k^2 \theta$ and

$$W = \int_0^{\infty}ik_i(\psi_i)_0 k^2 k'^2 (2\pi)^2 \exp\left\{-D\left[(1+N_S)k^2\left(t-t_o+\frac{\Delta t}{t+N_S}\right)+k'^2(1+N_S)(t-t_o)\right.\right.$$

$$+[(-fQ)/D](t-t_0)+(2R/D)\left(t-t_o+\frac{\Delta t}{2}\right)\right\}\left[\int_{-1}^{1}\exp[-2N_s Dkk'(t-t_0)\cos\theta]dk'$$

$$+\int_0^{\infty}[i(-k_i)\psi_i(-\hat{k},-\hat{k}')]_0(2\pi)^2 k^2 k'^2 \exp\{-D\left[(1+N_S)k^2\left(t-t_o+\frac{\Delta t}{1+N_S}\right)\right.$$

$$-k'^2(1+N_s)(t-t_0+\Delta t)+[(-fQ)/D](t-t_0)+\frac{2R}{D}(t-t_0+\frac{\Delta t}{2})]\}$$

$$\times \int_{-1}^{1}\exp[-2N_s Dkk'(t-t_0+\Delta t)\cos\theta](d\cos\theta)dk' \tag{32}$$

where dk' is written as $2\pi k'^2 d(\cos\theta)dk'$ and the quantity $(\psi_i)_0$ depends on the initial conditions of the turbulence. Now, following Deissler [1, 2], we take

$$(2\pi)^2 i[k_i\psi_i(\hat{k},\hat{k}')_0 = -\frac{1}{2}\delta_0(k^2 k'^4 - k^4 k'^2) \quad \text{and}$$

$$(2\pi)^2 i[-k_i\psi_i(-\hat{k},-\hat{k}')_0 = \frac{1}{2}\delta_0(k^2 k'^4 - k^4 k'^2) \tag{33}$$

Substituting equation (33) in (32) and completing the integration, we get

$$W = -\frac{\delta_o N_s \pi^{1/2}}{4D^{3/2}(t-t_o)^{3/2}(1+N_s)^{5/2}}\exp\left[-k^2 D\frac{1+2N_s}{1+N_s}\left(t-t_o+\frac{1+N_s}{1+2N_s}\Delta t\right)\right.$$

$$\left.+(-fQ)(t-t_0)-2R\left(t-t_o+\frac{\Delta t}{2}\right)\right]\left\{\frac{15k^4}{4N_s^2(t-t_o)^2 D^2}\frac{N_s}{1+N_s}+\left[5\left(\frac{N_s}{1+N_s}\right)^2-\frac{3}{2}\right]\right.$$

$$\times\frac{k^6}{N_s D(t-t_o)}+\left[\left(\frac{N_s}{1+N_s}\right)^3-\frac{N_s}{1+N_s}\right]k^8\right\}-\frac{\delta o N_s \sqrt{\pi}}{4D^3/2(t-t_o+\Delta t)^{3/2}(t+N_s)^{5/2}}$$

$$\times\exp\left[-k^2 D\frac{1+2N_s}{1+N_s}\left(t-t_o+\frac{N_s}{+1+N_s}\Delta t\right)+(-fQ)(t-t_0)-2R\left(t-t_o-\frac{\Delta t}{2}\right)\right]$$

$$\times\left\{\frac{15k^4}{4D^2 N_s^2(t-t_o+\Delta t)^2}\left(\frac{N_s}{1+N_s}\right)+\left[5\left(\frac{N_s}{1+N_s}\right)2-\frac{3}{2}\right]\frac{k^6}{N_s D(t-t_o+\Delta t)}\right.$$

$$\left.+\left[\left(\frac{N_s}{1+N_s}\right)^3-\frac{N_s}{1+N_s}\right]k^8\right\}. \tag{34}$$

This represents the transfer function arising due to the consideration of concentration at three- point and three- time. When $\Delta t = 0$ and $R = 0$, the expression for reduces to the case of pure mixing .It may also be noted that (for $\Delta t = 0$)

$$\int_0^\infty W.dk = 0 \tag{35}$$

This means that the conditions of continuity and homogeneity are satisfied. Physically, it was to be expected as *W* is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero. With the help of equations (31) and (34), one can get

$$G = \frac{N_0 k^2}{\pi}\exp\left\{-2(k^2 D+2R)\left(t-t_o+\frac{\Delta t}{2}\right)\right\}+\frac{\sqrt{\pi}N_s}{D^{3/2}(1+N_s)^{7/2}}\frac{\delta_o}{4}$$

$$\times\exp\left[-D\frac{1+2N_s}{1+N_s}k^2\left(t-t_o+\frac{(1+N_s)\Delta t}{1+2N_s}\right)+(-fQ)(t-t_0)-2R\left(t-t_o+\frac{\Delta t}{2}\right)\right]$$

$$\times\left[\frac{3k^4}{2N_s D^2(t-t_o)^{5/2}}+\frac{(7N_s-6)k^6}{3D(1+N_s)(t-t_o)^{3/2}}-\frac{4(3N_s^2-2N_s+3)k^8}{3(1+N_s)^2(t-t_o)^{1/2}}\right.$$

$$\left.+\frac{8D^{1/2}(3N_s^2-2N_s+3)k^9}{3(1+N_s)^{5/2}}F\left(k\sqrt{\frac{(t-t_o)D}{1+N_s}}\right)\right]+\frac{\sqrt{\pi}N_s}{D^{3/2}(1+N_s)^{7/2}}\frac{\delta_o}{4}$$

$$\times\exp\left[-D\left(\frac{1+2N_s}{1+N_s}\right)k^2\left(t-t_o+\frac{N_s}{1+2N_s}\Delta t\right)+(-fQ)(t-t_0)-2R\left(t-t_o+\frac{\Delta t}{2}\right)\right]$$

$$\times \left[\frac{3k^4}{2D^2 N_s (t-t_o+\Delta t)^{5/2}} + \frac{(7N_s-6)k^6}{3D(1+N_s)(t-t_o+\Delta t)^{3/2}} - \frac{4}{3}\frac{(3N_s^2-2N_s+3)k^8}{(1+N_s)^2(t-t_o+\Delta t)^{1/2}} \right.$$

$$\left. + \frac{8D^{1/2}(3N_s^2-2N_s+3)k^9}{(1+N_s)^{5/2}} F\left(k\sqrt{\frac{(t-t_o+\Delta t)D}{1+N_s}}\right) \right] \tag{36}$$

where, $F(\omega) = \exp(-\omega^2)\int_0^\omega \exp(x^2)dx$, $\omega = k\sqrt{\frac{(t-t_o)D}{1+N_s}}$ or $k\sqrt{\frac{(t-t_o+\Delta t)D}{1+N_s}}$

As in the previous section, by integrating equation (36) with respect to k, we obtain

$$\left\langle \frac{XX'}{2}(\Delta t, t_m)\right\rangle = \int_0^\infty G dk = \frac{N_o}{8D^{3/2}\sqrt{2\pi}\left(T+\frac{\Delta t}{2}\right)^{3/2}} \exp\left[-2R\left(T+\frac{\Delta t}{2}\right)\right]$$

$$+ \frac{\pi}{D^6(1+N_s)(1+N_s)^{5/2}} \exp[(fQ)]\exp[-2R(T+\frac{\Delta t}{2})]$$

$$\times \frac{\delta_0}{4}\left[\frac{9}{16T^{5/2}\left(T+\frac{1+N_s}{1+2N_s}\Delta T\right)^{5/2}} + \frac{9}{16(T+\Delta T)^{5/2}\left(T+\frac{1+N_s}{1+2N_s}\Delta T\right)^{5/2}} \right.$$

$$+ \frac{5N_s(7N_s-6)}{16(1+2N_s)T^{3/2}\left(T+\frac{1+N_s}{1+2N_s}\Delta T\right)^{7/2}} + \frac{5N_s(7N_s-6)}{16(1+2N_s)(T+\Delta T)^{3/2}\left(T+\frac{N_s}{1+2N_s}\Delta T\right)^{7/2}}$$

$$+ \frac{35N_s(3N_s^2-2N_s+3)}{8(1+2N_s)T^{1/2}\left(T+\frac{1+N_s}{1+2N_s}\Delta T\right)^{9/2}} + \frac{35N_s(3N_s^2-2N_s+3)}{8(1+2N_s)(T+\Delta T)^{1/2}\left(T+\frac{N_s}{1+2N_s}\Delta T\right)^{9/2}}$$

$$+ \frac{8N_s(3N_s^2-2N_s+3)(1+2N_s)^{5/2}}{3.2^{23/2}(1+N_s)^{11/2}} \sum_{n=0}^\infty \frac{1.3.5\ldots(2n+9)}{n!(2n+1)2^{2n}(1+N_s)^n} \times \left. \frac{T^{(2n+1)/2}}{\left(T+\frac{\Delta T}{2}\right)^{(2n+11)/2}} + \frac{(T+\Delta T)^{(2n+1)/2}}{\left(T+\frac{\Delta T}{2}\right)^{(2n+1)/2}} \right] \tag{37}$$

where, $T = t - t_o$. For $T_m = T + \Delta T/2$, equation (37) becomes

$$\left\langle \frac{XX'}{2}(t_m)\right\rangle = \exp(-2RT_m)\frac{N_0}{8\sqrt{2\pi}D^{3/2}T^{3/2}{}_m} + \frac{\delta_o\pi}{4D^6(1+N_s)(1+2N_s)}\exp[fQ]\frac{9}{16}\frac{1}{\left(T_m-\frac{\Delta T}{2}\right)^{5/2}\left(T_m+\frac{\Delta T}{1+2N_s}\right)^{5/2}}$$

$$+ \frac{9}{16(T_m+\frac{\Delta T}{2})^{5/2}(T_m-\frac{\Delta T}{2})^{5/2}} + \frac{5N_s(7N_s-6)}{16(1+2N_s)}\frac{1}{(T_m+\frac{\Delta T}{2})^{7/2}(T_m-\frac{\Delta T}{2})^{3/2}} + \frac{5N_s(7N_s-6)}{16(1-2N_s)}\frac{1}{\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{1+2N_s}\right)^{7/2}} +\ldots \tag{38}$$

If $\Delta t = 0$, then equation (38) reduces to the form

$$\left\langle \frac{X^2}{2}\right\rangle = \exp(-2RT_m)\left\{ \frac{N_o}{8\sqrt{2\pi}D^{3/2}}\frac{1}{T_m^{3/2}} + \frac{\delta_o\pi}{2D^6(1+N_s)(1+2N_s)}\exp[fQ]\left[\frac{9}{16T_m^5} + \frac{5}{16}\frac{N_s(7N_s-6)}{(1+2N_s)T_m^5}\right]\right\}$$

Therefore, $\left\langle X^2\right\rangle = \exp(-2RT_m)\left[\frac{N_o}{4\sqrt{2\pi}D^{3/2}}\frac{3}{T_m^{3/2}} + \exp[fQ]\frac{\delta_o\alpha}{D^6}\frac{1}{T_m^5}\right]$ (39)

where, $\alpha = \frac{\pi}{(1+N_s)(1+2N_s)}\left[\frac{9}{16} + \frac{5}{16}\frac{N_s(7N_s-6)}{(1+2N_s)} +\ldots\right]$.

Thus, the decay law for the concentration energy fluctuation of dusty fluid homogeneous turbulence in a first order reactant for multi-point and multi-time prior to the ultimate phase may be written as

$$\left\langle X^2\right\rangle = \exp(-2RT_m)\left\{AT_m^{-3/2} + \exp[fQ]BT_m^{-5}\right\} \tag{40}$$

where $A = \dfrac{3N_0}{4\sqrt{2\pi}D^{3/2}}$, $B = \dfrac{\delta_0 \alpha}{D^6}$

In equation (40) we obtained the concentration fluctuation energy of dusty fluid homogeneous turbulence. In the absence of dust particles the equation (40) becomes

$$\langle X^2 \rangle = \exp(-2RT_m)\left\{ AT_m^{-3/2} + BT_m^{-5} \right\} \tag{41}$$

Which was obtained earlier by Kumar and Patel [5].For large times, the last term of equation (41) becomes negligible and the decay law for the ultimate period becomes $\exp(-2RT_m)(AT_m^{-3/2})$ which in the case of pure-mixing is similar to the law obtained by Corrsin [12].

In Figs. 1-4 we observe that the variation of chemical reaction in presence of dust particle i.e. for *exp (fQ)* =.75, .50, .25, 0 causes significant changes in the concentration fluctuation decay of energy of homogeneous turbulence. In the presence of dust particles the energy decay of the fluid particles more rapidly which indicated in the Figs. 3-1 respectively. In Fig. 4, we observe that in the absence of dust particles energy decay more slowly than with the present of dust particles. It is noted that *y1,y2,y3,y4,y5,y6* and *y7* are solution curves of equation (40) but in the absence of dust particles *y1,y2,y3,y4,y5,y6 , y7* are represented by equation (41) at the different values of *R* and dust particles and plotted are shown from Figs. 1 -3 and Fig. 4 respectively.

VI. Figures
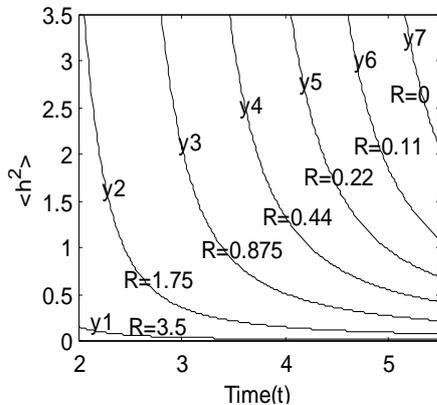
Fig. 1.Energy decay curves for exp (*Qf*) = 0.75.
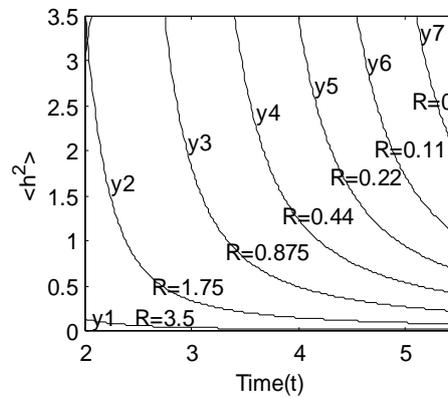
Fig. 2.Energy decay curves for exp (*Qf*) = 0.50.
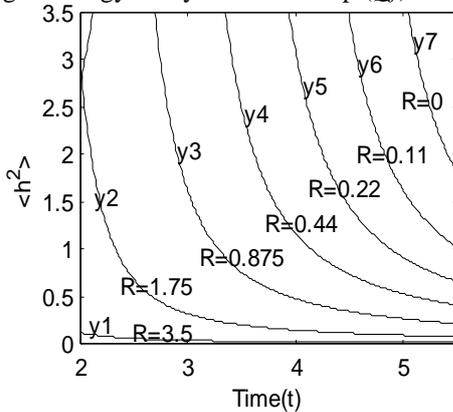
Fig. 3.Energy decay curves for exp (*Qf*) = 0.25.
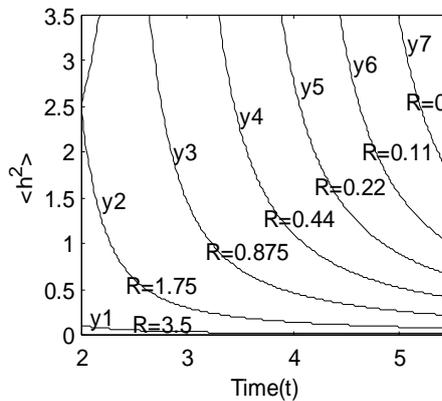
Fig. 4.Energy decay curves for exp (*Qf*) = 0.

VII. CONCLUSION

In the case of pure mixing, the concentration fluctuation decays with time in a natural manner. This study shows that if the concentration selected is the chemical reactant of the first order, then the effect is that the decomposition of the concentration fluctuation in homogeneous turbulence in the presence of dust particle for the case of multi-point and multi-time is much more rapid and the faster rate of decomposition is governed by $\exp(-2RT_m)$ .The decomposition of the concentration fluctuation in homogeneous turbulence is more slowly due to the absence of dust particles than any other type of chemical reactant as stated above. In a normal way, it takes a lot of time to get rid of a pollutant in the fluid. From the above figures and discussion, we conclude that in the absence of dust particles energy decay of the fluid particles more slowly but in the presence of dust particles the decomposition of the concentration fluctuation for the case of multi-point and multi-time in

homogeneous turbulence are decreases due to the increases of the first order chemical reaction and maximum at the point where the chemical reaction is zero.

REFERENCES

[1]   R. G. Deissler, On the decay of homogeneous turbulence before the final period. *Phys. Fluids* 1(2), 1958, 111-121.
[2]   R. G. Deissler, A theory of decaying homogeneous turbulence, Phys. Fluids 3(2), 1960, 176-187.
[3]   A.L. Loeffler, and R.G. Deissler, Decay of temperature fluctuations in homogeneous turbulence before the final period, Int .J. Heat Mass Transfer, 1, 1961, 312-324.
[4]   P. Kumar, and S.R. Patel, First-order reactant in homogeneous turbulence before the final period of decay. Phys .Fluids, 17, 1974, 1362-1368.
[5]   P. Kumar, and S.R. Patel, On first-order reactants in homogeneous turbulence, Int. J. Eng. Sci., 13,1975, 305-315.
[6]   M .S. A. Sarker, and N. Kishore, Decay of MHD turbulence before the final period, Int.J. Engeng. Sci., 29, 1991, 1479-1485.
[7]   S., Chandrasekhar, The invariant theory of isotropic turbulence in magneto-hydrodynamics, Proc. Roy.Soc., London, A204, 1951, 435-449.
[8]   M. S. A. Sarker, and M. A. Islam, Decay of MHD turbulence before the final period for the case of multi-point and multi-time, Indian J. pure appl. Math.,32,2001,1065-1076.
[9]   M.A.Aziz, M.A.K Azad, and M.S. Alam Sarker, First Order Reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system in presence of dust particle. Res. J. Math. Stat., 2, 2010, 56-68.
[10]   S., Corrsin, On the spectrum of isotropic temperature fluctuations in isotropic turbulence.J.Apll.Phys**,** 22, 1951, 469-473.
[11]   M. A. K. Azad, M. A. Aziz, and M. S. Alam Sarker, First Order Reactant in Magneto-hydrodynamic Turbulence before the Final Period of Decay in presence of dust particles, Bangladesh Journal of Scientific & Industrial Research, 45(1) 2010, 39-46.
[12]   M. A. K Azad, M. A. Aziz, and M. S. Alam Sarker, Statistical Theory of certain Distribution Functions in MHD Turbulent flow for Velocity and Concentration Undergoing a First Order Reaction in a Rotating System, Bangladesh Journal of Scientific & Industrial Research 46(1), 2011, 59-68.