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Some properties of Fuzzy Derivative (I)

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Abstract: *In [3], the fuzzy derivative was defined by using Caratheodory's derivative notion and a few basic properties of fuzzy derivative was proved. In this paper, we will a completion to prove for some properties of the subject and discussion Rolle's theorem and Generalized Mean -Value Theorem in fuzzy derivative and we given some applications of the Mean Value Theorem.*
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I. Preliminaries

Definition 1.1.[5] Let X be a vector space over the field F of real or complex numbers, (X,T) be a fuzzy topological space, if the two mappings $X \times X \to X$, $(x,y) \mapsto x+y$ and $X \times F \to X$, $(\alpha,x) \mapsto \alpha x$ are fuzzy continuous, where F is the induced fuzzy topology of the usual norm, then (X,T) is said to be a fuzzy topological vector space over the field F .

Definition 1.2. (Caratheodory). Let $f:(a,b) \subseteq R \to R$ be a function and $c \in (a,b)$, then f is said to be differentiable at a point c if there exist a function U_c that is continuous at $x=c$ and satisfies the relation $f(x)-f(c)=U_c(x)(x-c)$ for all $x \in (a,b)$.

We will usually write $U(x)$ instead of $U_c(x)$, since there to be little chance of confusion, but we must remember that the function U depend on the point c .

Definition 1.3. Let R be the field of real numbers and (R,T) be a fuzzy topological vector space over the field R . A function $f:R \to R$ is said to be fuzzy differentiable at a point c if there is a function U that is fuzzy continuous at $x=c$ and have $f(x)-f(c)=U(x)(x-c)$ for all $x \in R$.

$U(c)$ is said to be the fuzzy derivative of f at c and denoted $f'(c)=U(c)$.

II. Main Results

Theorem 2.1. If f is fuzzy differentiable at a point c , then f is fuzzy continuous at a point c .

Proof. Assume f is fuzzy differentiable at a point c , then there is a fuzzy continuous function, say φ and satisfies the relation

$f(x)-f(c)=\varphi(x)(x-c)$ for all $x \in R$ ………………………………………………. (1)

Since φ is fuzzy continuous at c , then $\varphi(x)$ is nearly equal to $\varphi(c)=f'(c)$ if x is near c .
Replacing $\varphi(x)$ by $f'(c)$ in (1), we obtain the equation $f(x)=f(c)+f'(c)(x-c)$
Which should be approximately correct when $(x-c)$ is small (i.e. If f is differentiable at c , then f is approximately a linear function near c .

Theorem 2.2. (Chain Rule).[3] If f is fuzzy differentiable at a point c and g is fuzzy differentiable at a point $f(c)$, then $h = g \circ f$ is also fuzzy differentiable at a point c and $h'(c) = g'(f(c))f'(c)$.

Theorem 2.3. If f and g are fuzzy differentiable at a point c, then

(1) $(f \pm g)'(c) = f'(c) \pm g'(c)$,

(2) $(fg)'(c) = f(c)g'(c) + g(c)f'(c)$,

(3) $\left(\dfrac{f}{g}\right)'(c) = \dfrac{f'(c)g(c) - g'(c)f(c)}{g^2(c)}, \ g(c) \neq 0$

Proof. We shall prove (2)

By using (Definition 3) there are two function φ and ψ both are fuzzy continuous at a point c and

$$\begin{aligned} f(x) &= f(c) + \varphi(x)(x-c) \\ g(x) &= g(c) + \psi(x)(x-c) \end{aligned} \quad \text{for all } x \in (a,b).$$

Now,

$$\begin{aligned} f(x)g(x) &= [f(c) + \varphi(x)(x-c)][g(c) + \psi(x)(x-c)] \\ &= f(c)g(c) + [f(c)\psi(x) + g(c)\varphi(x) + \varphi(x)\psi(x)(x-c)](x-c) \end{aligned}$$

Then $(fg)(x) = (fg)(c) + \eta(x)(x-c)$

Where $\eta(x) = f(c)\psi(x) + g(c)\varphi(x) + \varphi(x)\psi(x)(x-c)$, which is fuzzy continuous at c. If x is near c, then $\varphi(x)$ is nearly equal to $\varphi(c) = f'(c)$ and $\psi(x)$ is nearly equal to $\psi(c) = g'(c)$, finally $(fg)'(x) = \eta(x)\big|_{x=c} = f(c)g'(c) + g(c)f'(c)$.

Theorem 2.4. (Critical point theorem).[3] If f is fuzzy differentiable at a point c and $f(c)$ is extreme value, then c is a critical point (i.e., $f'(c) = 0$).

Theorem 2.5. (Rolle's theorem). Let f be fuzzy continuous on $[a,b]$ and fuzzy differentiable on (a,b). If $f(a) = f(b)$, then there is one interior point c at which $f'(c) = 0$.

Proof. We assume that for all $c \in (a,b)$, $f'(c) \neq 0$, since f is fuzzy continuous on a compact set $[a,b]$, it attains its maximum M and its minimum m somewhere in $[a,b]$. Neither extreme value is attained at an interior point (otherwise f' would vanish there) so both are attained at the end points. Since $f(a) = f(b)$, then $M = m$, and hence f is constant on $[a,b]$. This contradicts the assumption that f' is never 0 on (a,b). There for $f'(c) = 0$ for some c in (a,b).

Theorem 2.6. (Generalized Mean – Value Theorem). Let f and g are fuzzy continuous functions on $[a,b]$, and fuzzy differentiable on (a,b), assume also that there is no interior point x at which both $f'(x)$ and $g'(x)$ are infinite. Then for some interior point c we have $f'(c)[g(b) - g(a)] = g'(c)[f(b) - f(a)]$.

Proof. Let $h(x) = f(x)[g(b) - g(a)] - g(x)[f(b) - f(a)]$. Then $h'(x)$ is finite if both $f'(x)$ and $g'(x)$ are finite, and $h'(x)$ is infinite if exactly one of $f'(x)$ and $g'(x)$ are infinite. (The hypothesis excludes the case of both $f'(x)$ and $g'(x)$ being infinite). Also, h is fuzzy continuous on $[a,b]$ and $h(a) = h(b) = f(a)g(b) - g(a)f(b)$. By Rolle's Theorem we have $h'(c) = 0$ for some interior point and this proves the assertion.

Corollary 2.7. (Mean – Value Theorem). If f is fuzzy continuous on $[a,b]$ and fuzzy differentiable on (a,b), then there exists $c \in (a,b)$ such that $f(b) - f(a) = f'(c)(b-a)$.

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