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A Hybrid Rough Set And Graph-Theoretic Approach For Redundancy Detection In Experimental Engineering Systems

Yomna Dawood, M. Shokry, Abdallah A Galal, Manar Omran
(Department Of Physics And Engineering Mathematics, Faculty Of Engineering, Tanta University, Tanta, Egypt)

Abstract:
The analysis of experimental systems with multiple parameters demands a formalized mathematical approach that can account for the relationships among the various operating conditions. In this paper, a new approach using a combination of rough set theory and graph theory is proposed for representing and analyzing experimental data of desiccant-based air-conditioning system. The experimental data set is first transformed into a discrete form using a coding process before a graph is constructed for each parameter based on the defined edges using similarity relations among the operating conditions. A unified form of the graphs is achieved using the intersection of the graphs for each parameter. Furthermore, a redaction mechanism is proposed for evaluating the influence of each parameter on the entire structure. Toward this end, a combination of the concepts of rough set theory approximation in a graph environment using a neighborhood system is proposed along with several theoretical properties. The results show that the parameter C4 has no effect on the entire connectivity structure, and the experimental device can be manufactured simpler by removing this parameter without loss of information, and this reduces the computation cost. In addition, the results have demonstrated that derived propositions are valid for different graphs, and this confirms that the proposed model is stable. These outcomes have proved that the proposed framework has established a connection between rough set theory and graph theory, and this model can be used as an efficient mathematical tool for analyzing similarity relations and identifying redundancy in experimental systems.
Keywords**:** *Graph Theory, Rough Set, Data-to-Graph Transformation, Graph Reduction, Information Systems, Discrete Mathematics.*

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I. Introduction

Graph theory is considered to be one of the most important tools of discrete mathematics to model and analyze systems that exhibit relationships between their components. In fact, a graph G = (V, E), where V is a set of vertices and E is a set of edges, is a natural model to represent pairwise interactions in a wide range of applications, including scientific and engineering problems [1]. Due to the strong theoretical foundation of graph theory, it has been well developed and successfully applied to analyze the structural properties of systems, like connectivity, adjacency, and equivalence relations, which are crucial to understand different applications, especially complex systems [2]. In other words, graph theory offers a useful tool to transform applications into a discrete mathematical models, where combinatorial and algebraic methods can be used to analyze these systems. Based on this, different engineering problems such as thermal processes, energy networks, and flow systems can be modeled as a set of interconnected components, where the relationship between the system components can be well represented using a graph model [4]. Additionally, the emergence of network science has highlighted the importance of graphs in dealing with large systems, where the nodes of the graphs represent the entities, and the edges represent the interaction between the entities, hence giving a deeper understanding of the system organization and behavior [5].

In addition, graph theory has also been used in dealing with data-driven applications, where the data is represented as a graph to enable easier clustering, classification, and pattern recognition, among other applications [6]. From a mathematical aspect, graphs also enable the extension of other theories, like topology and algebraic structures, to practical applications, hence giving a connection between theory and practical applications [7]. Furthermore, the idea of similarity relations in data analysis can be represented by a graph structure, with edges being constructed based on the idea of equivalence and proximity between elements [8]. This is particularly useful in dealing with experimental data, as the establishment of relationships between different conditions is essential for decision-making [9]. Thus, it can be seen that the idea of graph theory provides a unified and rigorous approach to the representation of complex relational systems and is an essential tool in the study of mathematics and engineering [10].

The application of graph theory has been extensively investigated in the past by various scholars as an effective tool for modeling and analyzing real-world systems in various domains. Harary [11] examined the application of graph theory for modeling and analyzing engineering and mathematical systems and found that complex systems can be effectively modeled by considering the components of the system and the interactions between the components represented by vertices and edges of the graphs, respectively. Similarly, Bejan et al. [12] examined the application of graph theory for analyzing thermal systems and found that the heat transfer process in the systems can be effectively represented by the graphs and analyzed systematically. Further, Newman [13] examined the application of graph theory for analyzing complex networks and found that complex interconnected systems such as social and communication networks can be effectively analyzed by graph theory. In the framework of data-driven applications, Erciyes [14] in his study on the transformation of datasets into graph structures found that the representation of data in the form of a graph, based on similarity relations, improves the clustering and pattern recognition properties of the data. In addition, Arafa et al. [15] extended graph theory concepts into nano topology, and found that it is possible to utilize graph theory to define topological properties and approximation concepts, thereby creating a connection between graph theory and advanced mathematical concepts. These studies, therefore, demonstrate the versatility of graph theory in transforming different types of systems into a well-structured mathematical model, which is useful in analyzing experimental data and developing new theoretical concepts.

In addition, graph theory can be combined with other mathematical theories like topology and fuzzy sets, which are currently used in mathematics. El-Azab et al. [16] proposed the concept of soft sub-graphs and used it to study the cardiovascular system from the perspective of biomedical engineering. The results revealed the importance of using graph theory to understand complex relationships in the human body. In another study, El-Azab et al. [17] were able to show how graph theory can provide a clear insight into the flow of interactions in the human body by proposing specific kinds of sub-graphs to study the flow of the hepatic portal system. Kozae et al. [18] studied the possibility of using intuitionistic fuzzy set theory to model the spread of COVID-19 in medical and health-related applications. The research indicated that the mathematical models can be utilized to improve the decision-making process for complex medical situations and proved the usability of fuzzy-based representations to more accurately manage imprecise and incomplete information associated with epidemic systems.

Apart from graph theory, rough set theory has also been identified as a significant mathematical discipline to tackle problems of uncertainty and imprecision in data analysis. Rough set theory, as suggested by Pawlak [19], is based on the idea of "approximation" of a set using lower and upper bounds determined by the indiscernibility relation in an information system. Pawlak [19] showed that an information system is equivalent to the pair S = (U, A), where U is a finite set and A is the set of attributes, and knowledge in the information system is specified in terms of equivalence relations due to indistinguishable objects. Following this idea, Yao [20] studied the relationships between rough sets and information granulation and showed that the "approximation space" is a powerful tool to tackle problems of uncertainty in data. Furthermore, Skowron and Rauszer [21] studied the use of rough set theory in decision analysis and found that the use of lower and upper approximation has significant potential in classification and rule extraction. In this direction, Polkowski [22] has also extended rough set theory to more general frameworks and emphasized its potential in data mining and knowledge discovery. This shows that rough set theory has a solid foundation in mathematics in handling uncertain information, and when combined with graph theory, it has potential in representing complex systems in a comprehensive manner.

The integration of graph theory and rough set theory has been explored in different studies as a powerful tool for modeling relational data in an uncertain environment. In a study by Lin [23], rough set models based on neighborhood systems were explored, and it was demonstrated that the underlying relational structure of an approximation space can be represented in terms of a graph-based neighborhood system. Similarly, Zhu [24] explored generalized rough sets using covering and neighborhood structures. He demonstrated that such models can be represented in terms of a graph-like relation between data elements. In addition, Banerjee and Pal [25] explored rough set-based models in a relational environment, and it was demonstrated that such models can be improved by representing data relationships. Moreover, Qian et al. [26] explored rough set-based models in complex data analysis, and it was demonstrated that such models can be improved by integrating relational structures with approximation operators. Overall, such studies confirm that integrating graph theory and rough set theory provides a powerful mathematical tool for modeling complex systems where relational structures and uncertainty are fundamental. Nasef et al. [27-29] developed a number of approaches for the analysis and simplification of complex systems by using graph representations. Specifically, it was demonstrated that the application of Rough Set approaches can lead to the efficient reduction of complex electrical transmission networks, which enables the removal of extraneous information without the loss of essential system characteristics. Moreover, they provided a new idea for the study of discrete structures in topological spaces by establishing a connection between graph theory and nano-topology.

In spite of such significant development in graph theory and rough set theory, and their individual application in modeling system complexities and in handling uncertainties, there is a gap in literature in addressing their combined application in actual experimental engineering problems. Most of the existing literature is based on enhancing rough set theory in abstract relational structures or applying graph theory independently to model system complexities, without addressing a unified framework to combine both in actual experimental data analysis. Specifically, there is limited discussion in literature on transforming actual experimental data into a graph structure by a systematic approach and then applying rough set theory to analyze the data in such a transformed framework. To the best of our knowledge, there is no existing literature on applying such a framework in actual experimental data to analyze desiccant-based air-conditioning system complexities. Thus, this paper aims to introduce a framework to combine rough set theory and graph theory in a single framework to analyze system complexities. Specifically, classical definitions of rough set theory are extended to graph structures, and new propositions are formulated based on graph neighborhoods and subgraphs. Moreover, an algorithm is proposed to map experimental data to graphs through three major steps: data coding to map continuous data to discrete data, construction of graphs for all parameters and finding their intersection to obtain relationships between operating conditions and applying a redaction process by removing one parameter to analyze its effect. Finally, the proposed propositions are applied to the constructed graphs to validate the proposed framework. The proposed framework provides a novel link between theoretical rough set models and actual graphs, allowing for the analysis of experimental systems in a rigorous mathematical framework while providing insights for system simplification and cost reduction.

II. New Methodology Of Integrating Rough Set On Graph Theory

Recently, the integration of rough set theory and graph theory has been receiving considerable attention owing to its ability to provide a unified mathematical structure to represent relational structures in an uncertain environment. While rough set theory, as developed by Pawlak in [19], is primarily based on set approximations using equivalent relation structures derived from an information system, graph theory provides a natural representation of relationships between objects through a set of vertices and edges. By integrating rough set theory and graph theory, one can easily extend set approximations from a conventional information system to a more general representation of relationships between objects, which can be represented as a graph. This integration of the two theories becomes particularly important when relationships between objects play a major role in the representation. This is because a graph provides a flexible structure to represent relationships between objects, whereas rough set theory provides a set of tools to handle uncertain situations.

In this section, the traditional definitions of rough set theory, as proposed by Pawlak [19], are utilized and extended to graph structures. More precisely, the definitions of lower and upper approximations are reformulated in terms of graph theory, namely neighborhood systems. Based on this reformulation, new definitions of approximation operators on graphs are proposed, and several fundamental properties are established. These definitions and propositions are based on the application of rough set theory to graph structures, as described in the mathematical development in the following sections.

Definition 2.1. Let G = (V, E) be a simple connected undirected graph. For any vertex v ∈ V, the neighborhood of v, denoted by $N_G(v)$, is defined as: $N_G(v) = \{u \in V : uv \in E\}$.

Definition 2.2. Let G (V, E) be a simple connected graph, and let $H_1(V_1, E_1)$ be a sub graph of G. Then, lower approximation operators are defined as: $L^1(H_1, G) = \{v \in V_1 :\ N_G(v) \subseteq V_1\}$,
$L^2(H_1, G) = \cap \{\ N_G(v) : v \in V_1$ and $N_G(v) \subseteq V_1 \}$
Additionally, the upper approximation operators are defined as:

$$U^1(H_1, G) = \{v \in V_1 : N_G(v) \cap V_1 \neq \emptyset\}$$

$U^2(H_1, G) = \cup \{\ N_G(v) : v \in V_1$ and $N_G(v) \cap V_1 \neq \emptyset \}$

Proposition 2.1. Let $G = (V, E)$ be a simple connected graph and let $H_1 = G[V_1]$ be a connected induced subgraph of G with $|V_1| \geq 2$. Then, for $i = 1,2$, the following properties hold:
(i) $L^i(H_1, G) \subseteq V_1 \subseteq U^i(H_1, G)$
(ii) $G_1[\ L^i(H_1, G)] \subseteq H_1 \subseteq G_1[\ U^i(H_1, G)\ ]$, where $G_1[\ L(H_1, G)\ ]$ is vertices induced subgraph of lower approximation, and $G_1[\ U^i(H_1, G)\ ]$ is vertices induced subgraph of upper approximation

Proof
(i) Case $i = 1$,
1) $L^1(H_1, G) \subseteq V_1$

Let $v \in L^1(H_1, G) \leftrightarrow v \in V_1 \wedge N_G(v) \subseteq V_1 \to v \in V_1 \to L^1(H_1, G) \subseteq V_1$

2) $V_1 \subseteq U^1(H_1, G)$

Let $v \in V_1 \to \exists u \in V_1$ such that $uv \in E_1 \to uv \in E \to u \in N_G(v) \to N_G(v) \cap V_1 \neq \emptyset \to v \in U^1(H_1, G) \to V_1 \subseteq U^1(H_1, G)$

Case $i = 2$,

1) $L^2(H_1, G) \subseteq V_1$

$v \in L^2(H_1, G) \leftrightarrow v \in \bigcap\{N_G(v_i): v_i \in V_1, N_G(v_i) \subseteq V_1\} \to v \in N_G(v_i)\ \forall v_i$ with $N_G(v_i) \subseteq V_1 \to N_G(v_i) \subseteq V_1 \to v \in V_1 \to L^2(H_1, G) \subseteq V_1$.

2) $V_1 \subseteq U^2(H_1, G)$

$v \in V_1 \to \exists u \in V_1$ such that $uv \in E_1 \to uv \in E \to v \in N_G(u) \to N_G(u) \cap V_1 \neq \emptyset \to v \in \bigcup\{N_G(u): u \in V_1, N_G(u) \cap V_1 \neq \emptyset\} \to v \in U^2(H_1, G) \to V_1 \subseteq U^2(H_1, G)$.

(ii) From definition of vertex-induced subgraph, $G_1[L^i(H_1, G)] \subseteq H_1 \subseteq G_1[U^i(H_1, G)]$ i=1,2.

Proposition 2.2. Let $G = (V, E)$ be a simple connected graph, and let $H_1(V_1, E_1)$ and $H_2(V_2, E_2)$ be two connected subgraphs of G with $|V_1| \geq 2$ and $|V_2| \geq 2$. Let $H_1 \cup H_2$ denote the connected subgraph whose vertex set is $V_1 \cup V_2$. Then, for $i = 1,2$, $U^i(H_1 \cup H_2, G) = U^i(H_1, G) \cup U^i(H_2, G)$.

Proof

(i) $v \in U^i(H_1 \cup H_2, G) \leftrightarrow N(v) \cap (V_1 \cup V_2) \neq \emptyset \leftrightarrow (N(v) \cap V_1 \neq \emptyset) \vee (N(v) \cap V_2 \neq \emptyset) \leftrightarrow v \in U^i(H_1, G) \vee v \in U^i(H_2, G) \leftrightarrow v \in U^i(H_1, G) \cup U^i(H_2, G)$.

Hence, $U^i(H_1 \cup H_2, G) = U^i(H_1, G) \cup U^i(H_2, G)$.

Case $i = 1, v \in U^1(H_1 \cup H_2, G) \leftrightarrow N_G(v) \cap (V_1 \cup V_2) \neq \emptyset \leftrightarrow (N_G(v) \cap V_1 \neq \emptyset) \vee (N_G(v) \cap V_2 \neq \emptyset) \leftrightarrow v \in U^1(H_1, G) \vee v \in U^1(H_2, G) \leftrightarrow v \in U^1(H_1, G) \cup U^1(H_2, G)$.

Hence, $U^1(H_1 \cup H_2, G) = U^1(H_1, G) \cup U^1(H_2, G)$.

Case $i = 2$, $v \in U^2(H_1 \cup H_2, G) \leftrightarrow v \in \bigcup\{N_G(u): u \in V_1 \cup V_2, N_G(u) \cap (V_1 \cup V_2) \neq \emptyset\} \leftrightarrow v \in \bigcup_{u \in V_1 \cup V_2} N_G(u) \leftrightarrow v \in \left(\bigcup_{u \in V_1} N_G(u)\right) \cup \left(\bigcup_{u \in V_2} N_G(u)\right) \leftrightarrow v \in \bigcup\{N_G(u): u \in V_1, N_G(u) \cap V_1 \neq \emptyset\} \cup \bigcup\{N_G(u): u \in V_2, N_G(u) \cap V_2 \neq \emptyset\} \leftrightarrow v \in U^2(H_1, G) \cup U^2(H_2, G)$.

Hence, $U^2(H_1 \cup H_2, G) = U^2(H_1, G) \cup U^2(H_2, G)$.

Therefore, for $i = 1,2$, $U^i(H_1 \cup H_2, G) = U^i(H_1, G) \cup U^i(H_2, G)$

Proposition 2.3. Let $G = (V, E)$ be a simple connected graph and let $H_1(V_1, E_1)$ and $H_2(V_2, E_2)$ be two connected subgraphs of G.

Then, (i) $L^1(H_1 \cap H_2, G) = L^1(H_1, G) \cap L^1(H_2, G)$.

Moreover, if $\{u \in V_1: N_G(u) \subseteq V_1\} = \{u \in V_2: N_G(u) \subseteq V_2\}$,

then (ii) $L^2(H_1 \cap H_2, G) = L^2(H_1, G) \cap L^2(H_2, G)$.

Proof

(i) Case $i = 1$, $v \in L^1(H_1 \cap H_2, G) \leftrightarrow v \in V_1 \cap V_2 \wedge N_G(v) \subseteq V_1 \cap V_2 \leftrightarrow (v \in V_1 \wedge N_G(v) \subseteq V_1) \wedge (v \in V_2 \wedge N_G(v) \subseteq V_2) \leftrightarrow v \in L^1(H_1, G) \wedge v \in L^1(H_2, G) \leftrightarrow v \in L^1(H_1, G) \cap L^1(H_2, G)$.

Hence, $L^1(H_1 \cap H_2, G) = L^1(H_1, G) \cap L^1(H_2, G)$.

(ii) Case $i = 2$, Let $S_1 = \{u \in V_1: N_G(u) \subseteq V_1\}, S_2 = \{u \in V_2: N_G(u) \subseteq V_2\}$.

By assumption, $S_1 = S_2$.

Then, $v \in L^2(H_1 \cap H_2, G) \leftrightarrow v \in \bigcap\{N_G(u): u \in V_1 \cap V_2, N_G(u) \subseteq V_1 \cap V_2\} \leftrightarrow v \in \bigcap\{N_G(u): u \in S_1\} \leftrightarrow v \in \bigcap\{N_G(u): u \in V_1, N_G(u) \subseteq V_1\} \leftrightarrow v \in L^2(H_1, G)$.

Similarly, $v \in L^2(H_1 \cap H_2, G) \leftrightarrow v \in L^2(H_2, G)$.

Thus, $v \in L^2(H_1, G) \wedge v \in L^2(H_2, G) \leftrightarrow v \in L^2(H_1, G) \cap L^2(H_2, G)$.

Hence, $L^2(H_1 \cap H_2, G) = L^2(H_1, G) \cap L^2(H_2, G)$.

Proposition 2.4. Let $G = (V, E)$ be a simple connected graph, and let $H_1(V_1, E_1)$ and $H_2(V_2, E_2)$ be two connected subgraphs of G such that $V_1 \subseteq V_2$. Then the following properties hold:

(i)$L^1(H_1, G) \subseteq L^1(H_2, G)$,
(ii)$L^2(H_2, G) \subseteq L^2(H_1, G)$,
(iii)$U^i(H_1, G) \subseteq U^i(H_2, G), i = 1,2$,
(iv)$G[L^1(H_1, G)] \subseteq G[L^1(H_2, G)]$,
(v)$G[L^2(H_2, G)] \subseteq G[L^2(H_1, G)]$,
(vi)$G[U^i(H_1, G)] \subseteq G[U^i(H_2, G)], i = 1,2$.

Proof
(i) $L^1(H_1, G) \subseteq L^1(H_2, G)$
$v \in L^1(H_1, G) \leftrightarrow v \in V_1 \wedge N_G(v) \subseteq V_1 \rightarrow v \in V_2 \wedge N_G(v) \subseteq V_2 \leftrightarrow v \in L^1(H_2, G)$.
Hence, $L^1(H_1, G) \subseteq L^1(H_2, G)$.

(ii) $L^2(H_2, G) \subseteq L^2(H_1, G)$
Let $A_1 = \{N_G(u): u \in V_1,\ N_G(u) \subseteq V_1\}, A_2 = \{N_G(u): u \in V_2,\ N_G(u) \subseteq V_2\}$.
If $N_G(u) \in A_1$, then $u \in V_1 \wedge N_G(u) \subseteq V_1 \rightarrow u \in V_2 \wedge N_G(u) \subseteq V_2$.
Hence, $A_1 \subseteq A_2$.
Therefore, $L^2(H_2, G) = \cap A_2 \subseteq \cap A_1 = L^2(H_1, G)$.

(iii) $U^i(H_1, G) \subseteq U^i(H_2, G)$
Case $i = 1$, $v \in U^1(H_1, G) \leftrightarrow N_G(v) \cap V_1 \neq \emptyset \rightarrow N_G(v) \cap V_2 \neq \emptyset \leftrightarrow v \in U^1(H_2, G)$.
Hence,$U^1(H_1, G) \subseteq U^1(H_2, G)$.

Case $i = 2$, $v \in U^2(H_1, G) \leftrightarrow \exists u \in V_1$ such that $v \in N_G(u) \wedge N_G(u) \cap V_1 \neq \emptyset \rightarrow u \in V_2 \wedge N_G(u) \cap V_2 \neq \emptyset \rightarrow v \in U^2(H_2, G)$.
Hence, $U^2(H_1, G) \subseteq U^2(H_2, G)$.

Proposition 2.5. Let G (V,E) be a simple connected graph ,and $H_1(V_1, E_1)$ be a subgraph of G .We denote by G−H_1 the vertex-complement of H_1 in G, defined by V(G−H_1) = V \ V_1 ,Then, for i = 1, 2, the following properties hold:
(i) L^i (G−H_1, G) = V\U^i (H_1, G)
(ii) U^i (G−H_1, G) = V\ L^i (H_1, G)
(iii) L^i (L^i (H_1, G), G) $\subsetneq$ L^i (H_1, G), U^i (H_1, G) $\subsetneq$ U^i (U^i (H₁, G), G)
(iv) L^i (U^i (H_1, G), G) $\subsetneq$ U^i (H_1, G), U^i (L^i (H_1, G), G) $\subsetneq$ L^i (H_1, G)

Proof
(i) v∈L^i (G−H_1, G) ↔N(v)⊆V\V1↔ v∈ V\U^i (H_1, G)
(ii) v∈U^i (G−H_1, G) ↔N(v)∩(V\V1) ≠ ∅↔v∈ V\ L^i (H_1, G)
(iii) Let $v \in L^i(L^i(H_1, G), G) \rightarrow N(v) \subseteq L^i(H_1, G) \subseteq V_1 \rightarrow v \in L^i(H_1, G)$,
Additionally, let $v \in U^i(H_1, G) \rightarrow N(v) \cap V_1 \neq \emptyset \rightarrow N(v) \cap U^i(H_1, G) \neq \emptyset \rightarrow v \in U^i(U^i(H_1, G), G)$.
(iv) Let $v \in L^i(U^i(H_1, G), G) \rightarrow N(v) \subseteq U^i(H_1, G) \rightarrow N(v) \cap V_1 \neq \emptyset \rightarrow v \in U^i(H_1, G)$,
Additionally, let $v \in U^i(L^i(H_1, G), G) \rightarrow N(v) \cap L^i(H_1, G) \neq \emptyset \rightarrow N(v) \subseteq V_1 \rightarrow v \in L^i(H_1, G)$

III. Overview Of The Application Of Desiccant Based Air Conditioning System

Recently, much focus has been given to the reduction of energy consumption, inclusion of renewable energy sources, and reduction of environmental impacts of conventional air-conditioning systems. In this regard, desiccant-based air-conditioning systems have been found to be a promising solution to replace conventional air-conditioning systems, as these systems have the potential to control latent and sensible loads individually and use low-grade thermal energy sources. This has become a significant area of concern in the development of energy systems.

Nevertheless, a major limitation in the conventional rotary type of desiccant wheel systems is associated with the heat that is generated during the process of adsorbing moisture. The heat affects the adsorbing capacity of the desiccant material as well as the temperature of the air, thereby resulting in increment of the sensible cooling load.

To overcome these drawbacks, Abdelgaied et al. [30-32] have introduced a solar-assisted solid desiccant air-conditioning system using a multi-stage desiccant dehumidification process with intercooling, thermal recovery, and solar regeneration. In this system, the desiccant dehumidifier channel is made up of consecutive beds of silica gel, followed by a heat exchanger to remove the heat of adsorption. This results in simultaneous dehumidification and cooling of the process air, as opposed to the traditional system in which the temperature of the air rises due to adsorption. In this system, thermal recovery and solar collectors are also included. An in-depth description of this system has been provided in [30,31].

The experimental study of this system provides a set of measured data corresponding to different operating conditions. These conditions are defined by variations in regeneration temperature, process air flow rate, and regeneration air flow rate, and are denoted by the set V={B_1,B_2,…,B_7}.

In addition, the system behavior is characterized by a set of measured parameters corresponding to temperature and humidity ratios at different locations in the system, denoted by C={C_1,C_2,…,C_8}.

The dataset is summarized in table 1, while the definitions of the parameters and operating conditions are provided in tables 2 and 3, respectively.

The data provided in this experiment is the basis upon which the transformation to a graph-theoretic representation is based. The aim is to analyze the relationships between the various conditions of operation in a structured mathematical framework, in order to identify any similarity and reduce any redundancy in the system parameters.

Table no 1. Experimental data of the system at various conditions.

	C1	C2	C3	C4	C5	C6	C7	C8
B1	32	14.5	29.9	4.6	18.1	4.6	12.1	7
B2	32.6	14.6	29.7	5	18	5	12.4	7.2
B3	32.5	14.4	29.5	5.7	18	5.7	13.2	7.6
B4	33.7	14.8	29.7	5.4	18.1	5.4	11.4	7.1
B5	33.2	14.7	28.2	3.9	17.8	3.9	11.1	6.6
B6	32.3	14.0	29.6	5.7	22.0	5.7	13.5	8.1
B7	33.3	13.8	29.5	6.1	25.3	6.1	14.4	8.7

Table no 2. Temperature and humidity sensers used in the system (C1-C8).

Parameter	Definition
C1	represents temperature sensor inlet to desiccant dehumidifier
C2	represents humidity ratio sensor inlet to desiccant dehumidifier
C3	represents temperature sensor outlet from desiccant dehumidifier
C4	represents humidity ratio sensor outlet from desiccant dehumidifier
C5	represents temperature sensor outlet from heat exchanger
C6	represents humidity ratio sensor outlet from heat exchanger
C7	represents temperature sensor supplied to cooling room
C8	represents humidity ratio sensor supplied to cooling room

Table no 3. Different operating conditions of regeneration temperature, process air flow rate, and regeneration air flow rate (B1-B7).

Parameter	Definition
B1	Q_p=240 m^3/h, Q_r=240 m^3/h, T_{reg}=100 °C
B2	Q_p=240 m^3/h, Q_r=240 m^3/h, T_{reg}=90 °C
B3	Q_p=240 m^3/h, Q_r=240 m^3/h, T_{reg}=80 °C
B4	Q_p=180 m^3/h, Q_r=240 m^3/h, T_{reg}=90 °C
B5	Q_p=120 m^3/h, Q_r=240 m^3/h, T_{reg}=90 °C
B6	Q_p=240 m^3/h, Q_r=180 m^3/h, T_{reg}=90 °C
B7	Q_p=240 m^3/h, Q_r=120 m^3/h, T_{reg}=90 °C

IV. Algorithm For Data To Graph Transformation

In order to analyze the experimental dataset within a rigorous mathematical framework, it is necessary to transform the measured data into a graph-theoretic representation. As the experimental data set is in the form of real numbers corresponding to various operating conditions and parameters, it is not possible to apply graph theory directly to the experimental data set. Hence, an algorithm is presented to transform the experimental data set to a graph structure. The algorithm is intended to maintain the relational characteristics of the experimental data set and to apply graph theory concepts and rough set approximations, as developed in the previous section, to the experimental data set. The steps involved in the algorithm are coding experimental data, constructing graphs corresponding to individual parameters and determining their intersection, and finally, the redaction step to determine the influence of individual parameters. The experimental system is, therefore, transformed to a set of graphs representing the similarity relationships among the operating conditions, and hence, it is ready for mathematical analysis and validation of the propositions.

Step 1: Coding the Experimental Data

The experimental data given in Table 1 is converted to coded form as the first step in the suggested algorithm, allowing the data to be incorporated into a graph theory framework. The experimental data given in the original form, i.e., the parameters C1, C2, …, C8, are measured in various ranges and are given in continuous form. A direct comparison is not appropriate to establish relationships between the various operating conditions. Therefore, the coding step is incorporated to convert the experimental data to a comparable form as given in table 4. Formally, for each parameter C_i∈C, a mapping f_i:V→{1,2,…,k_i} is defined, where V={B_1,B_2,…,B_7} represents the set of operating conditions. The mapping f_i assigns an integer code to each operating condition based on the relative magnitude of its corresponding value under parameter C_i. This is achieved by arranging the measured values of each parameter in ascending order and assigning integer codes such that the smallest value is given code 1 and the largest value is assigned the highest code.

This process associates each condition with a discrete value for all parameters according to its relative position for that parameter. This process maintains the relative structure of the data set while eliminating the inconsistencies that may result from different scales and units of measurements. The coded data matrix in Table 5 represents a discrete information system in which equal codes imply that the two operating conditions belong to the same equivalence class for a given parameter.

This coded data structure forms the basis for the construction of the graph structure in the next step of the process, with the vertices and edges defined according to the equality of the coded values for the different parameters. This process of coding the experimental data set represents a systematic process for transforming the experimental data into a discrete mathematical structure.

Table no 4. Coding intervals used for transforming experimental data.

Parameter	Code	Value Interval	Parameter	Code	Value Interval
C1	1	$32 \le V_i < 32.5$	C5	1	$17 \le V_n < 19$
	2	$32.5 \le V_i < 33$		2	$19 \le V_n < 21$
	3	$33 \le V_i < 33.5$		3	$21 \le V_n < 23$
	4	$33.5 \le V_i < 34$		4	$23 \le V_n < 25$
				5	$25 \le V_n < 27$
C2	1	$13.5 \le V_j < 14$	C6	1	$3.5 \le V_p < 4$
	2	$14 \le V_j < 14.5$		2	$4 \le V_p < 4.5$
	3	$14.5 \le V_j < 15$		3	$4.5 \le V_p < 5$
				4	$5 \le V_p < 5.5$
				5	$5.5 \le V_p < 6$
				6	$6 \le V_p < 6.5$
C3	1	$28 \le V_k < 28.3$	C7	1	$11 \le V\_q < 11.5$
	2	$28.3 \le V_k < 28.6$		2	$11.5 \le V\_q < 12$
	3	$28.6 \le V_k < 28.9$		3	$12 \le V\_q < 12.5$
	4	$28.9 \le V_k < 29.1$		4	$12.5 \le V\_q < 13$
	5	$29.1 \le V_k < 29.4$		5	$13 \le V\_q < 13.5$
	6	$29.4 \le V_k < 29.7$		6	$13.5 \le V\_q < 14$
	7	$29.7 \le V_k < 30$		7	$14 \le V\_q < 14.5$
C4	1	$3.5 \le V_m < 4$	C8	1	$6.5 \le V_l < 7$
	2	$4 \le V_m < 4.5$		2	$7 \le V_l < 7.5$
	3	$4.5 \le V_m < 5$		3	$7.5 \le V_l < 8$
	4	$5 \le V_m < 5.5$		4	$8 \le V_l < 8.5$
	5	$5.5 \le V_m < 6$		5	$8.5 \le V_l < 9$
	6	$6 \le V_m < 6.5$			

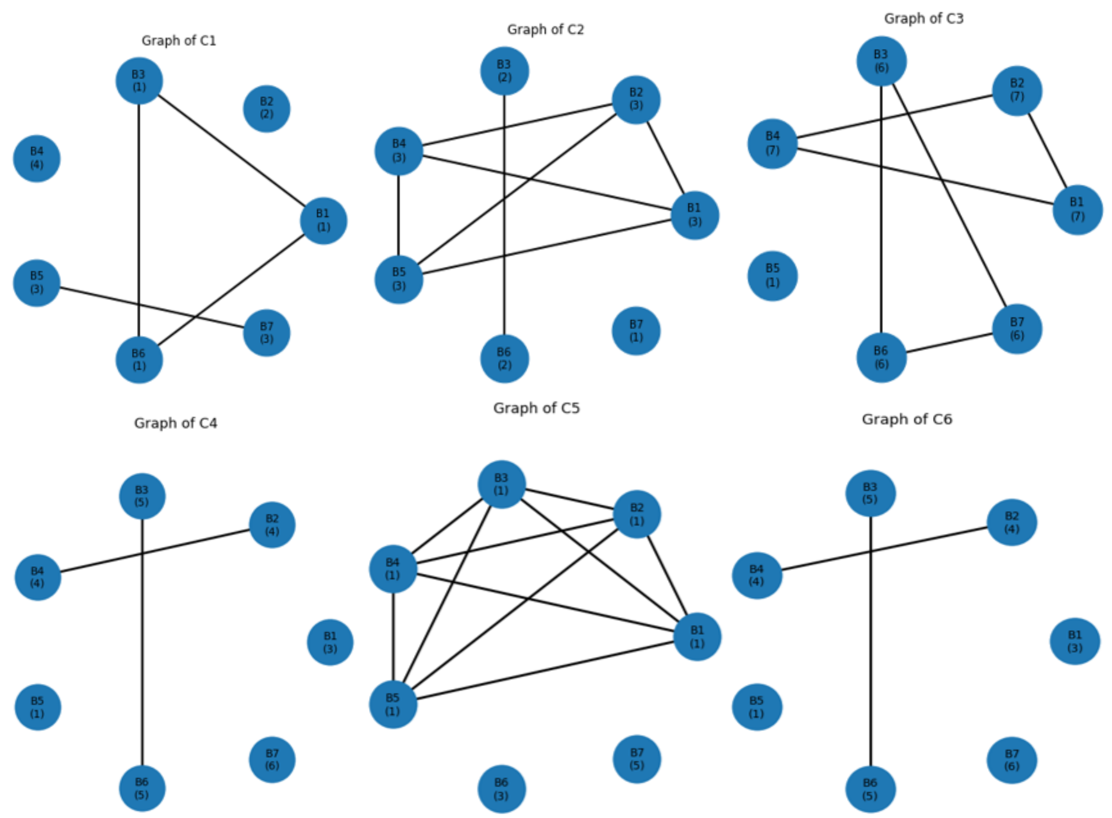
Table no 5. Final data after implementing the coding technique.

	C1	C2	C3	C4	C5	C6	C7	C8
B1	1	3	7	3	1	3	3	2
B2	2	3	7	4	1	4	3	2
B3	1	2	6	5	1	5	5	3
B4	4	3	7	4	1	4	1	2
B5	3	3	1	1	1	1	1	1
B6	1	2	6	5	3	5	6	4
B7	3	1	6	6	5	6	7	5

Step 2: Graphs Construction

After converting the experimental measurements into coded values, the dataset was represented using graph theory to analyze the relationships between the different operating conditions. For each parameter $C_i (i = 1,2, … ,8)$, a separate graph $G_{C_i}$was constructed. In these graphs, the vertex set Vconsists of the seven operating conditions $B_1, B_2, \ldots, B_7$, where each vertex represents one experimental operating case defined by a specific combination of regeneration temperature, process air flow rate, and regeneration air flow rate. An edge is drawn between two vertices $B_a$and $B_b$if the coded values of the corresponding parameter C_i for these two operating conditions are identical. In other words, two vertices are connected when the operating conditions belong to the same coded class for that particular parameter. Mathematically, for a graph $G_{C_i} = (V, E_{C_i})$, the vertex set is defined as $V = \{B_1, B_2, \ldots, B_7\}$, and the edge set $E_{C_i}$is defined such that $(B_a, B_b) \in E_{C_i}$whenever the coded values of C_i for B_a and B_b are equal. As a result, vertices sharing the same code form connected components within the graph, while vertices with unique codes appear as isolated vertices. This procedure was repeated for each of the eight parameters C_1–C_8, producing eight graphs that describe the similarity relationships among the operating conditions based on each measured variable as provided in figure 1. These graphs provide a discrete mathematical representation of the experimental dataset and serve as the foundation for subsequent analysis.

To further analyze the common relationships among the operating conditions, an intersection graph as presented in figure 2 was constructed from the individual graphs corresponding to the parameters $C_1, C_2, \ldots, C_8$. The intersection graph represents the common edges shared by all these graphs and therefore highlights the strongest similarities among the operating conditions. Formally, if $G_{C_1}, G_{C_2}, \ldots, G_{C_8}$denote the graphs associated with the coded parameters C_1–C_8, the intersection graph $G_I$is defined as $G_I = G_{C_1} \cap G_{C_2} \cap \cdots \cap G_{C_8}$, where the vertex set of G_I remains the same as that of the original graphs, namely $V = \{B_1, B_2, \ldots, B_7\}$, and the edge set consists only of those edges that appear simultaneously in all graphs G_{C_i} In other words, any two vertices in the intersection graph are connected if they are connected in all individual graphs defined by parameters C1, C2, ..., C8. This approach makes it possible to identify certain operating conditions that show identical coded behavior in all variables, thus revealing certain patterns in the experimental data set. This intersection graph is a concise representation of the strongest possible relations between certain variables and is a basis for further processing, such as reduction of the graph, where certain parameters are excluded.



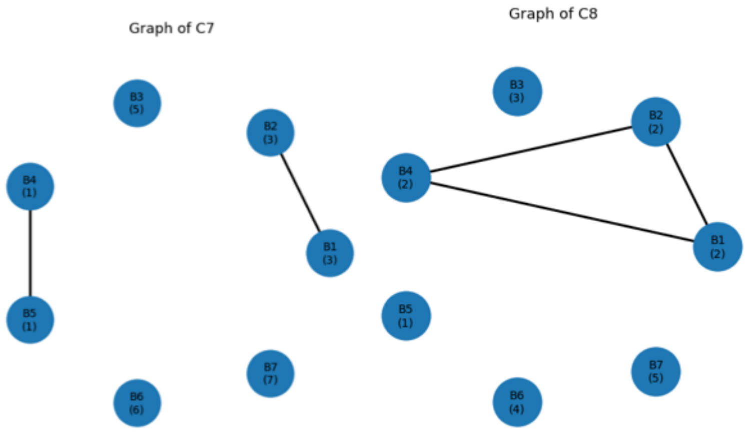


Figure 1. Constructed graphs for each parameter $C_i(i = 1, 2, \ldots, 8)$.

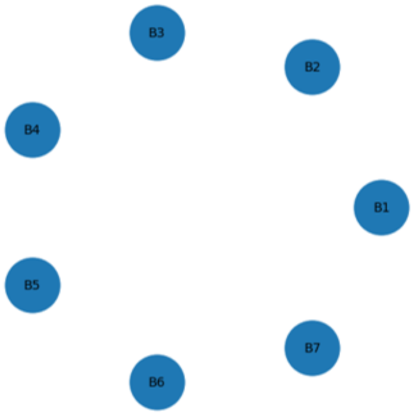


Figure 2. Intersected graph for all parameters $C_i(i = 1, 2, \ldots, 8)$.

Step 3: Graph Reduction

In order to reduce the cost of experimental systems, new methodology of graph reduction was applied to test the influence of individual parameters on the intersection graph. The redaction method was applied by removing one parameter from the graph formation process. In our investigation, the parameter "C4" representing the humidity ratio at point 2 (the outlet from the desiccant dehumidifier) was removed from the analysis. Consequently, the intersection graph was reconstructed using only the remaining graphs corresponding to the parameters $C_1, C_2, C_3, C_5, C_6, C_7$, and C_8. Let $G_{C_i} = (V, E_{C_i})$ denote the graph associated with the coded parameter C_i. The redacted intersection graph is therefore defined as $G_R = G_{C_1} \cap G_{C_2} \cap G_{C_3} \cap G_{C_5} \cap G_{C_6} \cap G_{C_7} \cap G_{C_8}$.

In this graph, the vertex set remains unchanged, $V = \{B_1, B_2, \ldots, B_7\}$, representing the seven operating conditions of the experimental system. However, the edge set contains only those edges that appear simultaneously in all graphs except G_{C_4}. By removing the constraint imposed by C_4, additional connections between vertices may appear if the corresponding operating conditions share identical coded values across the remaining parameters. This redaction process enables the identification of relationships among operating conditions that may be masked by the inclusion of a specific variable. Therefore, the redacted graph provides insight into the structural contribution of the eliminated parameter and allows the assessment of its impact on the overall connectivity pattern of the experimental dataset. The obtained results show that the redacted intersection graph G_R in figure 3 is identical to the original intersection graph G_I defined in Step 2, that is, $G_R = G_I$.

This indicates that parameter C_4 does not affect the overall connectivity structure of the dataset. Consequently, C_4 can be considered a redundant parameter in the context of the proposed graph representation. From a practical point of view, this means that the system can be manufactured by removing this parameter without loss of information, and this may be reflected as a reduction in the computation cost.

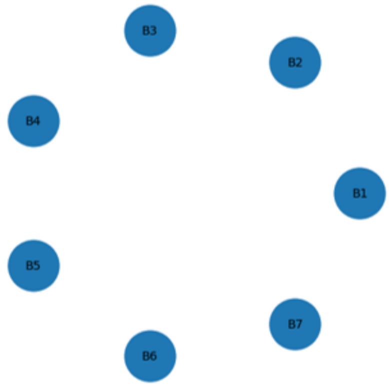


Figure 3. Intersected graph without C4.

V. Applying Derived Properties On Approximation Graphs Of Desiccant Based Air Conditioning System

In this section, the previously derived propositions introduced in Section 2 will be utilized for the validation of the generated graphs constructed from the coded experimental data. The application of the previously introduced propositions to the generated graphs is performed in a systematic manner by selecting appropriate graphs and subgraphs that capture different levels of complexity in the relational structure of the operating conditions. More specifically, Propositions 1 and 2 will be applied to the constructed graph G_{C1} presented before in figure 1, which captures the similarity relations for a single parameter. This constitutes a fundamental validation of the approximation operators for the basic structure of the graph. Afterwards, Propositions 3 and 5 will be applied to the union graph G_{C1}, G_{C2}, and G_{C3} as seen in figure 4 which incorporates multiple parameters and hence possesses a richer structure for the validation of the intersection and complement properties. In addition, Proposition 4 is also employed for the union graph G_{C1}, and G_{C3} as indicated in figure 5 to investigate the properties of approximation operators under inclusion relationships between subgraphs. The above process from individual graphs to union graphs ensures that the theoretical properties are verified under increasing complexity. The significance of the above process lies in the fact that it provides a link between theoretical models and actual graph representations obtained from experimental data, thereby validating the effectiveness of the proposed rough set-graph model in capturing the similarity between real-world systems. As a result of this application process, it is noticed that the generated propositions are satisfied by the generated graphs, thus proving the consistency of the approximation operators when generalized for the graph structures. The results show that the lower and upper approximation operators retain their primary characteristics for different types of graphs, such as individual graphs and union graphs. The application for union graphs indicates that the generated propositions still work when the complexity of the neighborhood increases due to the combination of different parameters. This indicates that the proposed approach is robust for different graph operations such as union, intersection, and complement operations and thus can be regarded as a stable tool for the analysis of similarity relations for the coded experimental data set. The successful verification of the generated propositions for the constructed graphs thus proves the effectiveness of the combination of rough set theory and graph theory for the representation of real-world experimental systems. The corresponding illustrative examples are presented below to demonstrate the application of each proposition on the constructed graphs.

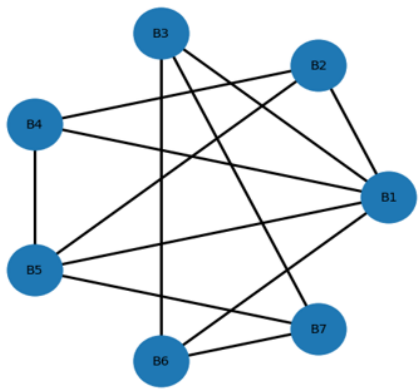


Figure 4. Union graph of C1, C2, and C3.

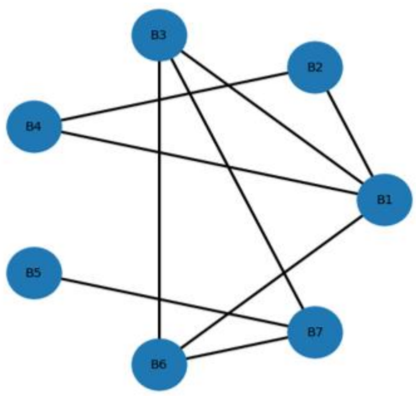


Figure 5. Union graph of C1 and C3.

Example of Proposition 1
Let $G = (V, E)$ be a simple connected graph such that: $G = C_1$.
Consider the subgraph $H_1 = G_1[\{B_1, B_3, B_6\}]$.
Thus, $V(H_1) = \{B_1, B_3, B_6\}$.
From the graph, the neighborhoods are: $N(B_1) = \{B_3, B_6\}, N(B_3) = \{B_1, B_6\}, N(B_6) = \{B_1, B_3\}$.

For $i = 1$, for each vertex of $V(H_1)$, we have $N(B_1) \subseteq V(H_1), N(B_3) \subseteq V(H_1), N(B_6) \subseteq V(H_1)$.
Hence, $L^1(H_1, G_1) = \{B_1, B_3, B_6\} = V(H_1)$, and $U^1(H_1, G_1) = \{B_1, B_3, B_6\} = V(H_1)$.
Therefore, $L^1(H_1, G_1) \subseteq V(H_1) \subseteq U^1(H_1, G_1)$.

Verification for $i = 2$, $L^2(H_1, G_1) = \bigcap\{N(u): u \in V(H_1),\ N(u) \subseteq V(H_1)\}$.
Since $N(B_1) \subseteq V(H_1)$, $N(B_3) \subseteq V(H_1)$, and $N(B_6) \subseteq V(H_1)$.
Then, $L^2(H_1, G_1) = N(B_1) \cap N(B_3) \cap N(B_6) = \emptyset$, and $U^2(H_1, G_1) = N(B_1) \cup N(B_3) \cup N(B_6) = \{B_1, B_3, B_6\}$.
Since $V(H_1) = \{B_1, B_3, B_6\}$.
Then, $L^2(H_1, G_1) \subseteq V(H_1) \subseteq U^2(H_1, G_1)$.
Example of Proposition 2
Let $G = (V, E)$ be a simple connected graph such that: $G = C_1$.
Take the two subgraphs: $H_1 = G_1[\{B_1, B_3\}], and\ H_2 = G_1[\{B_1, B_6\}]$.
Thus, $V(H_1) = \{B_1, B_3\}, V(H_2) = \{B_1, B_6\}$.
Hence, $V(H_1) \cup V(H_2) = \{B_1, B_3, B_6\}$.
Then, $H_1 \cup H_2 = G_1[\{B_1, B_3, B_6\}]$.
From the graph, the relevant neighborhoods are:
$N(B_1) = \{B_3, B_6\}, N(B_3) = \{B_1, B_6\}, N(B_6) = \{B_1, B_3\}$.

For $i = 1$, $U^1(H_1 \cup H_2, G_1) = \{v \in V(G_1): N(v) \cap \{B_1, B_3, B_6\} \neq \emptyset\}$.
Since only the vertices B_1, B_3, B_6 have neighborhoods intersecting $\{B_1, B_3, B_6\}$.
Then, $U^1(H_1 \cup H_2, G_1) = \{B_1, B_3, B_6\}$.
Therefore, $U^1(H_1, G_1) = \{v \in V(G_1): N(v) \cap \{B_1, B_3\} \neq \emptyset\}$.
Since, $N(B_1) \cap V(H_1) \neq \emptyset, N(B_3) \cap V(H_1) \neq \emptyset, N(B_6) \cap V(H_1) \neq \emptyset$.
Then, $U^1(H_1, G_1) = \{B_1, B_3, B_6\}$.
Similarly, $U^1(H_2, G_1) = \{v \in V(G_1): N(v) \cap \{B_1, B_6\} \neq \emptyset\}$.
Since, $N(B_1) \cap V(H_2) \neq \emptyset, N(B_3) \cap V(H_2) \neq \emptyset, N(B_6) \cap V(H_2) \neq \emptyset$,
Then, $U^1(H_2, G_1) = \{B_1, B_3, B_6\}$.
Therefore, $U^1(H_1, G_1) \cup U^1(H_2, G_1) = \{B_1, B_3, B_6\}$.
Hence, $U^1(H_1 \cup H_2, G_1) = U^1(H_1, G_1) \cup U^1(H_2, G_1)$.

For $i = 2$, $U^2(H_1 \cup H_2, G_1) = N(B_1) \cup N(B_3) \cup N(B_6)$,
Since, $N(B_1) \cap (V(H_1) \cup V(H_2)) \neq \emptyset, N(B_3) \cap (V(H_1) \cup V(H_2)) \neq \emptyset, N(B_6) \cap (V(H_1) \cup V(H_2)) \neq \emptyset$.
Hence, $U^2(H_1 \cup H_2, G_1) = \{B_3, B_6\} \cup \{B_1, B_6\} \cup \{B_1, B_3\} = \{B_1, B_3, B_6\}$.
Then, $U^2(H_1, G_1) = \bigcup\{N(u): u \in V(G_1), N(u) \cap V(H_1) \neq \emptyset\}$.
Since, $N(B_1) \cap V(H_1) \neq \emptyset, N(B_3) \cap V(H_1) \neq \emptyset, N(B_6) \cap V(H_1) \neq \emptyset$,
Then: $U^2(H_1, G_1) = N(B_1) \cup N(B_3) \cup N(B_6) = \{B_1, B_3, B_6\}$.
Similarly, $U^2(H_2, G_1) = \bigcup\{N(u): u \in V(G_1), N(u) \cap V(H_2) \neq \emptyset\}$.
Since, $N(B_1) \cap V(H_2) \neq \emptyset, N(B_3) \cap V(H_2) \neq \emptyset, N(B_6) \cap V(H_2) \neq \emptyset$,
Then: $U^2(H_2, G_1) = N(B_1) \cup N(B_3) \cup N(B_6) = \{B_1, B_3, B_6\}$.
Therefore, $U^2(H_1, G_1) \cup U^2(H_2, G_1) = \{B_1, B_3, B_6\} \cup \{B_1, B_3, B_6\} = \{B_1, B_3, B_6\}$.
Hence, $U^2(H_1 \cup H_2, G_1) = U^2(H_1, G_1) \cup U^2(H_2, G_1)$.

Example of Proposition 3
Let $G = (V, E)$ be a connected graph such that: $G = C_1 \cup C_2 \cup C_3$.
Additionally let $H_1 = (V_1, E_1)$ and $H_2 = (V_2, E_2)$ be two connected subgraphs of G, and
$V_1 = \{B_1, B_2, B_3, B_4, B_5\}, V_2 = \{B_1, B_2, B_4, B_5, B_6\}$.
Then, $V_1 \cap V_2 = \{B_1, B_2, B_4, B_5\}$,
Hence, $H_1 \cap H_2 = G[V_1 \cap V_2]$.
From the graph G, the neighborhoods of the vertices are given by:
$N(B_1) = \{B_2, B_3, B_4, B_5, B_6\}, N(B_2) = \{B_1, B_4, B_5\}$,
$N(B_3) = \{B_1, B_6, B_7\}, N(B_4) = \{B_1, B_2, B_5\}$,
$N(B_5) = \{B_1, B_2, B_4, B_7\}, N(B_6) = \{B_1, B_3, B_7\}$,

$N(B_7) = \{B_3, B_5, B_6\}$.

For $i = 1$, $L^1(H_1, G) = \{B_2, B_4\}$, and $L^1(H_2, G) = \{B_2, B_4\}$.
Since, $V_1 \cap V_2 = \{B_1, B_2, B_4, B_5\}$,
Then, $L^1(H_1 \cap H_2, G) = \{v \in V_1 \cap V_2 : N(v) \subseteq V_1 \cap V_2\}$.
Therefore, $L^1(H_1 \cap H_2, G) = \{B_2, B_4\}$.
Consequently, $L^1(H_1 \cap H_2, G) = L^1(H_1, G) \cap L^1(H_2, G)$,

For $i = 2$, $L^2(H_1, G) = N(B_2) \cap N(B_4)$.
Then, $L^2(H_1, G) = \{B_1, B_4, B_5\} \cap \{B_1, B_2, B_5\} = \{B_1, B_5\}$, and $L^2(H_2, G) = N(B_2) \cap N(B_4) = \{B_1, B_5\}$.
For $H_1 \cap H_2$, and $V_1 \cap V_2 = \{B_1, B_2, B_4, B_5\}$, the vertices that satisfy $u \in V_1 \cap V_2$, and $N(u) \subseteq V_1 \cap V_2$ are B_2 and B_4.
Hence, $L^2(H_1 \cap H_2, G) = N(B_2) \cap N(B_4) = \{B_1, B_5\}$.
Therefore, $L^2(H_1 \cap H_2, G) = L^2(H_1, G) \cap L^2(H_2, G)$,

Example of Proposition 4
Let $G = (V, E)$ be a connected graph such that: $G = C_1 \cup C_3$.
Additionally let $H_1 = (V_1, E_1)$, and $H_2 = (V_2, E_2)$ be connected subgraphs of G, and
$V_1 = \{B_1, B_2, B_4\}, V_2 = \{B_1, B_2, B_3, B_4\}$.

For $i = 1$, from the neighborhood structure of G, $L^1(H_1, G) = \{B_2, B_4\}, L^1(H_2, G) = \{B_2, B_4\}$,
Hence, $L^1(H_1, G) \subseteq L^1(H_2, G)$, and $U^1(H_1, G) = \{B_1, B_2, B_3, B_4, B_6\}, U^1(H_2, G) = \{B_1, B_2, B_3, B_4, B_6, B_7\}$,
Then, $U^1(H_1, G) \subset U^1(H_2, G)$.
Therefore, $G[L^1(H_1, G)] \subseteq G[L^1(H_2, G)], G[U^1(H_1, G)] \subseteq G[U^1(H_2, G)]$.

For $i = 2$, $L^2(H_1, G) = N(B_2) \cap N(B_4) = \{B_1\}, L^2(H_2, G) = \{B_1\}$,
Hence, $L^2(H_1, G) \subseteq L^2(H_2, G)$, and $U^2(H_1, G) = \{B_1, B_2, B_3, B_4, B_6\}, U^2(H_2, G) = \{B_1, B_2, B_3, B_4, B_6, B_7\}$,
Then, $U^2(H_1, G) \subset U^2(H_2, G)$.
Therefore, for $H_1 \subset H_2$ in $G = C_1 \cup C_3$, we obtain for $i = 1,2$,
$L^i(H_1, G) \subseteq L^i(H_2, G), U^i(H_1, G) \subseteq U^i(H_2, G)$,

Example of Proposition 5
Let $G = (V, E)$ be a connected graph such that: $G = C_1 \cup C_2 \cup C_3$.
Additionally let $H_1 = (V_1, E_1)$ be a subgraph of G with $V_1 = \{B_1, B_2, B_4, B_5\}$.
Let $G - H_1$ denote the vertex-complement of H_1 in G, that is $V(G - H_1) = V \setminus V_1 = \{B_3, B_6, B_7\}$.

Then, for $i = 1$, the following relations hold:
$L^1(G - H_1, G) = V \setminus U^1(H_1, G)$,
$U^1(G - H_1, G) = V \setminus L^1(H_1, G)$.

From the definition of the neighborhood in G, we have:
$N(B_1) = \{B_2, B_3, B_4, B_5, B_6\}, N(B_2) = \{B_1, B_4, B_5\}$,
$N(B_3) = \{B_1, B_6, B_7\}, N(B_4) = \{B_1, B_2, B_5\}$,
$N(B_5) = \{B_1, B_2, B_4, B_7\}, N(B_6) = \{B_1, B_3, B_7\}$,
$N(B_7) = \{B_3, B_5, B_6\}$.

Then,
$L^1(H_1, G) = \{B_2, B_4\}$.
$U^1(H_1, G) = V$.
$L^1(G - H_1, G) = \emptyset$.
$U^1(G - H_1, G) = \{B_1, B_3, B_5, B_6, B_7\}$.
Since, $U^1(H_1, G) = V$.
Then, $V \setminus U^1(H_1, G) = \emptyset = L^1(G - H_1, G)$.
Also, since, $L^1(H_1, G) = \{B_2, B_4\}$,
Then, $V \setminus L^1(H_1, G) = \{B_1, B_3, B_5, B_6, B_7\} = U^1(G - H_1, G)$.
Therefore, $L^1(G - H_1, G) = V \setminus U^1(H_1, G), U^1(G - H_1, G) = V \setminus L^1(H_1, G)$,

VI. Conclusion

In this paper, a framework that integrates both rough set theory and graph theory was proposed for the analysis of experimental data from a desiccant-based air-conditioning system. To begin with, a transformation of the experimental data into a discrete form was carried out by a coding scheme. This step successfully normalized the data and enabled meaningful comparison between operating conditions, where equal codes represented equivalent relations among the conditions. Based on the coded data, a set of graphs was constructed, where each graph captured the similarity structure corresponding to a single parameter. Intersection graph was developed to identify strong relations between different operating conditions based on all parameters. Process of redaction has been applied by removing the parameter C4 and reconstructing the intersection graph. The results showed that the redacted graph is similar to the original intersection graph, and this confirms that the parameter C4 has no effect on the entire connectivity structure. Therefore, the system can be made simpler by removing this parameter without loss of information, and this may be reflected as a reduction in the computation cost. On the other hand, the definitions of rough set were utilized in the graph theory and different propositions were derived. The derived propositions later were applied to the generated graphs. The results showed that these propositions are valid for different graphs, and this confirms that the proposed model is stable. These outcomes proved that the proposed framework has established a connection between rough set theory and graph theory, and this model can be used as an efficient mathematical tool for analyzing similarity relations and identifying redundancy in experimental systems. Finally, the results obtained in this study confirm that the proposed framework is efficient and applicable in practice for simplifying complex systems.

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