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-Ricci Solitons On Para-Kenmotsu Manifolds Admitting Schouten-Van-Kampen Connection

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Abstracts
The present paper is to study of η-Ricci solitons on para -Kenmotsu manifolds with respect to Schouten-Van-Kampen connection. Curvature properties of such manifolds are derived and employed to obtain several structural results. Conditions involving concircular, conharmonic and projective curvature tensors are discussed for para-Kenmotsu manifolds admitting η-Ricci solitons. Furthermore, the behaviour of η-Ricci solitons on submanifolds is analyzed in the context of both Levi-Civita and Schouten-Van-Kampen connections.

Keywords: *Para-Kenmotsu manifolds, Submanifolds, Schouten-Van- Kampen connection, Ricci solitons, -Ricci solitons, curvature tensor.*

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1. Introduction

In 1969, Takahashi [27] studied almost contact manifolds endowed with a corresponding Pseudo-Riemannian manifolds. Afterthat, these manifolds were studied and characterized by Kenmotsu K.[13]. Subsequently in 1976, Sato [20] discussed the concept of an almost para contact Riemannian manifolds. A special type of an almost para-contact Riemannian manifolds investigated by T.Adati and Matsumoto [1]. In 2008, U.C.De [7] studied on ϕ-symmetric Kenmotsu manifolds. LP-Sasakian manifolds studied by R.N.Singh et al. [25]. The notion of para-Kenmotsu manifolds was firstely introduced by Welyczko [28]. The geometric structure of para-Kenmotsu manifolds analogous to that of Kenmotsu manifolds. Moreover Sai Prasad et al. [21], Blaga [4] and Sinha et al. [24] have examined para-Kenmotsu manifolds and their special cases in detail. Since then, para-Kenmotsu manifolds have been extensively studied by many researchers Sardar A. [22] and T. Raghuwanshi et al. [19].

In 1982, Hamilton[11] introduced the concept of Ricci flow to find a canonical metric on a smooth manifolds. The Ricci flow is an evolution equation for metrics on Riemannian manifolds

(1.1)

Ricci solitons represent a natural generalization of einstein metric on a Riemannian manifolds (M^n, g) by

(1.2)

Afterthat, in 2013 U.C.De et al.[9] investigated Ricci solitons and gradient Ricci solitons in a Kenmotsu manifolds. recently, the geometry of Ricci solitons studied by several authors in contact and Lorentizian manifolds Bagewadi et al. [2] , Pandey et al.[17]. Ricci solitons and gradient Ricci solitons studied by De et al.[10]. patel et al.[18] and Siddiqui et al. [23]. η-Ricci solitons is an extension of Ricci solitons. The concept of η-Ricci solitons was first introduced by Cho. and Kimura [6]. Thereafter, $\eta-$ Ricci solitons were studied by Calin and Crasmareanu [5] on Hopf hypersurfaces in complex space form. An η - Ricci solitons is tuple (g,V,λ,μ) where V is s vector field on M^n. λ and μ are constant and g is Riemannian metric satisfying the equation

, (1.3)

where S is Ricci tensor associated to g.

if μ=0 then the notion of an η- Ricci solitons reduces to the notion of Ricci solitons (g,V,λ). We refer the works of Blaga [4], Krishnendu De and U.C. De [8] studied η-Ricci solitons on -Kenmotsu manifolds. Recently η-Ricci solitons is studied by several authors and obtained many intersting geometric properties .

On other hand The Schouten-Van-Kampen connection is one of the most natural connection. It was introduced in the third decade of last century for a study of non-holomorphic manifolds. It preserve by parallelism a pair of Complementary distribution on a smooth manifolds equipped with an affine connection. The author of uses the characterized connection and studies hyperdistribution in Riemannian manifolds. The Schouten-Van-Kampen connection ∇^* on an almost contact metric manifolds with respect Levi-Civita connection ∇ is defined by [15]

(1.4)

for any vector field on .Thus with the help of Schouten-Van- Kampen connection many condition are satisfies : Also the torsion tensor of is defined by

(1.5)

In 1978, Solve'v [26] explored hyperdistribution in Riemannian manifolds using Schouten-Van-Kampen connection. After that, Bejancu[3] studied Schouten-Van-Kampen connection on Foliated manifolds.Then the Olszak [15] has studied the Schouten-Van-Kampen connection to adapt to almost contact metric structure. He determined some classes of almost contact metric manifolds admitting Schouten-Van-Kampen connection.Thereafter some results of Kenmotsu manifolds admitting Schouten-Van-Kampen studied by Gurupadvva Ingalahalli et al.[12], Kumhar B.[14] et al. studied ϵ-Kenmotsu manifolds admitting Schouten-Van-Kampen connection and G.Pandey et al.[16]studied Impact of Schouten-Van-Kampen connection on LP-Sasakian manifolds.

The paper is organized as follows: first the introductory section. Section 2, is dedicated to presenting the basic definition and properties of para-Kenmotsu manifolds. In section 3, we give Some curvature tensor on Schouten-Van-Kampen connection of para-Kenmotsu manifolds. Next, we analyze an η- Ricci solitons on para-Kenmotsu manifolds admitting the Schouten-Van-Kampen connection in section 4. Thereafter, in section 5, we consider certain curvature tensor including the concircular,conharmonic and projective curvature tensor on Schouten-Van-Kampen connection of para-Kenmotsu manifolds admits η Ricci solitons. In last section, we study η-Ricci solitons on submanifolds of para-kenmotsu manifolds with respect to Levi-Civita connection and Schouten-Van-Kampen connection respectively.

2. Para-Kenmotsu Manifolds

An -dimensional Pseudo-Riemannian manifolds Ricci solitons represent a natural generalization of einstein metric on a Riemannian manifolds)is said to be an almost para contact manifolds with an almost contact structure if there exist a (1,1)-tensor field ,a vector field and a 1 - form s.t.

, (2.1)

, (2.2)

for all vector field on .In this case ,we have and . from equation (2.2) it can be easily deduce that for all vector field on An almost para contact mani- fold is called a para Kenmotsu manifolds if

(2.3)

for all vector fields on. Where is the Levi-Civita connection with respect to metric In a para Kenmotsu manifolds we have

, (2.4)

. (2.5)

Using equations (2.4) and (2.5), we find

, (2.6)

(2.7

(2.8)

for all vector field where is the Riemann curvature tensor, is the Ricci tensor of a para Kenmotsu manifolds .

(2.9)

for any vector field on .

3. Curvature Tensor On Schouten-Van-Kampen Connection Of Para-Kenmotsu Manifolds

The Schouten-Van-Kampen connection affiliated to the Levi-Civita connection is endowed by [15] , (3.1)
for any vector field on. Again using equations (2.4) and (2.5) in equation (3.1)
, (3.2)
putting in equation (3.2)
(3.3)

Suppose and are the curvature tensor withrespect to and Levi-Civita connection and Schouten-Van-Kampen connection respectively,
(3.4)
Again by equation (3.4) we have,
(3.5)

(3.6)

(3.7)
again, contracting equation (3.7)
(3.8)
By equation (3.5) if then,
(3.9)
from equation (3.7)
, (3.10)

(3.11)
Again, by first Bianchi identity and equation (3.5), we arrive at
(3.12)
So, we can state that,

Theorem 3.1. On a Schouten-Van-Kampen connection of para Kenmotsu manifolds the Riemannian curvature tensor, the Ricci tensor, and the scalar curvature tensor are assigned by the equations (3.5), (3.7) and (3.8) respectively and is satisfies the first Bianchi identity given by the equation (3.12).

-Ricci Solitons On Para-Kenmotsu Manifolds Admitting Schouten-Van-Kampen Connection

An -Ricci solitons on On Schouten-Van-Kampen connection of para-Kenmotsu manifolds is defined by
(4.1)
As we know that,
(4.2)
using equation (3.2), we obtain

(4.3)

Putting equations (3.7) and (4.3) in equation (4.1)
(4.4)

So , we can state that:
Theorem 4.1. On Schouten-Van-Kampen connection of para-Kenmotsu manifolds admits an -Ricci solitons is invariant if and only if it's satisfies equation (4.4).
Now, if we put in equation (4.1), we deduce that
(4.5)
which can be written as,
(4.6)
Using equation (3.2) we obtain,
(4.7)
By equations (2.4) and (4.7)
(4.8)
By the equations (4.6) and (4.8)
(4.9)
From the equation (3.7)

(4.10)

Therefore, we state that:

Theorem 4.2. An -Ricci solitons on Schouten-Van-Kampen connection of para-Kenmotsu manifolds is Einstein manifolds.

Again, we take into equation (4.8), and we have

(4.11)

From equation (3.7)

(4.12)

So, we state that

Theorem 4.3. An -Ricci solitons on para-Kenmotsu manifolds admitting Schouten-Van-Kampen connection is always shrinking.

5. Some Curvature Condition Of - Ricci Solitons On Para-Kenmotsu Manifolds With Respect To Schouten-Van-Kampen Connection

Concircular curvature tensor admitting Schouten-Van-Kampen connection is defined by

(5.1)

By equations (3.5), (3.8) and (5.1), we obtain

(5.2)

where concircular curvature tensor admitting on Schouten-Van-Kampen connection of para Kenmotsu manifolds.

Now, put in equation (5.2), we get

(5.3)

So, we obtain

Theorem 5.1. A concircular curvature tensor on Schouten-Van-Kampen connection of para-Kenmotsu manifolds determined through equation (5.3) .

Now, we take in equation (4.9) and contracting it, then we find

(5.4)

Using the equations (5.3) and (5.4)

(5.5)

Hence, we conclude the following:

Theorem 5.2. A concircular-curvature tensor of an -Ricci solitons on para- Kenmotsu manifolds admitting Schouten-Van-Kampen connection is given by equation (5.5).

Furthermore, Conharmonic-curvature tensor admitting Schouten-Van-Kampen connection is defined by

(5.6)

On putting equations (3.5) and (3.7) in equation (5.6)

(5.7)

where denote conharmonic curvature admitting Levi-Civita connection.

For in equation (5.7) we get,

(5.8)

From above, we conclude that

Theorem 5.3. A conharmonic- curvature tensor on Para-Kenmotsu manifolds with respect to Schouten-Van-Kampen connection is given by equation (5.8).

Also, by taking equation (5.7),

(5.9)

using equation (2.6) in equation (5.9)

(5.10)

Again, taking in equation (4.10) we get,

(5.11)

By using equations (5.10) and (5.11) we find,

(5.12)

which is conclude that,

Theorem 5.4. A conharmonic curvature tensor of an -Ricci solitons on para- Kenmotsu manifolds admitting Schouten-Van-Kampen connection is given by equation (5.12).

Moreover, the projective curvature tensor admitting Schouten-Van-Kampen connection is defined by

(5.13)

putting in equation (5.13)

(5.14)

By using equations (4.8) and (3.9) in equation (5.14)

(5.15)

Hence, we state the following:

Theorem 5.5. -Ricci solitons on para-Kenmotsu manifolds admitting Schouten-Van-Kampen connection is -projectively flat if and only if .

-Ricci Solitons On Submanifolds Of Para-Kenmotsu Manifolds With Respect To Levi-Civita Connection

The study of submanifolds of an almost contact manifolds is one of the utmost enthralling subject in differential geometry. According to the behaviour of the tangent bundle of a submanifolds with respect to the action of an almost contact structure of the ambient manifolds, there are two well defined types of submanifolds, firstly invariant submanifolds and another anti-invariant submanifolds. Let -dimensional be a submanifolds of para Kenmotsu manifolds , where , with induced metric . Also, let and be the Levi-Civita connection on the tangent bundle and the normal bundle of, respectively . Then the Gauss and Weingarten formulae are represented by

, (6.1)

(6.2)

for all in and in , where and are second fundamental form and the shape operator respectively for the immersion of into the second fundamental form and the shape operator are related by

(6.3)

for any in . A submanifolds of an almost contact metric manifolds is said to be invariant if the structure vector fields is tangent to for every vector field tangent to at every point of. Suppose we take -Ricci solitons on as

(6.4)

Put in equation (6.1) and by equation (2.4)

(6.5)

Since is invariant therefore in Then, on comparing tangential and normal components of (6.5), we obtain

(6.6)

By the equation (6.6)

(6.7)

By equations (6.4) and (6.7) we get

(6.8)

Again, using equation (6.6) and the curvature formula becomes

(6.9)

On contracting the equation (6.9)

. (6.10)

Put in equation (6.8) and use equation (6.10), then we obtain

(6.11)

So, we can state the following:

Theorem 6.1. If is invariant submanifolds of para-Kenmotsu manifolds admits an -Ricci solitons then is

1. Einstein manifolds.
2. Always shrinking.

7.-Ricci Solitons On Submanifolds Of Para-Kenmotsu Manifolds With Respect To Schouten-Van-Kampen Connection

Assuming that Levi-Civita connection and Schouten-Van-Kampen connection on submanifolds of para-Kenmotsu manifolds with Levi-Civita connection and SVK connection [15]. Here we denote the Second fundamental form with respect to SVK connection by . Then Gauss formula with respect to SVK connection can be respresented as

(7.1)

Now, using the equation (3.2) in equation (7.1)

(7.2)

By the equations (6.1) and (7.2)

(7.3)

(7.4)

and

(7.5)

Thus, we have

Theorem 7.1. Assuming be a para-Kenmotsu manifolds whose an invariant submanifolds equppied with two connection Levi-Civita and Schouten-Van-Kampen connections respectively and be the induced Levi-Civita and Schouten-van-Kampen connection on from , respectively. Then we obtain

1. admits Schouten-Van-Kampen connection.

2. The second fundamental form are identical admitting .

Again, we obtain the Riemann curvature tensor, Ricci tensor and scalar curvature tensor of .

Theorem 7.2. If be a para-Kenmotsu manifolds whose submanifolds is admitting Schouten-Van-Kampen , then the curvature tensor is obtain

(7.6)

Ricci tensor is determined

(7.7)

The scalar curvature is represented as

(7.8)

Next, assuming an - Ricci solitons

(7.9)

By taking identically step done in proving theorems (4.2) and (4.3) we get,

Theorem 7.3. Suppose be an - Ricci solitons on an invariant submanifolds of para-Kenmotsu manifolds admitting Schouten-Van-Kampen connection then is

1. Einstein manifolds.

2. Always shrinking.

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