IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-0661, p-ISSN: 2278-8727, Volume 21, Issue 5, Ser. 2 (Sept. – Oct. 2025), PP 29-33
www.iosrjournals.org

Baudhayana Theorem To Zariski Cancellation Problem

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Abstract
"Brahmins on the banks of Ganges are entitled to some credits"- F. Cajori
Mathematics has its origin in India prior to Rig Veda. Sulba-sutras of Katyayana, Apastamba and Baudhayana had been composed around 800 BCE. 'Pythagoras' theorem is being stated in 'Sulba sutras'. The Vedic era was good with numbers they worked on surds, mensuration, fractions and were able to approximate irrational numbers like $\sqrt{2}$. *The Decimal system and Arithmetic is India's gift to all. 'Zero' was formulated around 200 BCE before the time of Pingala . The decimal system helped in the progress 'trade and commerce' as well as 'astronomy' and 'mathematics'. 'Renaissance' occured in Europe after the adoption of Indian notations in 16th-17th century. 'Geometry' originated in the 'Vedic era'. Theorems (628 CE) on 'cyclic quadrilaterals' proposed by Brahmagupta is the 'base' in the progress of geometry. Indians started a conceptual use of symbols to represent unknown quantities and arithmetic operations. In India algebra achieved it's epitome. Indians studied equations (sami-karana) , coined negative numbers and the 'operations' for computational problems. Indians obtained the 'solutions' of 'linear' and 'quadratic' equations. Indian obtained formulas for geometric progression and arithmetic progression, summation of finite series,* ${}^{n}_{r}P$ *and* ${}^{n}_{r}C$. *Bhaskaracharya (1150 CE) by using concept of infinitesimal changes showed the world path of 'calculus' , which is backbone of all modern sciences. Estimations for '*π*' are given in Indian texts. The concept of integration as the 'limit of a sum' is mentioned in 'Yuktibhasa'. There was understanding of convergence in infinite series. Madhava had evaluated power series expansions : Sine and Cos Series. The great astronomer Nilakanta Somayaji defined the heliocentric model before Copernicus. India's gift to mankind is technically all modern science.*
Keywords: *India, Vedas, Mathematics, Scriptures, Arithmetic, Decimal system*

Date of Submission: 01-10-2025 Date of Acceptance: 11-10-2025

I. Introduction

Mathematics has been an important part of human knowledge for centuries. As per findings of famous algebraist 'A Seidenberg', mathematics has its origin in India prior to Rig Veda[1]. During medieval and ancient times works in mathematics were in 'Sanskrit sutras' in concise form to make it easy to memorize. 'Sruti' that is 'what is heard' through recitation was the way to pass on knowledge in ancient India. Till 500 BCE roughly 'Guru-shishya parampara' was the only way to transmit knowledge. Afterwards, transmission was through 'manuscript' too. "Bakhshali Manuscript", the birch bark obtained from Bakhshali in today's Pakistan in 1881 is probably written in the 700 CE and is primitive document from the undivided India [2].

History

"Basic mathematics" was applied by the inhabitants from the Indus Valley Civilization. The bricks were in proportional dimensions so as to have the stability in structure. They standardized the weight measure with the weight unit roughly 28 grams in ratios like 1/20, 1/10, 1/5, 1/2, 1, 2, 5, 10, 20, 50, 100, 200, and 500. The weights were in geometrical shapes like cones and cylinders, emphasizing on basic 'geometry'. Indus civilisation peasants also standardised measure of length by designing—the 'Mohenjo-daro ruler'—with length of unit roughly 1.32 inches or 3.4 centimetres. In Lothal (2200 BCE) and Dholavira; cylindrical shell objects are excavated which were probably used to evaluate angles in a plane and to calculate position of stars for navigation [2].

The Development

Pāṇini (520–460 BCE) was a sanskrit grammarian who used the 'Backus–Naur' concept which is used in 'programming languages' [2]. 'Sulba-sutras' of Baudhayana, Apastamba and Katyayana had been estimated to have been composed around 800 BCE. Sulba writers emphasised that they were merely writing the results, already known to the people of the early Vedic age. In the Sulba-sutras, we see an explicit statement of the 'Pythagoras' theorem and its applications in various geometric constructions. 'Pythagoras theorem' was available in other civilizations also for example 'Babylonian'. Vedic inhabitants were proficient in computation as well.

References of computing 'arithmetic of fractions', 'surds', 'rational approximations' and various results in 'mensuration' are also obtained [1].

Mathematical topics are also available in 'Jain texts' (600 BCE) . Jains' were fascinated with 'numbers' and 'infinities', they believed that numbers have deep meaning and that understanding can be applied on the space. They divided numbers into 'three' forms: infinite, innumerable and enumerable[2]. Later this developed as 'finite', 'countable' and 'uncountable'. 'Infinity' was also further classified, which helped them to decode "cosmology". Jain mathematicians obtained representations for exponents and powers of numbers which equipped them to introduce 'algebraic equations' (bījagaṇita samīkaraṇa). They used 'shunya' (empty in Sanskrit) to refer to zero. This word led to English word "zero", as it was coined in Arabic 'ṣifr' and then calqued as 'zephirum' into Latin used during middle ages, before 'zero' [2]. Virasena (800 CE) was a Jain mathematician, who says in '*Dhavala'* multiplicity of 2 as a factor, as the hypothesis of "*ardhaccheda"*, along with the rules to be followed in this operation. This is the same as the 'binary logarithm'[2]. The last of the noteworthy Jain mathematicians 'Mahavira Acharya' (800–870 CE) wrote a book '*Ganit Saar Sangraha'*. The scripture includes the knowledge of Solid and plane geometry, sum of arithmetic and geometric series, cube roots, square roots, cubes, zero and many more along with it stressed the 'square root' of 'negative' number fails to be obtained. He also obtained the formulas to evaluate the 'area' of a quadrilateral and an ellipse inside a 'circle' [2]. Their works also contained formulas for "permutations and combinations".

The Buddhist School, were not as famous as Jains but they were very good with Decimal place value system.

The 'Kerala School of Mathematics and Astronomy' buring 1400-1600 CE made contributions in Trignometry, calculated planetry positions and celestial events, scripted "Yuktibhasha" and " Tantrasangraha" also improved caalender systems on astronomical observations.

Ancient mathematicians

Pingala (300–200 BCE) blended poetry and mathematics, his work is written in '*Chhandas Shastra*' i.e. "The science of Meter". His work mentions the ideas of 'Fibonacci numbers' and his familiarity with 'zero' [1,2]. 'Halayudha' (1000 CE) refers in his work that Pingala had knowledge about the 'combinatorial identity' too [2].

400–1300 CE period is the "classical period" for Indian Mathematics. During this time many eminent mathematicians like, varahamihira, Aryabhata, Bhaskara I, Brahmagupta, Bhaskara II, Mahavira, Nilakantha Somayaji and Madhava of Sangamagrama provided growth to different sections of mathematics. Their work transmitted from Asia to Arab countries, and then 'Europe'. This golden period of mathematics is in the 'astral science' (jyotiḥśāstra) and constitute three sub-sections: 'horoscope' astrology (horā or jātaka), 'mathematical sciences' (gaṇita or tantra) and 'divination' (saṃhitā) [2].

Aryabhata's (476–550 CE) scripture 'Āryabhaṭīya' (499 CE) is the earliest mathematical prose which contains 'sutras' on 'mathematics' and 'astronomy' but without any proofs [1]. '*Aryabhatiya'* has 332 shlokas. The text contains trigonometry, results of simultaneous quadratic equations, the π value, estimation of time and duration of 'solar and lunar eclipse' among many results [2]. He stated "*kalyug*" started on 18 February, 3102 BCE. This date is accepted for Hindu Calender. His calculation for one year time period was 365.25858 days which is very close to accepted value of 365.25636!

Bhaskara I (600–680 CE) amplified the Aryabhata's studies further in his books named '*Aryabhatiya-bhashya'*, '*Mahabhaskariya'* and '*Laghu-bhaskariya'*. He obtained: value of the sine function, rule for evaluating the sine of an acute angle along with solutions of indeterminate equations etc [2].Bhaskara I comments on 'Āryabhaṭa' includes rules, 'reason' or base of any rule, examples, working and verification [2]. By 600 CE at the time of Bhaskara I, prose started mentioning 'derivations' (upapatti) [2].

Bhaskara II (1114-1185 CE) developed understanding of 'calculus' and 'equations'. He introduced "*Chakravala method*" for solving "*Pell's equation*": $x^2 - Ny^2 = 1$. He wrote '*Shiddhanta Siromani*' which has four sections like "*Lilawati: Arithmetic and geometry*", "*Bijganita: Algebra*", "*Goladhyaya: Spheres*" and "*Grahaganita: Planets*". He also obtained Sine value of many acute angles. His contributions were honoured by naming a satellite "Bhaskara II" by ISRO.

Out-Reach and Growth of Indian Mathematics

600 BC-300 CE, 'Greeks' contributed a lot in mathematics - they developed the approach of 'axiom', which is the foundation for 'modern' mathematics, built 'abstract' structure for 'Euclidean geometry', introduced 'trigonometry', 'number theory', and presented the inner beauty of 'pure mathematics'. 'Conic sections' and 'integration' were introduced by *Archimedes and Apollonius* based on the foundation provided by *Euclid*. But, 'creative mathematics' literally stopped in the "West" after this gleaming phase of the Greeks, until its modern revival. However, mathematics in India which started developing in 'Vedic-era' continued to thrive till 1600 CE, especially in the field of arithmetic, algebra and trigonometry [1].

Greece was the source of knowledge in "Europe" and "India" was playing the same role in south and central part of "Asia". These two countries were the power house of philosophy, political ideas, architecture, culture and grace because of their rich heritage.

Indian mathematics was well recognized by 'scholars'. For example, a manuscript found in a Spanish monastery (976 CE) reads: "The Indians have an extremely subtle and penetrating intellect, and when it comes to arithmetic, geometry and other such advanced disciplines, other ideas must make way for theirs. The best proof of this is the nine symbols with which they represent each number no matter how large" [1]. The 'Decimal place-value system' introduced in India is a priceless gift to the world. This magnanimous Indian innovation is unsurpassable for its utility in practical life [1]. 'Greeks' have no representation for numbers more than myriad 10^4 also 'Romans' could not nominate anything more than 'mile'(10^3). The concept of 'Sanskrit' numeral representation and 'Indian' passion with big numbers initially fuelled the designing of the 'decimal system' [1]. There is a mention and nomenclature for multiples of ten till 10^{18} during preliminary 'Vedic' compositions, the 'Ramayana' contains expressions as high as 10^{55} , also 'Jaina-Buddhist' writings exhibit recurrent mention of numbers up to10^{140}! for their reference about time along with space. These types of expressions cannot be seen in other civilizations 'works' [1]. The decimal system is easy to follow. It was adopted initially by 'Arabs' and they transmitted it to 'Europe' [2]. It has less number of symbols with small representations. It has an easy computational facility and can effortlessly represent arbitrarily large numbers, e.g., 888 in the decimal system is the twelve-digit Roman number DCCCLXXXVIII![1] '*Kharoṣṭhī* script'(400 BCE- 400 CE), 'Brāhmī script' (250 BCE) which is the base of many South Asian and South-east Asian scripts both have 'symbols' and 'systems', but they were not derived from a system based on place-value [2].

Due to inadequacy of the system of symbols, the 'Greeks' were not good in 'computational mathematics' - possible reasons liable for the fall of 'Greek mathematics'. *Leonardo Fibonacci of Pisa* (1170-1250 CE), the preliminary big European mathematician, led the way in expanding the Indian numeral system in 'Europe'. The Indian method of solving long multiplications and divisions was introduced in Europe around the 14th century CE [1]. Al-Khwarizmi was an 'Islamic scholar' whose role in spreading 'Indian mathematical' knowledge to the West cannot be forgotten and there must have been many like him [3]. In 1600-1700 CE 'Indian notation' with 'arithmetic' got acknowledged in 'Europe'. Decimal place value system quickened and advanced, 'trade and commerce' as well as 'astronomy' and 'mathematics'. There is no serendipity that the scientific and mathematical 'Renaissance' revived in 'Europe' only after adopting the Indian nomenclature. Infact, the decimal representation is the tower of strength of all modern society [1]. Europe was divided by two ideologies 'Abacists' and 'Algorists'. Algorists use to work on sand-board and computed using numbers and symbols however abacists worked with beads placed on strings and wires. Algorists started using decimal representation and slowly because of its ease of use abacists also accepted this system , bringing a closure to long said differences [5].

In the 700 CE, "mathematics of algorithms" i.e. arithmetic (*pāṭī-gaṇita,* measurement) and "mathematics of seeds", i.e. algebra (*bīja-gaṇita*). Brahamgupta (600 CE) proposed ' theorem of diagonals of a cyclic quadrilateral' and also gave 'Herons formula'[2].

Although 'geometry' was formulated at the beginning of Vedic era, its 'development' and 'axiomatisation' was done by the Greeks. The Greeks attained great heights in geometry by the era of *Apollonius* (260-170 BCE) and were ahead of any upcoming medieval or ancient society. However, development in geometry quickly reached standstill. During the era 300 CE *'Pappus'* - the famous name of 'Greek geometry' in developing Europe, results of *Brahmagupta's* about(628 CE) "cyclic quadrilaterals" formulated the 'stepping stones' in geometry and created a landmark. To proceed ahead new techniques were required, actually a new approach was need of the hour - algebra. 'Algebraization of geometry' established an algebra culture in 'European mathematics' around 1600 CE. Revival of geometry began in early 1700 CE by *Descartes and Fermat* . Actually, from 1700 CE in Europe the refinement and acclimatization of algebra also helped in development of 'number theory' along with 'calculus' [1]

In the 2000 CE "algebra" played an important part in modern mathematics not only as an independent branch but also as a vital instrument in other fields, there was a robust phase of 'algebraisation of mathematics' after evolving through many stages [1]. Indians initiated a methodical use of "symbols" to represent 'arithmetic operations' and 'unknown quantities'. The four operations were called as "yu", "ksh" "gu" and "bha" which were the first alphabets (or a small change) of the correlated Sanskrit words 'yuta' (addition), 'ksaya' (subtraction), 'guna' (multiplication) and 'bhaga' (division); similarly "ka" was used for 'karani' (root), To represent various unknown variables the first letter of different colours names were chosen. This symbolic representation was rapid advancement in mathematics; an innovation. Though Greek texts of '*Diophantus'* also mention the use of symbols, in India complete development in algebraic formalism was achieved. The Indians also classified and rigorously studied equations (sami-karana) , introduced 'negative numbers' results on 'surds', 'solutions' of "linear and quadratic equations", defined results on "arithmetic and geometric progression" and the formulas for rnP and rnC.[1]

Courtesy, early prevalence of 'negative numbers', Indians amalgamated the solutions of the different types of 'quadratic equations',i.e., $ax^2 + bx = c$, $ax^2 + c = bx$, $bx + c = ax^2$. The Indians pioneered that a 'quadratic equation' possesses two roots. *Sridharacharya* in 750 CE led way to the solution of "quadratic equation" by forming perfect 'square' - an innovation having profound results. *Halayudha* in the 1000 CE described "The *Pascal's* triangle" for fast computation of rnC as '*Meru-Prastara'* approximately 700 years before *Pascal* ! [1]

Algebraization of 'infinitesimal changes' created "calculus" during the time of Bhaskara II. Greeks were founders of 'trignometry' still they could not progress as they lacked algebraic rules and symbols [7]. '*Sulba Siddhanta*'(400 CE) contains the basis of modern trigonometry.

Indians formulated 'sine' ,'cosine' functions, conceived the formula of *Sin(A±B).* The 'interpolation formulas' for calculation of 'sines' intermediate angles were given by '*Brahmgupta'* (628 CE) and '*Govindaswami'* (880 CE).

Astonishing approximations for "**π**" are mentioned in our scriptures like 3.1416 of *Aryabhata* (499 CE), 3.14159265359 of *Madhava* (1400 CE) and 355/113 of *Nilakanta* (1500 CE). Ananonymous work *"Karanapaddhati"* probably narrated by *Putumana Somayajin* in the 1500 CE has the value up to seventeen decimal places 3.14159265358979324 which is true. The growth of 'originality' and 'excellence' in Indian trigonometry attained a pinnacle in the astounding discoveries of *Madhavacharya* (1340-1425 CE) about the power series expansions in trigonometric functions. *Madhava* had described the series 300 years before *Gregory (1667 CE),* [1]

$$\theta = tan\,\theta - \frac{tan^3\theta}{3} + \frac{tan^5\theta}{5} - \cdots \quad (|tan\theta| \leq 1)$$

His result, in "Yuktibhasa", uses the '**idea of integration'** as 'limit of a sum' aligning with the current methods along with 'term-by-term' integration. A critical step is using the result

$Lim\, n \to \infty(1^p + 2^p + \cdots + (n-1)^p) \quad /n^{p+1}$ = 1/(*p* + *1*).

The use of $|tan\theta| \leq 1$ shows magnitude of refinement in comprehension. Text on 'infinite series' includes concept of **convergence**. *Madhava* also obtained [1]

π/4 = 1 - 1/3 + 1/5 - 1/7 +…

Subsituting θ = π/4 in "*Madhava-Gregory"* series. This series resurfaced after 300 years by *Leibniz* in 1674 CE. '*Madhava'* obtained the following series too

$\pi/\sqrt{12} = 1 - 1/3.3 + 1/5.3^2 - 1/7.3^3 + \ldots$

Introduced in Europe by *A Sharp* (1717 CE). Madhava has also narrated **Sine and Cosine series**, 300 years ahead of Newton (1676 CE)**.** These series helped in calculations in "**astronomy" [1]**.

Contribution of Ancient Mathematics to Contemporary Mathematics

Vedic Mathematics works on mental maths and provides methods that allow humans to solve difficult calculations in their minds in fraction of time[3].Most of the results in basic 'arithmetic' are from 'India'. These are neat and systematic methodsusing addition, multiplication, subtraction and division, evaluating squares (cubes) and square roots (cube roots), financial problems of profit-loss, income-expenses, simple-compound interest, partnerships-discounts and cistern problems similar to what we solve nowdays.

Jain mathematics was based on reasoning.Jain mathematical principles are revant in artificial intelligence, finance, combinatorics, probability theory and even modern physics. Jain texts also make relevance for data sciences, cryptography and machine learning algorithms. The doctrine of "*Syadvada*" has text similar to the concept of 'Fuzzy Logic' Which helps to work along with uncertainity and thus made self driving Cars a reality.

Pingala's method of attaining long and short syllable (1 and 0) in combinatorics indicated "numerical progress", that helped in further electronics.

Sadly, the original work of many great minds like *Jayadeva, Madhava, Sridhara and Padmanabha* has not obtained. Subsequent mention of their works or reference to some of the propositions made by them, we get knowledge of their achievements. *Madhava's* results were acknowledged in scripts obtained during 1500 CE for example 'Tantra Samgraha' written by *Nilakanta Somayaji* (1445-1545 CE) an astronomer *by profession.* He developed the **'heliocentric model'** much before it was discovered by *Copernicus, the 'Yuktibhasa' scripture dated around 1540 CE of "Jyesthadeva"*(1500-1610 CE) and the *'Karanapaddhati*'. *Charles Whish* discovered these scriptures and were made available by 1835 CE. *Aryabhata, Brahmagupta and Bhaskaracharya* are the primitive mathematicians whose scriptures are obtained. They all were eminent 'astronomers' too [1].

Modern Indian Mathematicians

There was no development in area of mathematics from 1700-1900 CE - during this period India was ruling under 'British empire' and in general there was a pause in growth. Even after gaining Independence, the social political scenario made it very effortful to have or live in current India. Hence, mathematics taught in high school, more importantly "arithmetic" and "algebra", is more of based in India , we hardly see Indian origin names in higher education along with university courses.

However, some great minds contributed to Mathematics in the 19th century. India's greatest genius Srinivasa Ramanujan made lot of contributions to the analytical theory of numbers and worked on elliptic functions, continued fractions, and infinite series [3]. He developed formulas for partition functions prime numbers. He gave approximations for '*Eulers constant'*. He compiled 3900 results which were equations and identities.His results found application in cryptography and computer science. "22 December" his date of birth is celebrated as "National Mathematics day" in India.

Harish Chandra Mehrotra has worked on harmonic analysis and on semi simple Lie groups along with c-functions and many algebra field. He received "Cole Prize" in Algebra and "Ramanujan medal."

C R Rao a well known statisian contributed "Theory of Estimation" and many more results in Statistical theory. P C Mahalanobis introduced Mahalanobis Distance and founded ISI. He also designed large scale sample surveys in India. We also have prodigy like D R Kaprekar who without formal mathematical education described several classes of natural numbers.

Satyendra Nath Bose collaboration with Albert Einstein is well known. He made significant advances in quantum statistics, X- ray diffraction and electromagnetic waves [4]. His and contributions formed the base of Bose-Einstein statistics and bosons. He also gave "Plank's law" for blackbody radiations. Inspite of doing phenomenal work Bose never received Nobel prize which clearly represents India's undermined position among Western Researchers. He was awarded "*Padma Vibhushan".* The 'boson' term is named after him and his work is fundamental in quantum theory.

In 2011 'Prof. Kannan Soundararajan' published a reputed research paper on "*Multiplicative functions and the zeros of L- Functions*".

In 2013 'Prof. Arul Shankar' and 'Prof. Manjul Bhargava' provided the work on the "*average rank of elliptic curves ()"* through new techniques by using geometry of numbers and vector spaces.

In 2013 only, 'Prof. Mahesh Kakde' provided proof for the conjecture of the " *Non –Commutative Iwasawa Theory for Totally Real Fields*"

In 2014 "Prof. Neena Gupta" solved "*Zariski cancellation Problem for Positive Characteristic*" an open problem since 1949.

All this work was appreciated throughout the world. Indians are showing their potential to the world.

II. Conclusion

Mathematics is a significant part of our scientific and philosophical learning.India's expenditure on research and development is approximately 0.64% of GDP compared to the global average of 2.6% and about two percent on the publications. The big issue today is that the development of India lies in retaining the extraordinary talent of the country. India is home to one of the largest intellectual socities of the world, but only few of those graduating getting into research [5]. Brilliant minds must step forward for the future of mathematical Research.

We should not forget our culture and intellect our 'bloom' because of 'ignorance' and 'weakness' that India was going through under various civic and national disturbances. We are proud Indian, Proud of our Heritage. We need to relive the words of "Aryabhata" who also correctly proposed a spherical Earth that rotated on its own axis.

"By the grace of Brahma," he says, "I dived deep in the ocean of theories, true and false, and rescued the precious sunken jewel of true knowledge by the means of the boat of my own intellect"[6].

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