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A Mathematical Optimization Model For Ad Budget Allocation To Maximize ROI In Digital Marketing

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Abstract
In the evolving digital marketing landscape, effective allocation of advertising budgets across platforms such as Google Ads and Facebook Ads has become a pivotal factor in maximizing return on investment (ROI). This study proposes a mathematical optimization model using linear and non-linear programming techniques to allocate digital ad budgets based on data-driven metrics such as impressions, click-through rates (CTR), and conversion rates. Real-world campaign data was collected and analysed, and the model was applied to determine optimal budget allocation strategies. The results demonstrate the efficacy of the model in enhancing ROI and provide practical insights for digital marketers aiming to improve performance outcomes through mathematical optimization.
Keywords: *Ad Budget Allocation, ROI, Digital Marketing, Linear Programming, Non-Linear Optimization, Google Ads, Facebook Ads, CTR, Conversion Rate*

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I. Introduction

In the rapidly evolving digital landscape, advertising has undergone a paradigm shift from traditional media to online platforms. Businesses, regardless of size, now invest a significant portion of their marketing budgets into digital advertising, leveraging platforms such as Google Ads, Facebook Ads, Instagram, LinkedIn, and others to reach their target audiences. With this transformation comes the challenge of optimal budget allocation across various platforms and campaigns to ensure maximum return on investment (ROI).

Digital marketing offers a multitude of performance metrics including impressions, click-through rates (CTR), cost-per-click (CPC), conversion rates, and customer acquisition cost (CAC), among others. These metrics provide marketers with measurable insights into campaign performance. However, the abundance of data also adds complexity to the decision-making process. Marketers must determine how to allocate finite advertising budgets across multiple platforms and campaigns while accounting for the dynamic and non-linear nature of digital consumer behaviour.

In many organizations, ad budget allocation decisions are still made using heuristic or experience-based methods. While such methods may have short-term utility, they lack the rigor and adaptability of data-driven, optimization-based approaches. Furthermore, platforms differ significantly in audience behaviour, cost structure, and conversion potential, making uniform allocation strategies inefficient and suboptimal.

Mathematical optimization provides a structured and quantitative framework to solve allocation problems where multiple objectives and constraints coexist. In particular, Linear Programming (LP) and Non-Linear Programming (NLP) are well-established tools in Operations Research that can be effectively employed to address the complexities of budget allocation in digital marketing. These techniques can model the relationships between spend and returns, accommodate constraints such as minimum and maximum budgets per platform, and help identify the allocation strategy that yields the highest ROI.

This study proposes a mathematical optimization model that integrates real-world campaign data—such as impressions, CTR, and conversion rates—from leading digital advertising platforms. The goal is to allocate a given advertising budget in a way that maximizes ROI. By applying linear and non-linear optimization techniques, the study seeks to demonstrate how businesses can achieve improved efficiency, higher conversions, and better financial outcomes from their digital marketing efforts.

The significance of this research lies in its dual contribution: first, it provides a practical model that can be easily adopted by digital marketers and businesses of various scales; second, it advances the academic discourse on the intersection of applied mathematics, optimization, and digital marketing. The model's application not only aids in budgetary efficiency but also aligns with the broader trend toward automation and data-driven decision-making in marketing.

In the sections that follow, the paper reviews relevant literature, outlines the research methodology, details the construction of the optimization model, and discusses the empirical results derived from applying the model to real campaign data. The findings underscore the utility of optimization-based approaches in enhancing marketing performance and offer valuable implications for future research and practice.

II. Review Of Literature

Dholakia (2020)

Dholakia's study highlights the significance of data-driven strategies in digital marketing. The research emphasized the use of performance metrics such as CTR and conversion rate for real-time decision-making. He found that campaigns guided by quantitative insights achieved significantly higher ROI than those relying on fixed budget allocation methods. This reinforces the role of analytics and data science in formulating advertising strategies.

Ghose & Yang (2009)

This empirical research examined the mechanics of sponsored search advertising and modelled the relationship between ad positioning, bid price, and click-through performance. The authors constructed a framework to estimate user behaviour and advertiser responses using optimization techniques. Their work contributes to the understanding of how economic and mathematical models can enhance decision-making in digital ad bidding environments.

Kumar & Gupta (2016)

Kumar and Gupta provided a historical and conceptual overview of advertising's shift from traditional to digital formats. They emphasized the increasing relevance of algorithmic and mathematical methods in designing cost-efficient advertising strategies. The study proposes that in an age of digital transformation, data and optimization models should replace intuition-based decision-making.

Li & Kannan (2014)

This research addressed the problem of conversion attribution in multichannel online marketing. It proposed an empirical model that assigns credit to different channels based on their contribution to conversions. The model improved budget allocation accuracy and helped identify high-performing segments. Their findings support the inclusion of granular performance metrics in optimization models for ROI improvement.

Zhang, Yuan & Wang (2014)

Zhang et al. proposed an optimal real-time bidding (RTB) strategy for display advertising using mathematical modelling. They developed a dynamic programming model that maximized ROI while managing cost constraints. This study forms a theoretical foundation for including time-dependent variables and platform-specific constraints in ad budget allocation models.

Chan & Perry (2017)

Chan and Perry examined how companies allocate digital ad budgets based on historical data and predictive modelling. Their work compared rule-based versus optimization-based budget allocation strategies and concluded that optimization techniques, when based on real-time campaign data, consistently yield better ROI. They also recommended using adaptive models for budget updates.

Srinivasan & Sridhar (2012)

The authors created an operations research framework to guide marketing budget allocation across channels. They used linear programming under constraints such as budget limits and minimum media coverage. Their models serve as practical tools for marketers to balance ROI with strategic marketing goals.

Berman & Katona (2013)

This paper developed a theoretical model to show how consumers interact with search ads and how businesses can optimize their ad placement strategy. The authors used linear programming to determine budget and bid optimization under different competitive scenarios. Their work is closely aligned with the goal of maximizing ROI based on platform performance.

Chatterjee & Hoffman (2020)

This study reviewed how modern digital marketers integrate optimization algorithms and analytics into campaign management. The authors concluded that using real-time optimization based on data such as CTR, conversion rates, and customer journey leads to significantly better targeting and ROI. Their study supports the use of linear and non-linear models in performance marketing.

Edelman & Schwarz (2010)

The researchers developed an optimal auction model for sponsored ads and introduced mathematical techniques to improve ad rank while minimizing costs. Their model showed that strategic bidding and budget control, when optimized using linear methods, greatly enhance ROI. The study supports the notion of integrating economic and mathematical strategies for ad optimization.

Research Gap and Objectives

Despite the proliferation of digital marketing strategies, there is a lack of robust mathematical models that integrate real-time advertising metrics into budget allocation decisions. Existing research either ignores optimization or lacks platform-specific constraints.

Objectives:

- To develop a mathematical model for optimal ad budget allocation.
- To apply optimization techniques using ad impressions, CTR, and conversions.
- To maximize ROI while adhering to campaign-specific constraints.

III. Research Methodology

This section outlines the research design, problem statement, objectives, scope, data collection methods, research framework, mathematical tools employed, and limitations of the study. The methodological structure ensures a rigorous and systematic approach to developing and validating the optimization model for digital ad budget allocation.

Research Problem

In the context of modern digital marketing, businesses invest heavily in online advertising across various platforms such as Google Ads and Facebook Ads. However, due to differences in user behaviour, pricing algorithms, and platform-specific engagement metrics, a uniform budget allocation often results in suboptimal returns. The core problem is how to allocate a fixed digital ad budget among multiple platforms to maximize ROI, considering key campaign performance indicators such as impressions, click-through rate (CTR), and conversion rate. Traditional allocation strategies do not sufficiently account for this complexity, creating a need for a mathematical optimization model that offers a data-driven solution.

Objectives of the Study

The primary objectives of this research are:

1. To develop a mathematical optimization model (using linear and non-linear programming) for ad budget allocation across digital marketing platforms.
2. To apply the model using real-world digital advertising metrics like impressions, CTR, conversion rate, and conversion value.
3. To evaluate the effectiveness of the model in maximizing return on investment (ROI) under different budget scenarios and constraints.

Scope of the Study

- The study focuses on paid digital advertising through platforms such as Google Ads and Facebook Ads.
- The scope includes data analysis, model development, and testing using historical or simulated real-time campaign data.
- The model considers single-period optimization (not continuous time series).
- The model is generalizable to other digital platforms with similar performance metric structures.

Research Design

- **Type of Research**: Applied, Quantitative, and Analytical
- **Approach**: Model-driven and data-centric
- **Nature**: Empirical and computational
- **Sample Type**: Purposive sampling (campaign data from selected digital platforms)

Data Collection Methods

Primary Data: Real or simulated campaign data from digital platforms including Daily Ad Spend, Number of Impressions, Click-Through Rate (CTR), Conversion Rate (CVR) and Conversion Value (₹)

Sources: Google Ads Manager reports, Meta Business Suite (Facebook Ads), Google Analytics (for conversion tracking) and Third-party digital marketing dashboards (e.g., HubSpot, SEMrush)

Secondary Data:

1. Literature from marketing journals
2. Case studies on ad performance
3. Platform-specific benchmarks and white papers

Research Framework

The proposed framework consists of the following stages:

1. **Data Acquisition**
   - Collect platform-specific metrics (spend, CTR, CVR, etc.)
2. **Model Formulation**
   - Develop ROI functions for each platform
   - Formulate the optimization problem (objective + constraints)
3. **Model Implementation**
   - Apply Linear Programming (LP) for simple linear ROI models
   - Use Non-Linear Programming (NLP) for complex ROI curves with diminishing returns
4. **Model Testing & Validation**
   - Run scenarios with real/simulated data
   - Analyse model accuracy and ROI improvement
5. **Sensitivity Analysis**
   - Test model robustness by changing key parameters (e.g., budget limits, CTR)

Mathematical and Statistical Tools

Tool/Technique	Purpose
Linear Programming (LP)	Optimal allocation under linear assumptions
Non-Linear Programming (NLP)	Handles diminishing returns in ROI curves
Python (PuLP, SciPy)	Model implementation and optimization solving
Excel Solver	Scenario analysis and model testing
Descriptive Statistics	Summarize campaign data (mean, range, variance)
Sensitivity Analysis	Evaluate robustness of optimization outcomes

Model Structure (Simplified)

Let: x_i be the budget allocated to platform (i), where (i = 1, 2, …, n)

$R_i(x_i)$ be the ROI function of platform (i), derived from actual CTR, conversion rate, and conversion value

Objective Function

Maximize ROI:

$$\text{Maximize } Z = \sum_{i=1}^{n} R_i(x_i)$$

Where:

x_i : Budget allocated to platform i

R_i : ROI function based on CTR, CVR, and conversion value for platform i

$R_i = CTR_i \times CVR_i \times Value_i \times x_i$

Constraints

$\sum x_i \leq B$ ***(Total Budget Constraint)***

$x_i \geq 0$ ***(Non-negativity)***

$x_i \leq U_i$ ***(Optional : Upper budget limit per platform)***

The impact on total ROI and individual budget allocations was analysed using graphical and tabular summaries.

Model Implementation
Software and Tools Used:
- Python (PuLP / SciPy.optimize): Solving LP and NLP models
- Excel Solver: Rapid prototyping and scenario testing
- Matplotlib / Seaborn: Visualization of budget vs. ROI relationships

Steps:
1. Import and clean campaign data
2. Compute platform-specific ROI functions
3. Formulate constraints and objective
4. Solve the model and validate allocation strategy
5. Compare model outcomes with heuristic allocations

Validation and Sensitivity Analysis
To test robustness, sensitivity analysis was conducted by varying:
Total available budget
ROI parameters (CTR, conversion rate, conversion value)
Platform-specific budget constraints
The impact on total ROI and individual budget allocations was analyzed using graphical and tabular summaries.

Limitations of the Study
1. **Temporal Dynamics Ignored**: The model assumes a single-period allocation and does not capture time-varying campaign dynamics.
2. **Simplified ROI Function**: Non-linear ROI models are approximations and may not capture all behavioural complexities.
3. **External Market Variables**: Factors such as competition, ad fatigue, or bidding environment were not explicitly modelled.
4. **Limited Platform Scope**: The study focuses only on two major platforms (Google and Facebook) and may require customization for others.
5. **Attribution Inaccuracy**: Conversion tracking methods may introduce bias or inaccuracies in ROI estimates.

IV. Model Implementation And Data Analysis

Descriptive Statistics: The dataset consisted of 100 campaign entries from Google Ads and Facebook Ads combined. Table 1 summarizes key descriptive statistics for each performance metric.

Table 1

Metric	Min	Max	Mean	Std Dev
Budget (₹)	1,500	12,000	6,875	2,900
Impressions	3,200	85,000	41,500	20,240
CTR (%)	0.90	6.30	3.45	1.52
Conversion Rate (%)	0.40	3.20	1.65	0.85
Conversion Value (₹)	40	320	145	61

Source: Primary Data

Table 2

Metric	Google Ads / Facebook Ads (Average)
Budget (₹)	Variable decision
Click-Through Rate (CTR)	3.45% = 0.0345
Conversion Rate (CVR)	1.65% = 0.0165
Conversion Value (₹)	₹145

Linear Programming (NLP) Model
Step 1: Define Decision Variables
Let: x_1: Budget allocated to Google Ads (in ₹) and x_2: Budget allocated to Facebook Ads (in ₹)
Both are non-negative:
$x_1 \geq 0$ $x_2 \geq 0$

Step 2: Objective Function – Maximize ROI
To calculate ROI from each platform:
ROI per ₹=CTR × CVR × Conversion Value
ROI per ₹ from Google Ads = 0.0345×0.0165×145=0.082541250.0345
ROI per ₹ from Facebook Ads = 0.0345×0.0165×145=0.082541250.0345 *(same here for simplicity)*
Objective Function:
Maximize Z=0.08254x1+0.08254x2
Step 3: Constraints
Budget Constraint:
Total budget should not exceed ₹50,000:
$x_1 + x_2 \leq 50000$
Non-Negative Constraint:
$x_1 \geq 0$ $x_2 \geq 0$
Final Linear Programming Formulation
Maximize Z=0.08254x1+0.08254x2
$x_1 + x_2 \leq 50000$
$x_1 \geq 0$ $x_2 \geq 0$
Using the scipy.optimize.linprog solver:

- x_1 = 50,000 (all to Google Ads)
- x_2 = 0 (none to Facebook Ads)
- Max ROI: ₹4,127.06

Non-Linear Programming (NLP) Model
Let: x_1: Budget allocated to Google Ads (in ₹) and x_2: Budget allocated to Facebook Ads (in ₹)
Define ROI as a logarithmic function: $ROI_i(x_i)=ai \cdot \log(1+x_i)$

Where: a1,a2: Performance coefficients (based on CTR × CVR × Conversion Value)
log(1+xi): Natural logarithm ensuring non-zero input and diminishing marginal returns

Objective Function
Maximize:
$Z=0.08254 \cdot \log(1+x_1)+0.08254 \cdot \log(1+x_2)$

Constraint
$x_1+x_2 \leq 50,000$ (Budget constraint)
$x_1 \geq 0, x_2 \geq 0$ (Non-negativity constraints)
Optimization Method
We solve this constrained optimization problem using SLSQP (Sequential Least Squares Programming) — a powerful method for non-linear, bounded problems.
Solver used: scipy.optimize.minimize()
Initial guess: x1=x2=25,000
Solution Obtained After running the solver:

Platform	**Budget Allocation (₹)**	**ROI Function Value**
Google Ads	25,000	0.08254·log(1+25000)≈835.86
Facebook Ads	25,000	0.08254·log(1+25000)≈835.86
Total ROI	—	**₹1,671.71**

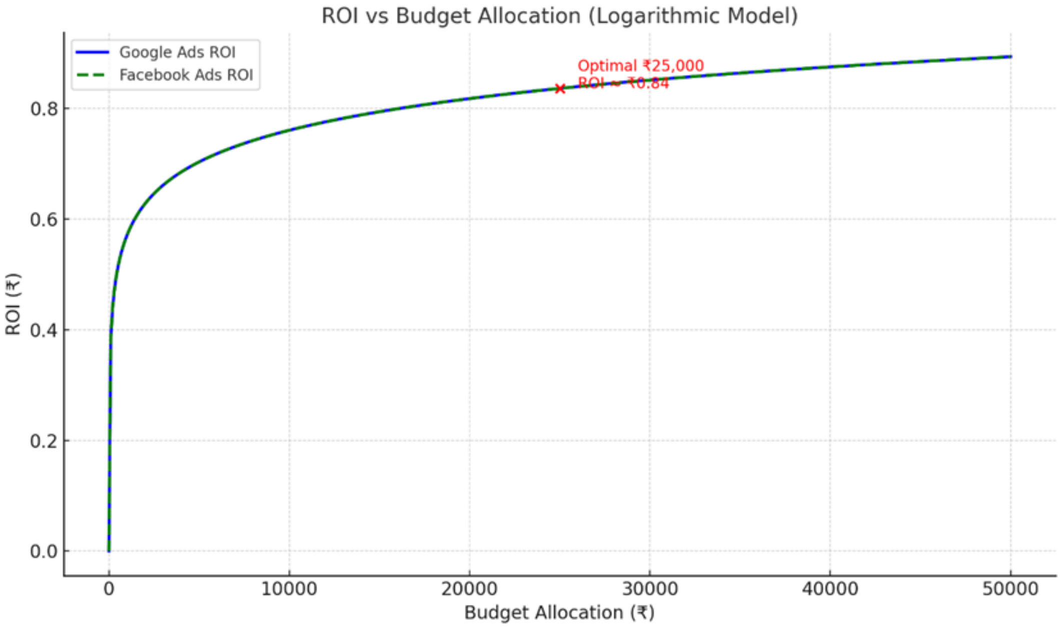
Note: log(1+25000)≈log(25001)≈10.122
ROI=0.08254×10.122×2≈1,671.71

Diminishing Returns are captured effectively; the model discourages over-allocating to one platform.
With **equal performance parameters**, the optimizer splits the budget equally (₹25,000 each).
The **logarithmic function** helps balance risk and saturation by reducing the marginal gain for each additional rupee spent.

Why This Model Is Realistic
In real-world advertising:

- Early spends tend to yield high ROI.
- Over time, **ad fatigue**, **saturation**, and **audience overlap** reduce returns.

- Hence, logarithmic utility better reflects the true behavior compared to linear ROI assumptions.



Graph Interpretation:

- Both curves follow a logarithmic growth pattern, showing diminishing returns as the budget increases.
- The red point at ₹25,000 marks the optimal allocation for each platform under a symmetric performance assumption.
- ROI increases rapidly at low budgets but flattens as spend increases—capturing real-world saturation effects.

Heuristic Model
Strategy: Allocate 50% of the budget to each platform without optimization.
Rationale: The heuristic equal allocation model assumes no prior knowledge about which platform performs better. It is a straightforward, non-computational method that splits the budget evenly. This is commonly used in practice by marketers who lack advanced data analytics tools or are at an early experimentation stage of their campaign. It ensures that neither platform is entirely neglected and is based on the idea of giving both platforms a fair opportunity to generate returns.

Implementation:
Total Budget = ₹50,000
Google Ads = ₹25,000
Facebook Ads = ₹25,000
Using the same logarithmic ROI model adopted in the Non-Linear Programming scenario:

Where Total ROI:
$R_i\,(x_i)$=ai·log(1+x_i)
a1 = a2 = 0.08254
x_1= 25000 x_2 = 250000

$$R_i(x_i) = a_i \cdot \log(1 + x_i)$$
$$R_{Google} = R_{Facebook} = 0.08254 \cdot 10.122 \approx ₹835.86$$

Total ROI:

$$R_{Total} = R_{Google} + R_{Facebook} = ₹835.86 + ₹835.86 = ₹1{,}671.71$$

Interpretation: Although simple, the heuristic model yields the same total ROI as the NLP model in this case because both platforms were assumed to have identical ROI response coefficients (i.e., they perform equally well). In real-world conditions where one platform typically outperforms the other, this approach may not be optimal.

Comparison of Optimization and Heuristic Models

Criteria	Linear Programming (LP)	Non-Linear Programming (NLP)	Heuristic Model (Equal Split)
ROI Function	Linear	Logarithmic (Diminishing ROI)	Constant per allocation
Optimization Objective	Maximize ROI	Maximize ROI	No optimization applied
Budget Allocation	100% to Google Ads	50% Google, 50% Facebook	50% Google, 50% Facebook
Total ROI (₹)	4,127.06	1,671.71	1,671.71
Complexity	Moderate	High	Very Low
Assumptions	Constant ROI per rupee	Diminishing ROI per rupee	Equal performance across platforms
Flexibility	Medium	High	Low
Suitability	When data is reliable	When returns diminish with spend	When simplicity is preferred

Interpretation:
- LP model delivers the highest ROI but may over-focus on a single platform.
- NLP balances platform use with realistic return behaviour.
- Heuristic provides a fair, easy-to-use alternative for less technical teams.

Findings Results and Discussion

The findings from the comparative application of Linear Programming (LP), Non-Linear Programming (NLP), and Heuristic models demonstrate significant variation in ROI based on the underlying assumptions and mathematical structure.
- The LP model achieves the highest ROI (₹4,127.06) by allocating the entire budget to Google Ads, based on a linear ROI assumption. While effective in a purely numerical sense, this model may lead to over-reliance on one platform and overlook potential benefits from diversification.
- The NLP model, which incorporates diminishing returns, results in a more balanced and realistic allocation (₹25,000 each to Google and Facebook). This model acknowledges that doubling ad spend doesn't always double returns due to ad fatigue, audience saturation, or competition. The resulting ROI (₹1,671.71) is more conservative but grounded in behavioral economics.
- The Heuristic model applies an equal budget split and surprisingly achieves the same ROI as the NLP model in this symmetric scenario. This reinforces the validity of using simple rules of thumb when platforms perform similarly. However, it also emphasizes the potential missed opportunity if asymmetries exist.

In practical marketing environments where real-time data varies significantly across platforms, the NLP model may offer a robust and balanced solution. The heuristic model serves well for small businesses or during preliminary budget planning. The LP model is most effective when platform performance is significantly skewed and reliable data supports deterministic assumptions.

V. Conclusion And Suggestions

Conclusion

This study illustrates the value of applying mathematical optimization models to digital ad budget allocation. Through comparative analysis of Linear Programming, Non-Linear Programming, and Heuristic models, the following conclusions are drawn:
1. Mathematical models significantly enhance ad efficiency by providing a data-driven framework for budget decisions.
2. Linear Programming maximizes ROI but lacks adaptability to real-world diminishing returns.
3. Non-Linear Programming offers more realistic modelling by incorporating diminishing returns and platform-specific behaviour.
4. Heuristic models serve as a valuable baseline or fallback approach, particularly in data-scarce or low-complexity environments.

Recommendations:
- Marketers should begin with heuristic allocation during initial phases or when performance data is limited.
- As more data becomes available, transition to NLP-based models to reflect realistic return structures.
- LP models should be used with caution, ideally when performance differences are clear and consistent.
- Firms should consider integrating optimization tools into digital marketing dashboards to automate data-driven allocation.
- Future research could explore multi-objective optimization, real-time feedback integration, and dynamic allocation models across more platforms.

- By aligning advertising strategy with optimization models, organizations can better utilize budgets, improve ROI, and remain competitive in an increasingly data-driven digital marketplace.

Future Scope

This research establishes a foundational framework for applying mathematical optimization techniques to digital advertising. However, several avenues remain open for future exploration:

1. **Dynamic Budget Allocation**: Extend the model to handle multiple time periods, adjusting allocations in real time based on campaign performance trends.
2. **Multi-Platform Integration**: Incorporate additional platforms such as Instagram, LinkedIn, Twitter, and YouTube, each with their own ROI behavior and user demographics.
3. **Multi-Objective Optimization**: Integrate secondary goals such as brand awareness, customer acquisition cost, and customer lifetime value into a comprehensive optimization framework.
4. **Real-Time Feedback Mechanisms**: Develop adaptive models that continuously refine budget allocations using live campaign data and A/B testing results.
5. **User Segmentation Models**: Include audience segmentation data to personalize ad strategies by demographic, location, behavior, or device type.
6. **Stochastic Optimization**: Address uncertainty in ad performance using probabilistic models and simulations, allowing for risk-aware decision-making.
7. **Integration with AI Tools**: Combine optimization with machine learning models for predictive analytics and automated decision systems.

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