IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-0661, p-ISSN: 2278-8727, Volume 21, Issue 2, Ser. 1 (Mar. – Apr. 2025), PP 01-12
www.iosrjournals.org

A Comparative Analysis Of Multicriteria Decision Making (MCDM) Methods In Optimal Gold Mining Sites Selection In Ghana

Bonya K.[1], Twum S. B.[2], Etwire C. J.[3]
Department Of Mathematics [1,2,3]
C. K. Tedam University Of Technology And Applied Sciences, Navrongo, Ghana

Abstract

The application of Multi-Criteria Decision Making (MCDM) methods in selecting the best locations for siting mines is appropriate and necessary given the many diverse and conflicting criteria that may have to be considered if the mining is done responsibly and equitably. In Ghana, where gold mining has assumed alarming scales, the need to balance social, economic, and environmental factors has become ever more crucial. A comparative assessment of three MCDM techniques to gold mining site selection in Ghana is presented in this article to evaluate how they perform in choosing the best location. Specifically, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), the Multi-Objective Optimization based on Ratio Analysis (MULTIMOORA), and the Weighted Sum Model (WSM) which are commonly utilized MCDM methodologies are used in the research. They are used to analyze actual data from potential gold mining locations in the Ashanti Region of Ghana, taking into account the mineral quality, social ramifications, economic viability, and environmental impact. The findings provide insights into the practical benefits of each of the techniques in selecting the best and worst mining sites, highlighting the advantages and disadvantages of each method in tackling the intricate, multi-dimensional decision problem. From the results of this research, it is clear that the mining industry in Ghana can benefit tremendously in terms of equitably meeting the expectations of stakeholders based on more informed, responsible, and sustainable decisions in selecting mining sites.

Keywords: *Multicriteria Decision-Making, Weighted Sum Model, Technique for Order of Preference by Similarity to Ideal Solution, Multi-Objective Optimization based on Ratio Analysis, Optimal Site Selection.*

Date of Submission: 15-02-2025 Date of Acceptance: 25-02-2025

I. Introduction

Selecting the optimal mining locations is a critical decision for the mining sector, given its potential to significantly impact the economy, environment, and society. This decision-making process is inherently complex due to the necessity of considering a wide range of factors, including social impact, economic viability, environmental sustainability, and geological potential [18]. The multidimensional nature of this challenge often leads to suboptimal outcomes when traditional decision-making techniques are employed. In this context, Multi-Criteria Decision Making (MCDM) techniques have emerged as powerful tools, offering a systematic approach to evaluating and ranking multiple competing criteria [12]. These methods provide a structured framework for assessing various aspects of site selection, enabling decision-makers to make well-informed choices that balance economic benefits with social and environmental responsibilities [9]. Among the various MCDM approaches, are models such as the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), the Weighted Sum Model (WSM), and the Multi-Objective Optimization based on Ratio Analysis (MULTIMOORA), which have gained prominence due to their successful application in resource management and site selection challenges. However, despite their widespread use, there has been limited comparative analysis of their effectiveness in mining site selection. This research aims to address this gap by conducting a comparative analysis of stated MCDM methods to determine their relative strengths, weaknesses, and suitability for mining sites selection in Ghana. Given that different methods may yield varying outcomes depending on the criteria considered and the specific context of the decision problem, this study will apply a consistent set of criteria across the different MCDM approaches to evaluate their performance. The insights gained from the comparative analysis are expected to guide the mining sector towards more informed and sustainable decision-making. By identifying the most appropriate MCDM techniques for handling the complex challenges associated with mining site selection, this research seeks to contribute to better-informed, environmentally sustainable practices in the industry.

II. The Models

The TOPSIS Model

Based on the premise that every criteria consistently affects utility in a positive or negative way, Hwang and Yoon (1981) developed the widely used MCDM technique known as the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). Due to this, positive and negative ideal solutions may be precisely defined and used as standards for comparing different options [11]. TOPSIS measures each alternative's proximity to these ideal solutions using the Euclidean distance technique [7]. TOPSIS guarantees that options with various sizes and units may be compared objectively by normalizing criteria into non-dimensional representations. The approach places the options in order of decreasing distance from the negative ideal solution (NIS) and increasing closeness to the positive ideal solution (PIS), with the optimum choice being the one that is furthest from the NIS and closest to the PIS [14]. TOPSIS has several limitations despite its simplicity and efficacy. Its emphasis on Euclidean distance may cause it to ignore the links between criteria, making it more difficult to consistently assign weights and evaluate alternatives.

Mathematical Concept of the TOPSIS Model

TOPSIS is an essential tool that helps decision-makers identify the optimal choice from a range of options [1]. TOPSIS examines and ranks choices according to how close they are to ideal solutions, a systematic approach that is intended to manage complicated problems containing several criteria [15]. Its methodical approach guarantees a thorough and objective evaluation, assisting decision-makers in selecting the best option from a range of options. To obtain precise and dependable results while applying TOPSIS, a set of clearly defined stages must usually be followed.

Step 1: Creating a decision matrix (*A*) is the first stage in putting the TOPSIS process into practice [21]. Using a clear, organized depiction of each alternative's performance across several criteria, this matrix arranges the alternatives in rows and the criteria in columns.

$$A = (a_{ij}) \qquad (1)$$

Step 2: Using the vector normalization approach is an important stage in the process of normalizing the decision matrix (*A*) [5]. This method guarantees that the criterion's initial orientation won't change throughout the normalization procedure. The following steps are taken to normalize the decision matrix:

$$N_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}} \quad (i, j, m = 1, 2, \ldots, n). \qquad (2)$$

Step 3: A weighted normalized decision matrix is calculated. Here, the DM assigns specific weights to each criterion based on their relative importance. The weighted normalized matrix, denoted as *V*, is then derived by multiplying the normalized decision matrix by these assigned weights. This step ensures that the decision-making process emphasizes the most critical criteria [16], leading to a more accurate and comprehensive evaluation of the alternatives.

$$V_{ij} = N_{ij} \times W_{ij} \qquad (3)$$

Step 4: The positive ideal solution (PIS) is determined by choosing the highest value for beneficial criteria and the smallest value for non-beneficial criteria [4], while the negative ideal solution (NIS) is determined by choosing the smallest value for beneficial criteria and the largest value for non-beneficial criteria.

PIS values: $A^+ = \{\text{best } v_{ij}\}$,

$$A^+ = \{v_1^+, v_2^+, \ldots, v_n^+\}. \qquad (4)(a)$$

NIS values: $A^- = \{\text{worst } v_{ij}\}$,

$$A^- = \{v_1^-, v_2^-, \ldots, v_n^-\}. \qquad (4)(b)$$

Step 5: At this point in the procedure, the computation of the separation measure is the main focus, and it can be described concisely in terms of the Euclidean distance for each option. One measure used to quantify the distance of a specific option *i* from the PIS is the Euclidean distance, which is a metric for evaluating element dissimilarity [8]. The following is a detailed calculation for this distance:

Distance from PIS: $$S_i^+ = \sqrt{\sum_{j=1}^{n} (V_{ij} - V_j^+)^2}\ . \qquad (5)(a)$$

Distance from NIS: $$S_i^- = \sqrt{\sum_{j=1}^{n} (V_{ij} - V_j^-)^2}\ . \qquad (5)(b)$$

Step 6: Determine the relative proximity to the optimal solution that is positive. This entails determining how near an alternative is to the specified PIS [27]. The relative proximity to the ideal Solution (C_i) can be calculated as follows:

$$C_i = \frac{S_i^-}{S_i^- + S_i^+}\ (0 \le C_i \le 1). \qquad (6)$$

Step 7: Rank the preference order

It is now possible to rank a group of options in preference based on the value of C_i in decreasing order. Identify the option that is closest to one (1).

The WSM

A simple approach to decision-making is the Weighted Sum Model (WSM), in which weights are allocated to criteria and alternatives are assessed on the bases of the weights. It is simple to use and to comprehend [23]. WSM works especially well in situations where there is broad consensus over the criteria and their significance, such as in location selection. Decision-makers give each criterion a numerical weight that represents its relative importance during the decision-making process [17]. Subsequently, the alternatives are graded by multiplying each result by the appropriate weight and adding up the products of their performance ratings for all criteria. This technique offers decision-makers a useful and effective tool to help them make well-informed decisions [25] based on weighted criteria in less complicated decision contexts where criterion relevance is clearly stated.

Mathematical Concept of the WSM

By multiplying the score of each criterion by its weight and adding together these weighted values, the WSM essentially integrates the different criteria. By changing the weights given to each criterion, this method enables DMs to take their priorities and preferences into consideration [20]. According to [13], the option with the highest weighted total is frequently regarded as the best option. It is a simple yet effective technique that is frequently applied in decision analysis, project appraisal, and other situations where several factors affect the choice that is made. The following steps are typically used when applying the WSM:

Step 1. Construction of a Decision Matrix

Before applying it, a decision/evaluation matrix must be created.

$$X = [\![x_{ij}]\!]_{m \times n}, \qquad (1)$$

where x_{ij} = performance of *i*th alternative for *j*th criterion; *m* = number of alternatives;

n = number of criteria.

Step 2. Normalization of Decision Matrix

Standardize the values found in each column of this matrix. Scaling each column's values to a conventional range, like between 0 and 1, is the usual process of normalization.

$$r_{ij} = \frac{x_{ij}}{\max x_{ij}} \text{ (for beneficial criteria),} \qquad (2)$$

$$r_{ij} = \frac{\min x_{ij}}{x_{ij}} \text{ (for non-beneficial criteria)} \qquad (3)$$

where r_{ij} is the normalized value of x_{ij} .

Step 3. Calculation of Preference Score for each Alternative (V_i)

The performance ratings of the alternatives are then multiplied by the corresponding weights for each criterion [10], and the sum of these products is used to evaluate them.

$$V_i = \sum_{j=1}^{n} w_j r_{ij}\ . \qquad (4)$$

where V_i = preference score of alternative *i*, w_j = weight value of criterion *j*,

r_{ij} = normalized value of alternative *i* for criterion *j*, n = *n*umber of criteria.

A higher V_i value suggests that option A_i is favoured.

The MULTIMOORA Model

When solving problems with numerous criteria, the MULTIMOORA method provides an organized approach to guarantee a consistent foundation for comparison, even when the criteria are expressed in various units or scales. It starts by normalizing the criteria. Each criterion's relative importance in the decision-making process is reflected by the weights assigned to it [22]. The incorporation of both additive and multiplicative forms during criteria aggregation is what sets MULTIMOORA apart and enables a more in-depth examination of the relationships and dependencies between criteria [6]. Decision-makers may now grasp the decision environment with greater nuance thanks to this dual-form examination. MULTIMOORA helps decision-makers identify the best choice that fits their goals and preferences by generating a ranked list of options after the aggregation process, based on the alternatives' overall performance across multiple parameters [26]. MULTIMOORA's ability to handle choice issues with competing objectives is one of its most significant strengths. The method is especially useful in circumstances where thorough analysis of trade-offs and opposing goals is required because it incorporates additive and multiplicative interactions, which provide a balanced and informed perspective.

Mathematical Concept of the MULTIMOORA Model

Using a variety of mathematical principles, the MULTIMOORA model methodically evaluates and ranks options in a situation involving decision-making [24]. An examination of the basic mathematical operations that are essential to the MULTIMOORA model is provided below:

Step 1. Construction of a Decision Matrix

Before applying it, a decision (evaluation) matrix must be created.

$$X = [\![x_{ij}]\!]_{m \times n}. \qquad (1)$$

where x_{ij} = performance of *i*th alternative for *j*th criterion; *m* = number of alternatives;
n = number of criteria.

Step 2. Normalization of Decision Matrix

To put all of the criteria's values on the same scale, normalize them. This is an essential step when working with criteria that vary in scales or units.

$$X^* = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}}, \qquad (2)$$

where x_{ij} = performance of *i*th alternative for *j*th criterion.

Step 3. Weight Assignment

According to [2], weights are allocated to every criterion, taking into account their respective significance in the decision-making process. These weights provide an indication of the priorities or preferences of the decision maker's criteria *j* given a weight that ensures

$$\sum_{j=1}^{n} w_j = 1. \qquad (3)$$

Step 4. Determination of Weighted Normalized Decision Matrix

The weighted normalized decision matrix, represented by Y, is obtained by multiplying the normalized matrix by the weight matrix allocated to every criterion.

$$Y = x_i \times w, \qquad (4)$$

where x_i = normalized matric value, *w* = criteria weight.

Step 5. The variations between the highest and lowest values for every option are obtained by using

$$Y_i = \sum \max x_{ij}^* - \sum \min x_{ij}^*. \qquad (5)$$

Step 6. Determine substitute values by applying the equation.

$$U_i = \frac{\sum \max X_{ij}^*}{\sum \min X_{ij}^*}. \qquad (6)$$

Step 7. Ranking of Alternatives

Sort the options according to the total points they have earned. According to [3], the option with the highest score is regarded as the most ideal.

III. Application

Ghana, a country in sub-Sahara Africa, is famous for its abundant mineral wealth, notably gold, which has played a crucial role in its economic development. The Ashanti Region of Ghana stands out for its vast gold reserves, drawing significant attention from mining firms. However, optimizing the right locations for gold extraction entails intricate decision-making due to factors like geology, environmental concerns, economic viability, and social impacts [19]. Table 1 shows the potential gold mining locations in the Ashanti Region of Ghana under six major criteria namely; Gold Grade and Ore Reserves, Operations Cost per ounce of Gold, Environmental Impact Assessment, Health and Safety Risks, Social Impact Assessment, and Legal and Regulatory Compliance.

Table 1: Potential gold mining sites in the A shanti Region of Ghana under six major criteria

Potential Mining Sites	Gold Grade and Ore Reserves	Operations cost per ounce of gold (US$)	Environmental Impact	Health and Safety Risks	Social Impact	Legal and Regulatory Compliance
LTA (S1)	High	2,324	Moderate	Moderate	Moderate	High
LTB (S2)	High	3,393	Moderate	Moderate	Low	High
LTC (S3)	High	2,155	Moderate	Low	Moderate	High
LTD (S4)	High	2,770	Moderate	Low	Low	Moderate
LTE (S5)	Very High	3,793	High	High	Low	Moderate
LTF (S6)	Very High	4,855	High	High	Moderate	High
LTG (S7)	High	2,619	Moderate	High	High	Moderate
LTH (S8)	Very High	4,355	Low	High	High	Moderate
LTI (S9)	High	2,226	High	Low	Moderate	High
LTJ (S10)	Very High	3,930	High	Low	Low	High

A 50-point scale was used for the conversion of the linguistic variables for beneficial criteria and a 5-point scale for the conversion of the linguistic variables for non-beneficial criteria resulting in Table 2 below.

Table 2: Potential gold mining sites in the A shanti Region of Ghana under six major criteria

Sites	Gold Grade and Ore Reserves (GG)	Operations cost per ounce of gold (US $) (OC)	Environmental Impact (EI)	Health and Safety Risks (HSR)	Social Impact (SI)	Legal and Regulatory Compliance (LRC)
S1	35	2,324	3	3	3	4
S2	35	3,393	3	3	2	4
S3	35	2,155	3	2	3	4
S4	35	2,770	3	2	2	3
S5	45	3,793	4	4	2	3
S6	45	4,855	4	4	3	4
S7	35	2,619	3	4	4	3
S8	45	4,355	2	4	4	3
S9	35	2,226	4	2	3	4
S10	45	3,930	4	2	2	4

Mathematical analysis using TOPSIS model

Step 1: Formation of a decision matrix

This leads to Table 3

Table 3: Decision Matrix $D = [\![x_{ij}]\!]_{m \times n}$

Sites	Gold Grade and Ore Reserves (GG)	Operations cost per ounce of gold (US $) (OC)	Environmental Impact (EI)	Health and Safety Risks (HSR)	Social Impact (SI)	Legal and Regulatory Compliance (LRC)
S1	35	2,324	3	3	3	4

S2	35	3,393	3	3	2	4
S3	35	2,155	3	2	3	4
S4	35	2,770	3	2	2	3
S5	45	3,793	4	4	2	3
S6	45	4,855	4	4	3	4
S7	35	2,619	3	4	4	3
S8	45	4,355	2	4	4	3
S9	35	2,226	4	2	3	4
S10	45	3,930	4	2	2	4
$\sum_{j=1}^{n} x_{ij}^2$	15450	113413386	113	98	84	132
$\sqrt{\sum_{j=1}^{n} x_{ij}^2}$	124.30	10649.57	10.63	9.90	9.17	11.49

Step 2: *Construction of Normalized Decision Matrix*

To normalize the decision matrix, divide each entry of each column by $\sqrt{\sum_{j=1}^{n} x_{ij}^2}$. This leads to Table 4.

Table 4: Normalized Decision Matrix (*NDM*) *R* = (r_{ij})

Sites	Gold Grade and Ore Reserves	Operations cost per ounce of gold (US $)	Environmental Impact	Health and Safety Risks	Social Impact	Legal and Regulatory Compliance
S1	0.28158	0.21822	0.28222	0.30303	0.32715	0.34813
S2	0.28158	0.31860	0.28222	0.30303	0.21810	0.34813
S3	0.28158	0.20236	0.28222	0.20202	0.32715	0.34813
S4	0.28158	0.26010	0.28222	0.20202	0.21810	0.26109
S5	0.36203	0.35616	0.37629	0.40404	0.21810	0.26109
S6	0.36203	0.45589	0.37629	0.40404	0.32715	0.34813
S7	0.28158	0.24593	0.28222	0.40404	0.43621	0.26109
S8	0.36203	0.40894	0.18815	0.40404	0.43621	0.26109
S9	0.28158	0.20902	0.37629	0.20202	0.32715	0.34813
S10	0.36203	0.36903	0.37629	0.20202	0.21810	0.34813

Step 3: *Computation of the weight matrix*

The weights (w_j) which represents the relative importance of criteria are calculated such that $\sum_{j=1}^{n} w_j = 1$.

Step 4: *Computation of Weighted Normalized Decision Matrix* (*WNDM*)

To get *WNDM*, multiply each column of *NDM* by the weights w_j, of the weight vector computed in step 3 and results in Table 5.

Table 5: Weighted Normalized Decision Matrix $U(u_{ij}) = r_{ij} \times w_j$

Sites	Gold Grade and Ore Reserves (0.37152)	Operations cost per ounce of gold (US $) (0.24739)	Environmental Impact (0.12934)	Health and Safety Risks (0.12991)	Social Impact (0.07458)	Legal and Regulatory Compliance (0.04726)
S1	0.10461	0.05399	0.03650	0.03937	0.02440	0.01645
S2	0.10461	0.07882	0.03650	0.03937	0.01627	0.01645
S3	0.10461	0.05006	0.03650	0.02624	0.02440	0.01645
S4	0.10461	0.06435	0.03650	0.02624	0.01627	0.01234
S5	0.13450	0.08811	0.04867	0.05249	0.01627	0.01234
S6	0.13450	0.11278	0.04867	0.05249	0.02440	0.01645
S7	0.10461	0.06084	0.03650	0.05249	0.03253	0.01234
S8	0.13450	0.10117	0.02434	0.05249	0.03253	0.01234
S9	0.10461	0.05171	0.04867	0.02624	0.02440	0.01645
S10	0.13450	0.09129	0.04867	0.02624	0.01627	0.01645

Step 5: *Calculation of Positive Ideal Solution* (PIS) *and Negative Ideal Solution* (NIS) Values
This is depicted in Table 6.

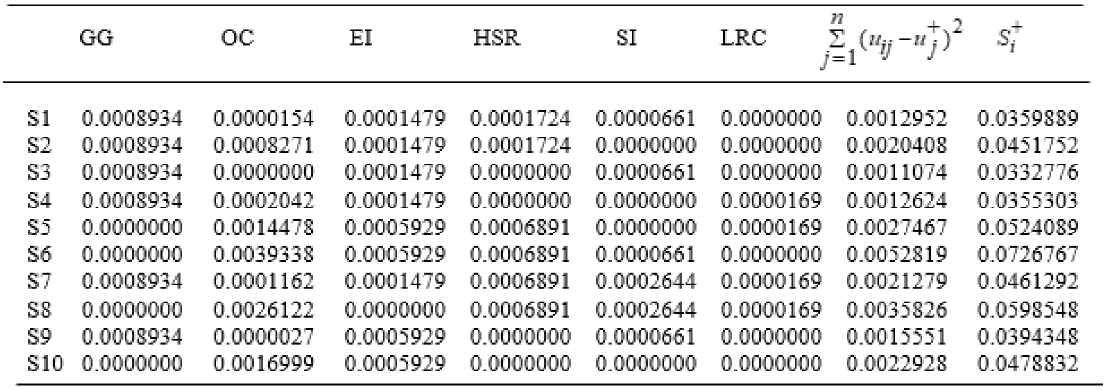
Table 6: Calculation of PIS (A^+) and NIS (A^-) Values

Sites	Gold Grade and Ore Reserves (0.37152)	Operations cost per ounce of gold (US $) (0.24739)	Environmental Impact (0.12934)	Health and Safety Risks (0.12991)	Social Impact (0.07458)	Legal and Regulatory Compliance (0.04726)
S1	0.10461	0.05399	0.03650	0.03937	0.02440	0.01645
S2	0.10461	0.07882	0.03650	0.03937	0.01627	0.01645
S3	0.10461	0.05006	0.03650	0.02624	0.02440	0.01645
S4	0.10461	0.06435	0.03650	0.02624	0.01627	0.01234
S5	0.13450	0.08811	0.04867	0.05249	0.01627	0.01234
S6	0.13450	0.11278	0.04867	0.05249	0.02440	0.01645
S7	0.10461	0.06084	0.03650	0.05249	0.03253	0.01234
S8	0.13450	0.10117	0.02434	0.05249	0.03253	0.01234
S9	0.10461	0.05171	0.04867	0.02624	0.02440	0.01645
S10	0.13450	0.09129	0.04867	0.02624	0.01627	0.01645

A^+	0.13450	0.05006	0.02434	0.02624	0.01627	0.01645
A^-	0.10461	0.11278	0.04867	0.05249	0.03253	0.01234

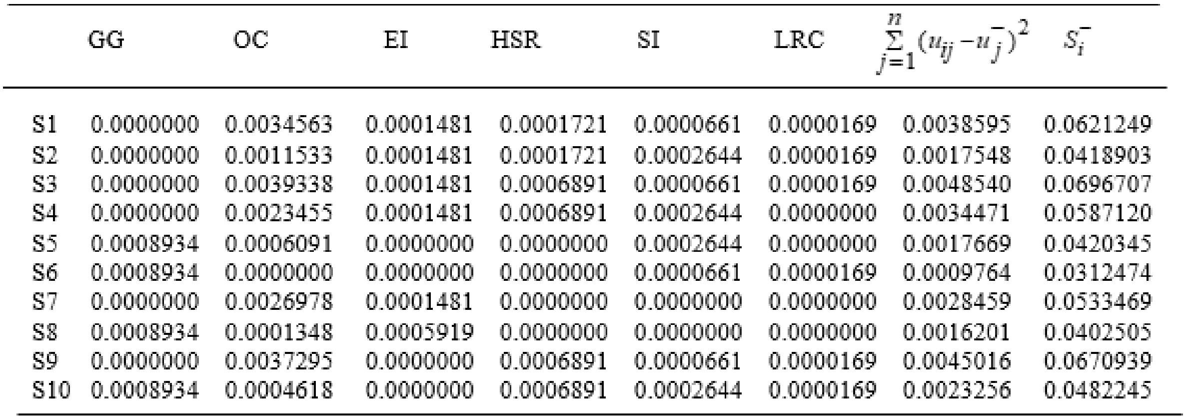
Step 6: *Determination of the separation measures for each alternative using weighted Euclidean distance metric.*
The results of this step are shown in Tables 7a and 7b.

Table 7a: Calculating separation from PIS (S_i^+); $S_i^+ = \sqrt{\sum_{j=1}^{n}(u_{ij}-u_j^+)^2}$



	GG	OC	EI	HSR	SI	LRC	$\sum_{j=1}^{n}(u_{ij}-u_j^+)^2$	S_i^+
S1	0.0008934	0.0000154	0.0001479	0.0001724	0.0000661	0.0000000	0.0012952	0.0359889
S2	0.0008934	0.0008271	0.0001479	0.0001724	0.0000000	0.0000000	0.0020408	0.0451752
S3	0.0008934	0.0000000	0.0001479	0.0000000	0.0000661	0.0000000	0.0011074	0.0332776
S4	0.0008934	0.0002042	0.0001479	0.0000000	0.0000000	0.0000169	0.0012624	0.0355303
S5	0.0000000	0.0014478	0.0005929	0.0006891	0.0000000	0.0000169	0.0027467	0.0524089
S6	0.0000000	0.0039338	0.0005929	0.0006891	0.0000661	0.0000000	0.0052819	0.0726767
S7	0.0008934	0.0001162	0.0001479	0.0006891	0.0002644	0.0000169	0.0021279	0.0461292
S8	0.0000000	0.0026122	0.0000000	0.0006891	0.0002644	0.0000169	0.0035826	0.0598548
S9	0.0008934	0.0000027	0.0005929	0.0000000	0.0000661	0.0000000	0.0015551	0.0394348
S10	0.0000000	0.0016999	0.0005929	0.0000000	0.0000000	0.0000000	0.0022928	0.0478832

Table 7b: Calculating separation from NIS (S_i^-); $S_i^- = \sqrt{\sum_{j=1}^{n}(u_{ij}-u_j^-)^2}$



	GG	OC	EI	HSR	SI	LRC	$\sum_{j=1}^{n}(u_{ij}-u_j^-)^2$	S_i^-
S1	0.0000000	0.0034563	0.0001481	0.0001721	0.0000661	0.0000169	0.0038595	0.0621249
S2	0.0000000	0.0011533	0.0001481	0.0001721	0.0002644	0.0000169	0.0017548	0.0418903
S3	0.0000000	0.0039338	0.0001481	0.0006891	0.0000661	0.0000169	0.0048540	0.0696707
S4	0.0000000	0.0023455	0.0001481	0.0006891	0.0002644	0.0000000	0.0034471	0.0587120
S5	0.0008934	0.0006091	0.0000000	0.0000000	0.0002644	0.0000000	0.0017669	0.0420345
S6	0.0008934	0.0000000	0.0000000	0.0000000	0.0000661	0.0000169	0.0009764	0.0312474
S7	0.0000000	0.0026978	0.0001481	0.0000000	0.0000000	0.0000000	0.0028459	0.0533469
S8	0.0008934	0.0001348	0.0005919	0.0000000	0.0000000	0.0000000	0.0016201	0.0402505
S9	0.0000000	0.0037295	0.0000000	0.0006891	0.0000661	0.0000169	0.0045016	0.0670939
S10	0.0008934	0.0004618	0.0000000	0.0006891	0.0002644	0.0000169	0.0023256	0.0482245

Step 7: *Calculation of the relative closeness to the ideal solution* (C_i); $C_i = \dfrac{S_i^-}{S_i^+ + S_i^-}$.

S1: $C_1 = \dfrac{0.0621249}{0.0359889+0.0621249}$ = 0.6331923; S2: $C_2 = \dfrac{0.0418903}{0.0451752+0.0418903}$ = 0.4811355

S3: $C_3 = \dfrac{0.0696707}{0.0332716+0.0696707}$ = 0.6767936; S4: $C_4 = \dfrac{0.0587120}{0.0355303+0.0587120}$ = 0.6229898

S5: $C_5 = \dfrac{0.0420345}{0.0524089+0.0420345}$ = 0.4450761; S6: $C_6 = \dfrac{0.0312474}{0.0726767+0.0312474}$ = 0.3006752

S7: $C_7 = \dfrac{0.0533469}{0.0461292+0.0533469}$ = 0.5362786; S8: $C_8 = \dfrac{0.0402505}{0.0598548+0.0402505}$ = 0.4020816

S9: $C_9 = \dfrac{0.0670939}{0.0394348+0.0670939}$ = 0.6298199; S10: $C_{10} = \dfrac{0.0482245}{0.0478832+0.0482245}$ = 0.5017756

Step 8. The set of alternatives can now be preference-ranked according to the descending order of the value C_i . From *Step* 7, it is observed that site S3 is the best potential gold mining site since it has the highest C_i value of 0.6767936, while site S6 is the worst potential gold mining site since it has the lowest C_i value of 0.3006752 based on the evaluation criteria. Hence S3 ≻ S1 ≻ S9 ≻ S4 ≻ S7 ≻ S10 ≻ S2 ≻ S5 ≻ S8 ≻ S6.

Mathematical analysis using WSM

Step 1: Formation of a decision matrix

The decision matrix is obtained as shown in Table 8.

Table 8: Decision Matrix $D = [\![x_{ij}]\!]_{m\times n}$

Potential Mining Sites	Gold Grade and Ore Reserves (GG)	Operations cost per ounce of gold (US $) (OC)	Environmental Impact (EI)	Health and Safety Risks (HSR)	Social Impact (SI)	Legal and Regulatory Compliance (LRC)
S1	35	2,324	3	3	3	4
S2	35	3,393	3	3	2	4
S3	35	2,155	3	2	3	4
S4	35	2,770	3	2	2	3
S5	45	3,793	4	4	2	3
S6	45	4,855	4	4	3	4
S7	35	2,619	3	4	4	3
S8	45	4,355	2	4	4	3
S9	35	2,226	4	2	3	4
S10	45	3,930	4	2	2	4

Step 2: Normalization of the decision matrix

Normalize the values in each column of the decision matrix by standardizing them.

$r_{ij} = \frac{x_{ij}}{\max x_{ij}}$ for beneficial criteria,

$r_{ij} = \frac{\min x_{ij}}{x_{ij}}$ for non-beneficial criteria,

where r_{ij} is the normalized value of x_{ij} . The results are shown in Table 9.

Table 9: Normalized Decision Matrix (NDM)

Potential Mining Sites	Gold Grade and Ore Reserves	Operations cost per ounce of gold (US $)	Environmental Impact	Health and Safety Risks	Social Impact	Legal and Regulatory Compliance
S1	0.77778	0.92728	0.66667	0.66667	0.66667	1.00000
S2	0.77778	0.63513	0.66667	0.66667	1.00000	1.00000

S3	0.77778	1.00000	0.66667	1.00000	0.66667	1.00000
S4	0.77778	0.77798	0.66667	1.00000	1.00000	0.75000
S5	1.00000	0.56815	0.50000	0.50000	1.00000	0.75000
S6	1.00000	0.44387	0.50000	0.50000	0.66667	1.00000
S7	0.77778	0.82283	0.66667	0.50000	0.50000	0.75000
S8	1.00000	0.49483	1.00000	0.50000	0.50000	0.75000
S9	0.77778	0.96810	0.50000	1.00000	0.66667	1.00000
S10	1.00000	0.54835	0.50000	1.00000	1.00000	1.00000

Step 3: *Computation of Weighted Normalized Decision Matrix* (*WNDM*)
This is obtained as in Table 10.

Table 10: Weighted Normalized Decision Matrix $U(u_{ij}) = r_{ij} \times w_j$

	Gold Grade and Ore Reserves (0.37152)	Operations cost per ounce of gold (US $) (0.24739)	Environmental Impact (0.12934)	Health and Safety Risks (0.12991)	Social Impact (0.07458)	Legal and Regulatory Compliance (0.04726)
S1	0.28896	0.22939	0.08623	0.08661	0.04972	0.04726
S2	0.28896	0.15712	0.08623	0.08661	0.07458	0.04726
S3	0.28896	0.24739	0.08623	0.12991	0.04972	0.04726
S4	0.28896	0.19246	0.08623	0.12991	0.07458	0.03545
S5	0.37152	0.14055	0.06467	0.06496	0.07458	0.03545
S6	0.37152	0.10981	0.06467	0.06496	0.04972	0.04726
S7	0.28896	0.20356	0.08623	0.06496	0.03729	0.03545
S8	0.37152	0.12242	0.12934	0.06496	0.03729	0.03545
S9	0.28896	0.23949	0.06467	0.12991	0.04972	0.04726
S10	0.37152	0.13566	0.06467	0.12991	0.07458	0.04726

Step 5. Calculation of Performance Score for each Alternative (P_i): $P_i = \sum_{j=1}^{n} w_j r_{ij}$.

This yields Table 11.

Table 11: Performance Score

S_i	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
P_i	0.7882	0.7408	0.8495	0.8076	0.7517	0.7079	0.7165	0.7609	0.8200	0.8236

Step 4. The alternatives can now be preference-ranked according to the descending order of the value of P_i. From *Step* 3, it is observed that site S3 is the best potential gold mining site since it has the highest P_i value of 0.84947, while site S6 is the worst potential gold mining site since it has the least P_i value of 0.70794 based on the evaluation criteria. Hence S3 ≻ S10 ≻ S9 ≻ S4 ≻ S1 ≻ S8 ≻ S5 ≻ S2 ≻ S7 ≻ S6.

Mathematical analysis using the MULTIMOORA Model

Step1: Formation of a decision matrix $D = [\![x_{ij}]\!]_{m \times n}$

This step leads to Table 12.

Table 12: Decision matrix under MULTIMOORA

Sites	Gold Grade and Ore Reserves	Operations cost per ounce of gold (US $)	Environmental Impact	Health and Safety Risks	Social Impact	Legal and Regulatory Compliance
S1	35	2,324	3	3	3	4
S2	35	3,393	3	3	2	4
S3	35	2,155	3	2	3	4
S4	35	2,770	3	2	2	3
S5	45	3,793	4	4	2	3
S6	45	4,855	4	4	3	4
S7	35	2,619	3	4	4	3
S8	45	4,355	2	4	4	3
S9	35	2,226	4	2	3	4
S10	45	3,930	4	2	2	4
$\sum_{j=1}^{n} x_{ij}^2$	15450	113413386	113	98	84	132

$\sqrt{\sum_{j=1}^{n} x_{ij}^2}$	124.30	10649.57	10.63	9.90	9.17	11.49

Step 2: *Construction of Normalized Decision Matrix*

To normalize the decision matrix, divide each value of each column by the corresponding $\sqrt{\sum_{j=1}^{n} x_{ij}^2}$ value. This leads to Table 13.

Table 13: Normalized Decision Matrix (*NDM*) $R = (r_{ij})$

Sites	Gold Grade and Ore Reserves	Operations cost per ounce of gold (US $)	Environmental Impact	Health and Safety Risks	Social Impact	Legal and Regulatory Compliance
S1	0.28158	0.21822	0.28222	0.30303	0.32715	0.34813
S2	0.28158	0.31860	0.28222	0.30303	0.21810	0.34813
S3	0.28158	0.20236	0.28222	0.20202	0.32715	0.34813
S4	0.28158	0.26010	0.28222	0.20202	0.21810	0.26109
S5	0.36203	0.35616	0.37629	0.40404	0.21810	0.26109
S6	0.36203	0.45589	0.37629	0.40404	0.32715	0.34813
S7	0.28158	0.24593	0.28222	0.40404	0.43621	0.26109
S8	0.36203	0.40894	0.18815	0.40404	0.43621	0.26109
S9	0.28158	0.20902	0.37629	0.20202	0.32715	0.34813
S10	0.36203	0.36903	0.37629	0.20202	0.21810	0.34813

Step 3: *Computation of Weighted Normalized Decision Matrix* (*WNDM*)

To get *WNDM*, multiply each column of *NDM* by the weights w_j, of the weight vector computed in step 2. This leads to Table 14.

Table 14: Weighted Normalized Decision Matrix $U(u_{ij}) = r_{ij} \times w_j$

Sites	Gold Grade and Ore Reserves (0.37152)	Operations Cost per ounce of gold (US $) (0.24739)	Environmental Impact (0.12934)	Health and Safety Risks (0.12991)	Social Impact (0.07458)	Legal and Regulatory Compliance (0.04726)
S1	0.10461	0.05399	0.03650	0.03937	0.02440	0.01645
S2	0.10461	0.07882	0.03650	0.03937	0.01627	0.01645
S3	0.10461	0.05006	0.03650	0.02624	0.02440	0.01645
S4	0.10461	0.06435	0.03650	0.02624	0.01627	0.01234
S5	0.13450	0.08811	0.04867	0.05249	0.01627	0.01234
S6	0.13450	0.11278	0.04867	0.05249	0.02440	0.01645
S7	0.10461	0.06084	0.03650	0.05249	0.03253	0.01234
S8	0.13450	0.10117	0.02434	0.05249	0.03253	0.01234
S9	0.10461	0.05171	0.04867	0.02624	0.02440	0.01645
S10	0.13450	0.09129	0.04867	0.02624	0.01627	0.01645

Step 4. Find the ratio of the sum of weighted normalized performance scores of beneficial criteria ($\sum \max x_{ij}^*$) *to the sum of weighted normalized performance scores of non-beneficial criteria* ($\sum \min x_{ij}^*$) *of each alternative from step 3*. This results is in Table 15.

Table 15: Ratio of scores of beneficial criteria to scores of non-beneficial criteria

Alternatives	$\sum \max x_{ij}^*$	$\sum \min x_{ij}^*$	$U_i = \frac{\sum \max X_{ij}^*}{\sum \min X_{ij}^*}$.
S1	0.12106	0.15426	0.78478
S2	0.12106	0.17096	0.70812
S3	0.12106	0.13720	0.88236
S4	0.11695	0.14336	0.81578
S5	0.14684	0.20554	0.71441
S6	0.15095	0.23834	0.63334

S7	0.11695	0.18236	0.64131
S8	0.14684	0.21053	0.69748
S9	0.12106	0.15102	0.80162
S10	0.15095	0.18247	0.82726

Step 5. Rank the set of alternatives according to the highest to the least value of U_i .

From *Step* 5, it is observed that site S3 is the best potential gold mining site since it has the highest value of 0.88236, while site S6 is the worst potential gold mining site since it has the lowest value of 0.63334 based on the evaluation criteria. Hence S3 ≻ S10 ≻ S4 ≻ S9 ≻ S1 ≻ S8 ≻ S5 ≻ S2 ≻ S7 ≻ S6.

Table 16 presents the rankings from the three models

Table 16: Ranking Order Comparison of the three models

Model	Ranking Order of Alternatives
TOPSIS	S3 ≻ S1 ≻ S9 ≻ S4 ≻ S7 ≻ S10 ≻ S2 ≻ S5 ≻ S8 ≻ S6
WSM	S3 ≻ S10 ≻ S9 ≻ S4 ≻ S1 ≻ S8 ≻ S5 ≻ S2 ≻ S7 ≻ S6
MULTIMOORA	S3 ≻ S10 ≻ S4 ≻ S9 ≻ S1 ≻ S8 ≻ S5 ≻ S2 ≻ S7 ≻ S6

IV. Results And Discussion

The alternative S3 is the top choice according to all three methods. Given that it remains consistent across all the three methods. S3 appears to be the best option irrespective of the any of the three models used. In other words, S3 performs well in all the assessed criteria. On the contrary, alternative S6 constantly performs worse than the other site choices, as seen by its lowest ranking (the last) in all three approaches.

The intermediate choices (S1, S9, S4, S7, S10, S2, S5, S8) have different ranks depending on the approach used. This variation emphasizes how each technique weighs the advantages and disadvantages of various possibilities differently. For example, S10 is ranked higher (2nd) by WSM than by TOPSIS (6th) and MULTIMOORA (2nd), suggesting that the techniques may highlight distinct characteristics of S10. Although S1 is ranked second by TOPSIS, it is placed lower (5th) by both WSM and MULTIMOORA, indicating that WSM and MULTIMOORA place less weight on factors that TOPSIS prioritizes.

The variation in rankings suggest that the approach utilized has an impact on the decision-making process, especially in the intermediate places. For decision-makers to appropriately interpret the outcomes, they must be aware of the underlying presumptions and focal points of each approach. This variance might also mean that some criteria matter more in one technique than in another. In TOPSIS, the distance to an ideal solution is taken into account, however, in WSM, alternatives with great performance in key criteria may be rated differently.

This research aimed to determine which gold mining site selection technique is best for Ghana by using the same input data for three distinct MCDM approaches and comparing the ranking of the possibilities for each method. Three MCDM methods; TOPSIS, WSM, and MULTIMOORA-were selected because they each take a distinct approach to calculating alternative values and allow for the application of criteria that can designate the optimal value, such as a certain minimum or maximum. Although the models did not produce the same results, S3 and S6 were consistently ranked as the best and worst alternatives in each of the methods. In Ghana's Ashanti Region, S3 is the most preferred location for gold mining operations based on six factors (or criteria) taken into account in this work. S6 on the other hand is the least preferred site for gold mining.

Since S3 consistently ranks highest among all MCDM techniques, it ought to be given priority for gold mining. Its viability must be confirmed by feasibility studies, which include socioeconomic and environmental evaluations. If S3 has difficulties, S10, S9, and S4 have to be taken into account as backups. To evaluate the effects of methodological variances and criterion weights, sensitivity analysis is required. Involving stakeholders is essential for sustainability and wider acceptability. To lessen adverse impacts, environmental impact assessments must be carried out. To improve the robustness of decision-making, future studies should investigate hybrid MCDM techniques such fuzzy MCDM or AHP-TOPSIS.

References

[1] Acuña-Soto, C., Liern, V., And Pérez-Gladish, B. (2021). Normalization In TOPSIS-Based Approaches With Data Of Different Nature: Application To The Ranking Of Mathematical Videos. Annals Of Operations Research, 296(1), 541-569.

[2] Akram, M., Khan, A., And Ahmad, U. (2023). Extended MULTIMOORA Method Based On 2-Tuple Linguistic Pythagorean Fuzzy Sets For Multi-Attribute Group Decision-Making, Granular Computing, 8(2), 311-332.

[3] Aranizadeh, A., Kazemi, M., Barahmandpour, H., And Mirmozaffari, M. (2020). MULTIMOORA Decision Making Algorithm For Expansion Of HVDC And EHVAC In Developing Countries (A Case Study). Iranian Journal Of Optimization, 12(1), 63-71.

[4] Balioti, V., Tzimopoulos, C., And Evangelides, C. (2018, July). Multi-Criteria Decision Making Using TOPSIS Method Under Fuzzy Environment. Application In Spillway Selection. In Proceedings (Vol. 2, No. 11, P. 637). MDPI.

[5] Behzadian, M., Otaghsara, S. K., Yazdani, M., And Ignatius, J. (2012). A State-Of The-Art Survey Of TOPSIS Applications. Expert Systems With Applications, 39(17), 13051-13069.

[6] Brauers, W. K. M., And Zavadskas, E. K. (2012). Robustness Of MULTIMOORA: A Method For Multi-Objective Optimization. Informatica, 23(1), 1-25.

[7] Dymova, L., Sevastjanov, P., And Tikhonenko, A. (2013). A Direct Interval Extension Of TOPSIS Method. Expert Systems With Applications, 40(12), 4841-4847.

[8] Elsayed, E. A., Dawood, A. S., And Karthikeyan, R. J. I. J. E. T. T. (2017). Evaluating Alternatives Through The Application Of TOPSIS Method With Entropy Weight. Int. J. Eng. Trends Technol, 46(2), 60-66.

[9] Erdin, C., And Akbaş, H. E. (2019). A Comparative Analysis Of Fuzzy TOPSIS And Geographic Information Systems (GIS) For The Location Selection Of Shopping Malls: A Case Study From Turkey. Sustainability, 11(14), 3837.

[10] Farooq, M. U., And Saqlain, M. (2021). The Selection Of LASER As Surgical Instrument In Medical Using Neutrosophic Soft Set With Generalized Fuzzy TOPSIS, WSM And WPM Along With MATLAB Coding. Neutrosophic Sets And Systems, 40(1), 3.

[11] Ginting, G., Fadlina, M., Siahaan, A. P. U., And Rahim, R. (2017). Technical Approach Of TOPSIS In Decision Making. Int. J. Recent Trends Eng. Res, 3(8), 58-64.

[12] Jozaghi, A., Alizadeh, B., Hatami, M., Flood, I., Khorrami, M., Khodaei, N., And Ghasemi Tousi, E. (2018). A Comparative Study Of The AHP And TOPSIS Techniques For Dam Site Selection Using GIS: A Case Study Of Sistan And Baluchestan Province, Iran. Geosciences, 8(12), 494.

[13] Kaddani, S., Vanderpooten, D., Vanpeperstraete, J. M., And Aissi, H. (2017). Weighted Sum Model With Partial Preference Information: Application To Multi-Objective Optimization. European Journal Of Operational Research, 260(2), 665-679.

[14] Keshavarz-Ghorabaee, M., Amiri, M., Zavadskas, E. K., Turskis, Z., And Antucheviciene, J. (2018). A Comparative Analysis Of The Rank Reversal Phenomenon In The EDAS And TOPSIS Methods. Economic Computation And Economic Cybernetics Studies And Research, 52(3).

[15] Khoshi, A., Gooshki, H. S., And Mahmoudi, N. (2018). The Data On The Effective Qualifications Of Teachers In Medical Sciences: An Application Of Combined Fuzzy AHP And Fuzzy TOPSIS Methods. Data In Brief, 21, 2689-2693.

[16] Liang, D., And Xu, Z. (2017). The New Extension Of TOPSIS Method For Multiple Criteria Decision Making With Hesitant Pythagorean Fuzzy Sets. Applied Soft Computing, 60, 167-179.

[17] Miljković, B., Žižović, M. R., Petojević, A., And Damljanović, N. (2017). New Weighted Sum Model. Filomat, 31(10), 2991-2998.

[18] Namin, F. S., Ghadi, A., Amd Saki, F. (2022). A Literature Review Of Multi Criteria Decision-Making (MCDM) Towards Mining Method Selection (MMS). Resources Policy, 77, 102676.

[19] Okyere, M., Ayitey, J. Z., And Ajabuin, B. A. (2021). Large Scale Mining In Ghana: A ``Review Of The Implications On The Host Communities. Journal Of Degraded And Mining Lands Management, 9(1), 3193.

[20] Onajite, O., And Oke, S. A. (2021). The Application Of WSM, WPM And WASPAS Multicriteria Methods For Optimum Operating Conditions Selection In Machining Operations. Jurnal Rekayasa Sistem Industri, 10(1), 1-14.

[21] Selimi, A., Milošević, M., And Saračević, M. (2018). AHP–TOPSIS Model As A Mathematical Support In The Selection Of Project From Aspect Of Mobility–Case Study. J. Appl. Math. Comput.(JAMC), 2, 257-265.

[22] Sintaro, S., And Setiawansyah, S. (2024). Kombinasi Multi-Objective Optimization On The Basis Of Ratio Analysis (MOORA) Dan PIPRECIA Dalam Seleksi Penerimaan Barista. Jurnal Ilmiah Informatika Dan Ilmu Komputer (JIMA-ILKOM), 3(1), 13-23.

[23] Suryadi, D., Widjaja, W., Sungkar, M. S., Kraugusteeliana, K., Adhicandra, I., And Sujito, S. (2023). Evaluating Location Alternatives For A New Manufacturing Plant Using Weighted Sum Model Method. Journal Of Applied Science, Engineering, Technology, And Education, 5(1), 46-51.

[24] Tian, C., Peng, J. J., Long, Q. Q., Wang, J. Q., And Goh, M. (2022). Extended Picture Fuzzy MULTIMOORA Method Based On Prospect Theory For Medical Institution Selection. Cognitive Computation, 14(4), 1446-1463.

[25] Tunas Bangsa Pematangsiantar, S. T. I. K. O. M. (2017). Comparison Of Weighted Sum Model And Multi Attribute Decision Making Weighted Product Methods In Selecting The Best Elementary School In Indonesia. International Journal Of Software Engineering And Its Applications, 11(4), 69-90.

[26] Zavadskas, E. K., Baušys, R., Leščauskienė, I., And Omran, J. (2020). M-Generalised Q-Neutrosophic MULTIMOORA For Decision Making. Studies In Informatics And Control, 29(4),

[27] Zavadskas, E. K., Mardani, A., Turskis, Z., Jusoh, A., And Nor, K. M. (2016). Development Of TOPSIS Method To Solve Complicated Decision-Making Problems-An Overview On Developments From 2000 To 2015. International Journal Of Information Technology And Decision Making, 15(03), 645-682.