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Numerical Evaluation Of Risk-Averse Vendors Optimal Quantity With Perishable Items On A Proposed Single-Period Loss Function With Conditional Value At Risk

N.K Oladejo
Department Of Industrial Mathematics, School Of Mathematical Sciences
C.K. Tedam University Of Technology And Applied Sciences

Abstract
This paper deal with Numerical evaluation of risk-averse vendors in optimizing quantity with perishable items on a proposed single-period loss function with Conditional Value at Risk. Here, the retailer's optimal price and cycle length are determined by forming an economic order quantity model of the retailer's possible loss considering the perishability of the items and other market factors. The model is optimized to obtain the retailer's optimal trade cycle length and price. The retailer's profit or loss is then calculated with the EOQ model. The risk involved in the business operation is optimized using the Conditional Value at Risk (CVaR) and the optimal quantity of the risk-averse vendor is calculated by optimizing the Conditional Value at Risk on a proposed single-period loss function. Some numerical examples are provided in the work to examine the effectiveness of the model with sensitivity analysis to determine and identify the robustness and the behaviour of the models when there is change in the parameters.

Keywords: *Optimizing, Numerical, Quantity, risk-averse, conditional value, perishable, vendor*

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I. Introduction

Retailers are confronted with inventory decision problems and normally wish to make decisions that will result in a maximum profit or minimum loss. Many researchers have developed models to ease the process of inventory decisions. (Oladejo and Chinto (2023)) and (Chinto and Oladejo (2023)).The study of inventory optimization dates back to the 19^{th} century when Edgeworth,1888 developed a model to optimize cash reserves to satisfy random withdrawals. (Morse & Kimball, 1951) formulated a single-period model also known as the newsvendor model. The original assumptions of the classical news-vendor model are no longer suitable to meet the actual needs in business practice, so many extensions as proposed by Khouja (1999), Dada et al, (2003). and Arikan & Fichtinger (2017) The news-vendor model was further extended by Marcias-Lopez et al. (2021) to reduce the difficulty of inventory management of perishable goods.

There are a lot of factors affecting the business of retailers dealing with perishable goods such as the risk of loss due to deterioration resulting from overage, risk of opportunity loss if the retailer cannot satisfy customers' demands and short-selling season with highly volatile demand. Perishable goods are very sensitive to time, as time pass by the goods loses their value and after a specific time (life span), they are either discarded or salvaged.

Several Mathematical models have been developed by researchers to determine the economic ordering quantity (EOQ) which is the order quantity that minimizes the holding and other related costs to maximize profit. Inventory decay has a significant effect on the inventory cost of perishable items. The economic order quantity (EOQ) model considers the deterioration rate which is a very vital variable since it has a great impact on demand and generally affects the revenue of the retailer negatively. Retailers have to sell all their goods in a short period to avoid deterioration and have a lot to lose if inventory decisions are not efficiently and accurately made. They face the risk of being unable to satisfy all customers leading to opportunity loss or deterioration when order quantity exceeds the demand realized.

Chung (1990) presented a simple method for finding solutions to inventory problems. It showed that the description of the ordering policy can be done with a single equation irrespective of the sign of the covariance term. Huang (2003) proposed a model for retailers' optimal ordering policies in the EOQ model when trade is conducted on credit bases. He proposed a theorem to determine efficient ordering policies. This theorem was substantiated with numerical examples.

Valliathal and Ulhayakumar (2010) developed an economic order quantity model for optimal pricing and replenishment policies considering backlogging and shortages. The existence of a unique optimal solution to the optimization problem was examined and sensitivity analysis was conducted to determine the stability of the model.

Agi and Soni (2022) presented a model for the optimal price and inventory control of products considering the price and stock level-dependent demand. The model a solution procedure for determining the impact of physical deterioration and freshness degradation of perishables on retailers' optimal decisions.

An inventory model for perishable goods with a stock level-dependent demand rate was examined by Duan *et al.* (2010).It was assumed that backlogging rate depends on waiting time and the amount produced already backlogged simultaneously. The cases where holding inventory is profitable or otherwise were also studied in the course of developing the model. Shelf space inventory-dependent demand, backlogging, and rate of deterioration rate were considered for effective inventory decisions. The resulting model was also studied with numerical examples to ensure its effectiveness as a guide for retailers' inventory decisions.

Xu *et al.* (2017) investigated the optimal risk of loss using the news-vendor model and the conditional value at risk (CVaR) as a risk measure. It shows that the news vendor's optimal order quantity is dependent on the density function of market demand when the news vendor exhibits risk-averse preference. The research was able to find a measure for hedging against the risk of opportunity loss from the newsvendor's order decisions.

Wang *et al.* (2019) proposed a model for retailers serving economically constrained consumers by offering layaway. It was found that the optimal order quantity of risk-averse retailers using the conditional value at risk (CVaR) as a risk measure and the risk-neutral case was seen as a special case of risk-aversion.

In this paper, we optimizes inventory decisions and numerically evaluate the risk-averse retailers with perishable items using the Conditional Value at Risk (CVaR) model risk-averse vendors optimal quantity with perishable items on a proposed single-period loss function with conditional Value at Risk

Assumption and Parameters

In forming our Economic Order Quantity (EOQ) models we apply inventory optimization such as loss minimization or profit maximization to formulate of the retailers' loss function and determine the optimal replenishment cycle length which is then optimized using the CVaR on the newsvendor model with the assumptions that

i. Items deteriorate at a constant rate as time goes by
ii. Items have a specific shelf life (n) after which they cannot be sold.
iii. Items can neither be sold nor repaired at the end of the shelf life.
iv. Demand for items is price and inventory-level dependent.
v. Salvage value and deterioration expenses are considered for items that deteriorate throughout the trade cycle.
vi. Stock period (T) does not surpass the item's shelf life (n) since items are unsellable after the shelf life.
vii. The holding cost of each unit per unit time is h.
viii. The time range planning is infinite. The start time is zero and therefore, the replenishment rate is immediate

Notations and Parameters

Parameter	Description
Q	Order quantity at the beginning of a trade cycle.
P	The selling price of a unit item
C	The cost price of a unit item
S	salvage value per unit item
θ	Deterioration rate per unit item
φ	Salvage ratio
H	Holding cost of the item per unit of time
W	maximum shelf space
c_d	Deterioration cost
T	Replenishment cycle length.
F	Description
d(p)	Price-dependent demand
I(t)	Stock quantity at time t.
π(p, T)	Loss function per trade cycle.

II. Formation Of The Retailer's Loss Function

Here we consider the rate of change in inventory with time(t) such that change in inventory level depends on deterioration and demand as well as in price and inventory level given as:

$$\frac{dI(t)}{dt} = -\frac{n-t}{n}d(p) - \omega I(t) - \theta I(t) \qquad (1)$$

with an integrating factor of the form; $e^{\int(\theta+\omega)dt} = e^{(\theta+\omega)t}$

integrate (1) in the interval $0 \leq t \leq T$, to attain the quantity of goods left after a period of time(t) yields:

$$e^{(\theta+\omega)t}\left(\frac{dI}{dt} + (\theta+\omega)I\right) = -\frac{n-t}{n}De^{(\theta+\omega)t}$$

$$\left(e^{(\theta+\omega)t}I\right)^{\iota} = -\frac{n-t}{n}De^{(\theta+\omega)t}$$

$$e^{(\theta+\omega)t}I = -\frac{D}{n}\int_{t}^{T}(n-\ t)\,e^{(\theta+\omega)t}dt\ \ where\ \ 0 \le t \le T$$

$$e^{(\theta+\omega)t}I = -\frac{D}{n}\left[\frac{n-t}{\theta+\omega}e^{(\theta+\omega)t} + \int\frac{e^{(\theta+\omega)t}}{\theta+\omega}dt\right]$$

$$e^{(\theta+\omega)t}I = -\frac{D}{n}\left[\frac{n-t}{\theta+\omega}e^{(\theta+\omega)t} + \frac{e^{(\theta+\omega)t}}{(\theta+\omega)^2}\right]$$

$$I = -\frac{D}{n}\left[\frac{n-t}{\theta+\omega} + \frac{1}{(\theta+\omega)^2}\right]$$

Inserting the limits we obtain the quantity of inventory for that time interval and.the quantity of goods in store at time $t \ge 0$ is given as:

$$I(t) = -\frac{D}{n}\left[\frac{n-t}{\theta+\omega} + \frac{1}{(\theta+\omega)^2}\right]_t^T$$

Considering the lower limit as t we obtain the quantity of goods left after time t

$$I(t) = -\frac{D}{n}\left[\frac{-T+t}{\theta+\omega}\right]$$

Solving equation (1) in the interval [0, T] gives I (t), the stock level at time t as.

$$I(t) = \frac{d(p)}{n}\left[\frac{T-t}{\theta+\omega}\right] \qquad (2)$$

Order quantity (q) occurs at the time$t = 0$, therefore $I(0) = q$.
Hence:

$$q = \frac{d(p)}{n}\left[\frac{T}{\theta+\omega}\right] \qquad (3)$$

We develop the retailer's loss function to find the retailer's optimal selling price and cycle length by finding the difference between the revenue generated and the cost incurred as following:

$\pi(p,T) = (PC + DC + HC) - (PR + SV)$ (4)

Where: PC is purchase cost: DC is direct cost; HC; Holding cost; PR: Purchase and SV is Salvage Value)

Thus:

$$\pi(p,T) = d(p)\left[\frac{Tc}{n(\theta+\omega)} + \frac{hT^2}{2n(\theta+\omega)} + \frac{c_d\theta T}{n(\theta+\omega)} - \frac{c_d n\omega T^2}{n(\theta+\omega)} - c_d\theta T + \frac{c_d\theta T^2}{2n} - pT + \frac{pT^2}{n} - \frac{p\omega T^2}{2n(\theta+\omega)} - \frac{s\varphi T}{n(\theta+\omega)} + s\varphi T - \frac{s\varphi T^2}{2n} + \frac{s\varphi\omega T^2}{\theta+\omega}\right] \qquad (5)$$

Simplifying (5) gives:

$$\pi(p,T) = d(p)\left[\left(\frac{c + c_d\theta - s\varphi}{n(\theta+\omega)} + s\varphi - c_d\theta - p\right)T + \left(\frac{h - 2np\omega - 2nc_d\theta\omega + 2s\varphi n\omega}{2n(\theta+\omega)} + \frac{c_d\theta + p - s\varphi}{2n}\right)T^2\right] \qquad (6)$$

Retailer's optimal cycle length and price

Given the loss function as $\pi(p,T)$ the convex of p and T in order to have a unique solution. Thus, the loss function should be a convex function for the existence of a global minimum. For the loss function to have a global minimum or a unique solution, it should satisfy the necessary conditions of convexity. That is there should be a solution or values of the p and T for which the following conditions are satisfied.

$$\left.\begin{aligned}\frac{d_n(p,T)}{dT} &\equiv 0\\ \frac{d_n(p,T)}{dp} &\equiv 0\end{aligned}\right\} \qquad (7)$$

Where p and T are price and trade cycle length respectively.For the loss function to be convex, the following conditions must be satisfied.

$$\left.\begin{aligned}\frac{d^2{}_n(p,T)}{dT^2}. > 0\\ \frac{d^2{}_n(p,T)}{dp^2} > 0\end{aligned}\right\} \qquad (8)$$

$$\left.\frac{\partial^2{}_n(p,T)}{\partial T\partial p} = \frac{\partial^2{}_n(p,T)}{\partial p\partial T}.\right\} \qquad (9)$$

$$\left(\frac{d^2{}_n(p,T)}{dT^2}\right)\left(\frac{d^2{}_n(p,T)}{dp^2}\right) - \left(\frac{\partial^2{}_n(p,T)}{\partial T\partial p}\right)\left(\frac{\partial^2{}_n(p,T)}{\partial p\partial T}\right) < 0 \qquad (10)$$

If the above conditions are satisfied, then the function can be described as convex and the optimal solution can be obtained by solving the equations (6) simultaneously. If the resulting solution satisfies the conditions 7,8 and 9 then the function $\pi(p,T)$ is strictly convex in both variables with a positive definite Hesian matrix.and if this is true, then solution (p,T) is the optimal solution.
Defferentiating the loss function in equation (9)gives the following;

$$\frac{\partial\pi}{\partial T} = d(p)\left[\frac{c+c_d\theta-s\varphi}{n(\theta+\omega)} + s\varphi - c_d\theta - p + 2T\left(\frac{h-2np\omega-2nc_d\theta\omega-2s\varphi n\omega}{2n(\theta+\omega)} + \frac{c_d\theta+p-s\varphi}{2n}\right)\right] \qquad (11)$$

$$\frac{\partial^2\pi}{\partial T^2} = d(p)\left[\frac{h-2np\omega-2nc_d\theta\omega+2s\varphi n\omega}{n(\theta+\omega)} + \frac{c_d\theta+p-s\varphi}{n}\right] \qquad (12)$$

$$\frac{\partial\pi}{\partial p} = d(p)^{\iota}\left[\left(\frac{c+c_d\theta-s\varphi}{n(\theta+\omega)} + s\varphi - c_d\theta - p\right)T + \left(\frac{h-2np\omega-2nc_d\theta\omega+2s\varphi n\omega}{n(\theta+\omega)} + \frac{c_d\theta+p-s\varphi}{n}\right)T^2\right] + d(p)\left[\left(\frac{1}{2n} - \frac{n\omega}{\theta+\omega}\right)T^2 - T\right] \qquad (13)$$

$$\frac{\partial^2\pi}{\partial p^2} = d(p)^{u}\left[\left(\frac{c+c_d\theta-s\varphi}{n(\theta+\omega)} + s\varphi - c_d\theta - p\right)T + \left(\frac{h-2np\omega-2nc_d\theta\omega+2s\varphi n\omega}{n(\theta+\omega)} + \frac{c_d\theta+p-s\varphi}{n}\right)T^2\right] + 2d(p)^{\iota}\left[\left(\frac{1}{2n} - \frac{n\omega}{\theta+\omega}\right)T^2 - T\right] \qquad (14)$$

From the equations above the condition in (6) is satisfied since

$$\frac{\partial^2\pi}{\partial p\partial T} = d(p)^{\iota}\left[\frac{c+c_d\theta-s\varphi}{n(\theta+\omega)} + s\varphi - c_d\theta - p + 2T\left(\frac{h-2np\omega-2nc_d\theta\omega-2s\varphi n\omega}{2n(\theta+\omega)} + \frac{c_d\theta+p-s\varphi}{2n}\right) + d(p)\left(\frac{T}{n} - \frac{2nT}{\theta+\omega} - 1\right)\right] \qquad (15)$$

and

$$\frac{\partial^2\pi}{\partial T\partial p} = d(p)^{\iota}\left[\frac{c+c_d\theta-s\varphi}{n(\theta+\omega)} + s\varphi - c_d\theta - p + 2T\left(\frac{h-2np\omega-2nc_d\theta\omega-2s\varphi n\omega}{2n(\theta+\omega)} + \frac{c_d\theta+p-s\varphi}{2n}\right) + d(p)\left(\frac{T}{n} - \frac{2nT}{\theta+\omega} - 1\right)\right] \qquad (16)$$

The parameters of the function determines whether the rest of the conditions and other conditions are satisfied. If the other conditions are not satisfied then, the function is not strictly convex and optimal solution will not exist or will not be unique. $\frac{d\pi(p,T)}{dT} = 0$ and $\frac{d\pi(p,T)}{dp} = 0$

Value at Risk (VaR) and Conditional Value at Risk (CVaR)

Here we let $\{y\}$ represents the vector of market uncertainty and $f(y)$ a probability density function with cumulative distribution function $F(y)$
Let $\alpha \in (0,1)$ be the confidence level of a retailer who can only tolerate loss up to β.
Then the risk-averse retailer with a confidence level of α has a value at risk as:

$$VaR_\alpha(x) = \inf\{\beta \in R : Pr\{l(x) \le \beta\} \ge \alpha\} \qquad (17)$$

Where $pr\{l(x) \le \beta\}$ represents the probability that loss does not exceed the threshold β and α denotes the confidence level.
The conditional value at risk (CVaR) is the average of losses beyond the risk-averse retailer's threshold.
Given the confidence level $\alpha \in (0,1)$ and a loss threshold β,
Then the CVaR is given as;

$$CVaR_\alpha(x,\beta) = (1-\alpha)^{-1} \int_{l(x)\geq\beta}^{t} f(y)dy \qquad (18)$$

To optimize the risk of the retailer, we seek to find the optimal inventory decision that leads to a minimal risk beyond the threshold.

The CVaR guides the vendor to make decision that will not lead to unbearable loss when there is an unforeseen shock in the business environment. Thus we consider CVaR proposed by Rockafellar and Uryasev, (2000) with an auxiliary function:

$$P_\alpha(q,\beta) = \beta + (1-\alpha)^{-1} \int_{y\in R} (l(q)-\beta)^+ f(y)dy \qquad (19)$$

Where β is the loss threshold and α is the confidence level and $CVaR_\alpha(x,\beta)$ can be minimized by minimizing the auxiliary function$P_\alpha(q,\beta)$.

Thus:

$$P_\alpha(q,\beta) = \beta + (1-\alpha)^{-1} \int_0^\infty [(\theta - s\psi)]. \qquad (20)$$

$$P_0(q,\beta) = \beta + (1-\alpha)^{-1} \int_0^\infty \left[(\theta - s\phi)(q-D) + (H+c)q + \mu(D-q) - p\min\{q,D\} - \beta\right]^+ dF(y).. \quad (21)$$

We let the function $P_\alpha(q,\beta)$ be convex and hence has a unique minimizer according to (Rockafella &Erseyav 2001). At the minimum, the first derivative with respect to the order quantity q is zero. Thus, if q^* is the optimal order quantity of the risk-averse vendor

Then $\frac{\partial P_\alpha(q^*,\beta)}{\partial q} = 0$.

Solving $\frac{\partial P_\alpha(q^*,\beta)}{\partial q} = 0$ gives the optimal order quantity for risk-averse retailer with perishable goods.

III. Optimizing The Retailers' Operational Risk.

According to Rockafellar and Uryasev (2000) Risk-averse vendors make their decisions by considering the riskiness of the operations. Their decisions are mostly affected by their degree of risk resistance. The risk-averse retailer's optimal order quantity can be determined by optimizing the conditional value at risk (CVaR) over the single period loss function..

To determine the optimal solution of $P_\alpha^*(q,\beta)$, we consider the following cases.

i. If $\beta \geq (c - s\varphi)q$;

Conditional Value at Risk CVaR optimization can expressed in the form;

$$P(q,\beta) = \beta + \frac{1}{(1-\alpha)} \int_{q+\frac{\beta}{c-s\varphi}}^{\infty} [(H+c)q + \mu(D-q) - pq - \beta]dF(y) \qquad (22)$$

Since $P(q,\beta)$ is convex in both variables for the best possible threshold for the risk-averse retailer, the first partial derivative with respect to β is equal to zero.

Thus:

$$\frac{\partial P}{\partial \beta} = 1 + (1-\alpha)^{-1} \int_{q+\frac{\beta}{c-s\varphi}}^{\infty} -dF(y) \qquad (23)$$

$$\frac{\partial P}{\partial \beta} = 1 - (1-\alpha)^{-1}\left[1 - F\left(q + \frac{\beta}{c-s\varphi}\right)\right] \qquad (24)$$

For $\frac{\partial P}{\partial \beta} = 0$, then the optimal value for β is obtained as:

$$\beta^* = (c - s\varphi)(F^{-1}(\alpha) - q) \qquad (25)$$

ii. If $0 < \beta \leq (c - s\varphi)q$

Here, the retailer's threshold is between zero, thus break even and the losss incured when no item is sold only a fraction of goods is salvaged. The vendor decide not to venture into a business that has the potential of loss in that interval. The retailers unacceptable loss falls between zero and $(c - s\varphi)q$

$$P(q,\beta) = \beta + (1-\alpha)^{-1} \int_{0}^{q-\frac{\beta}{c-s\varphi}} [(\theta - s\varphi)(q-D) + (H+c)q - pD - \beta] dF(y)$$

$$+(1-\alpha)^{-1} \int_{q+\frac{\beta}{c-s\varphi}}^{\infty} [\mu(D-q) - pq - \beta] dF(y) \qquad (26)$$

$$\frac{\partial P}{\partial \beta} = 1 - (1-\alpha)^{-1} F\left(q - \frac{\beta}{c-s\varphi}\right) - (1-\alpha)^{-1}\left[1 - F(q + \frac{\beta}{c-s\varphi})\right] \qquad (27)$$

Considering the optimal point in this case $\left(\frac{\partial P}{\partial \beta} = 0\right)$;

$$F\left(q + \frac{\beta}{c-s\varphi}\right) = F\left(q - \frac{\beta}{c-s\varphi}\right) + \alpha \qquad (28)$$

iii. $\beta \leq 0$

Here the retailer considers the largest unacceptable loss to be zero. Thus, the vendor chooses the threshold as zero or less than zero. This means the retailer will consider the break even or some amount of profit as unbearable and hence will not venture into business with low or no profit.

Thus:

$$P(q,\beta) = \beta + (1-\alpha)^{-1} \int_{0}^{q} [(\theta - s\varphi)(q-D) + (H+c)q - pD - \beta] dF(y) + (1-\alpha)^{-1} \int_{q}^{\infty} [(H+c)q + \mu(D-q) - pq - \beta] dF(y) \qquad (29)$$

$$\frac{\partial P}{\partial \beta} = 1 - (1-\alpha)^{-1} F(q) - (1-\alpha)^{-1}[1 - f(q)] \qquad (30)$$

Thus: it follows that.

$1 = 1 - \alpha$ When $\frac{\partial P}{\partial \beta} = 0$

This means there is no optimal solution when $\beta \leq 0$ hence P has an optimal solution when $\beta > 0$. the optimal solution satisfies P, therefore;

$$P(q,\beta^*) = \beta^* + (1-\alpha)^{-1} \int_{0}^{q-\frac{\beta^*}{c-s\varphi}} [(\theta - s\varphi)(q-D) + (H+c)q - pD - \beta^*] dF(y)$$

$$+ (1-\alpha)^{-1} \int_{q+\frac{\beta^*}{c-s\varphi}}^{\infty} [\mu(D-q) - pq - \beta^*] dF(y) \qquad (31)$$

The optimal order quantity can be calculated in the same way since the function is also convex in q. The best order quantity of risk-averse retailer is the quantity that optimizes risk but also optimize profit of the business operation.

$$\frac{\partial P}{\partial q} = (1-\alpha)^{-1}[\theta - s\varphi + H + c] F\left(q - \frac{\beta^*}{c-s\varphi}\right) + (1-\alpha)^{-1}(-\mu - p)\left[1 - F\left(q + \frac{\beta^*}{c-s\varphi}\right)\right] \qquad (32)$$

Then, it follows that $\frac{\partial P}{\partial q} = 0$ gives.

$$(\theta - s\varphi + H + c) F\left(q - \frac{\beta^*}{c-s\varphi}\right) = (\mu + p) - [\mu + p] F\left(q + \frac{\beta^*}{c-s\varphi}\right) \qquad (33)$$

From (28) it follows that:

$$(\theta - s\varphi + H + c)\left[F\left(q - \frac{\beta^*}{c-s\varphi}\right)\right] + (\mu + p)\left[F\left(q - \frac{\beta^*}{c-s\varphi}\right) + \alpha\right] = \mu + p \qquad (29)$$

$$F\left(q - \frac{\beta^*}{c-s\varphi}\right) = \frac{(1-\alpha)(\mu + p)}{\theta - s\varphi + H + \mu + p} \qquad (34)$$

$$\Rightarrow q_\alpha^* - \frac{\beta^*}{c - s\varphi} = F^{-1}\left(\frac{(1-\alpha)(\mu+p)}{c_d\theta - s\varphi + H + c + \mu + p}\right) \qquad (35)$$

It then follows that:

$$F\left(q_\alpha^* + \frac{\beta^*}{c - s\varphi}\right) = \frac{\mu + p + \alpha(c_{d\theta} - s\varphi + H + c)}{c_{d\theta} - s\varphi + H + c + \mu + p} \qquad (36)$$

$$\Rightarrow q_\alpha^* + \frac{\beta^*}{c - s\varphi} = F^{-1}\left(\frac{\mu + p + \alpha(c_{d\theta} - s\varphi + H + c)}{c_{d\theta} - s\varphi + H + c + \mu + p}\right) \qquad (37)$$

Hence (31) and (32) yields;

$$q_\alpha^* = \frac{1}{2}\left[F^{-1}\left(\frac{(1-\alpha)(\mu+p)}{c_d\theta - s\varphi + H + c + \mu + p}\right) + F^{-1}\left(\frac{\mu + p + \alpha(c_{d\theta} - s\varphi + H + c)}{c_{d\theta} - s\varphi + H + c + \mu + p}\right)\right] \quad (38)$$

Theorem1:

The optimal order quantity q_α^* of a risk-averse retaailer dealing with perishable goods minimizes CvaR of of the retail loss (Chinto and Oladejo,2023):

$$q_\alpha^* = \frac{1}{2}\left[F^{-1}\left(\frac{(1-\alpha)(\mu+p)}{c_d\theta - s\varphi + H + c + \mu + p}\right) + F^{-1}\left(\frac{\mu + p + \alpha(c_{d\theta} - s\varphi + H + c)}{c_{d\theta} - s\varphi + H + c + \mu + p}\right)\right] \qquad (39)$$

When $\alpha = 0$, the risk-averse news vendor turns to a risk-neutral.

It follows that when $\alpha = 0$ the theorem reduces to the expected loss optimization model (ELOM). The confidence level reflects the degree of risk aversion of the retailer. It also depicts the probability that the retailer does not cross the threshold. For instance, if the confidence level is 95% it means that a dealer in this line of business has 95% chance of staying in the safe zone.

Numerical evaluation of risk-averse vendors optimal quantity with perishable items

Here we consider simulation of retail market in order to determine retailer's optimal quantity and assummed that the density function of the market is exponential or normal. Here we find out how the risk-averse vendor behave in these markets as shown in the illustration with table 1 below

Illustration 1: we consider a risk-averse newsvendor problem with the stochastic demand subjects to an exponential distribution $e(\lambda)$. We seek the order quantity that minimizes risk of operations and optimizes efficiency of the business as the perishable product is sold at GH¢19.53, purchase cost GH¢5.00, deterioration cost GH¢2.00, deterioration rate 0.05, salvage price GH¢4.00, salvage ratio 0.8, holding cost of unit per cycle GH¢0.74 and a shortage cost of GH¢2.00. The results in the table 1 below give the optimal order quantity and confidence level of a risk-averse retailer.

Table 1 a: Optimal quantities with exponential distribution for illustration 1.

D	α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
E(0.01)	q_α^*	221	197	184	177	175	175	179	186	201	231
E(0.02)	q_α^*	111	98	92	89	87	87	89	93	100	115
E(0.03)	q_α^*	74	66	61	59	58	58	60	62	67	77
E(0.04)	q_α^*	55	49	46	44	44	44	45	47	50	58
E(0.05)	q_α^*	44	39	37	35	35	35	36	37	40	46

The risk-averse vendor in the illustration 1 above has optimal quantities as shown in table 1b below if the market distribution is normal.

Table 1b : Optimal order quantities with normal distribution for illustration 1.

D	α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N(1000,100)	q_α^*	1123	1107	1096	1087	1080	1073	1067	1061	1055	1047
N(100,50)	q_α^*	162	153	148	144	140	137	134	131	127	124
N(100,25)	q_α^*	131	127	124	122	120	118	117	115	114	112

Illustration 2: Here we consider the vendor deals with fresh goods with stock level coefficient of 0.5 and a cost c of GH¢ 7 , holding cost per unit h 29 pesewas, deterioration cost $c_d = GH$¢2 and deterioration rate θ of 0.1. If the retailer can sell the item at GH¢ 23.65 , salvage a ratio of 0.8 at GH¢4 and suffer a shortage cost of GH¢2. The risk sensitive retailer's optimal order quantity for goods with exponential distribution is calculated as shown in the table 2 a below:

Table 2a: Optimal order quntities with exponential distribution for illustration 2.

D	α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
E(0.01)	q_α^*	195	176	166	161	159	160	164	172	187	217
E(0.002)	q_α^*	973	882	831	805	795	799	820	862	936	1084
E(0.005)	q_α^*	389	353	333	322	318	320	328	345	374	335

Table 2b below shows the optimal order quantity risk-averse vendors of with different confidence levels and parameters as stated in illustration 2. It shows that the risk-averse vendor's optimal quantity falls to some extent and rises as the confidence level approaches 1.

Table 2b: Optimal order quantities of a risk-averse vendor in a market with different confidence

D	α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N(1000.100)	q_α^*	1107	1094	1084	1077	1070	1064	1058	1053	1048	1041
N(100,50)	q_α^*	153	147	142	138	135	132	129	127	124	121
N(100,25)	q_α^*	127	123	121	119	118	116	115	113	112	110

Table 2b.above shows that the risk averse retailer's optimal order quantity is inversely related to the confidence level in a market with normal distribution. This confirms the results in table 3 and also the results in Xu et al. (2017)

The following graphs shows the relationship between order quantity and confidence level in a market when the probability density function is exponential. It shows that the retailer's order quantity falls as the confidence level increases from 0 to 0.4 and then increase with the confidence level. The risk-averse retailers optimal order quantity coincide with the risk–neutral vendors order quantity when the confidence level is zero. This means when the confidence level is between 0 and 0.4 the risk-averse retailer's order quantity is less than that of the order quantity of the risk-neutral vendor

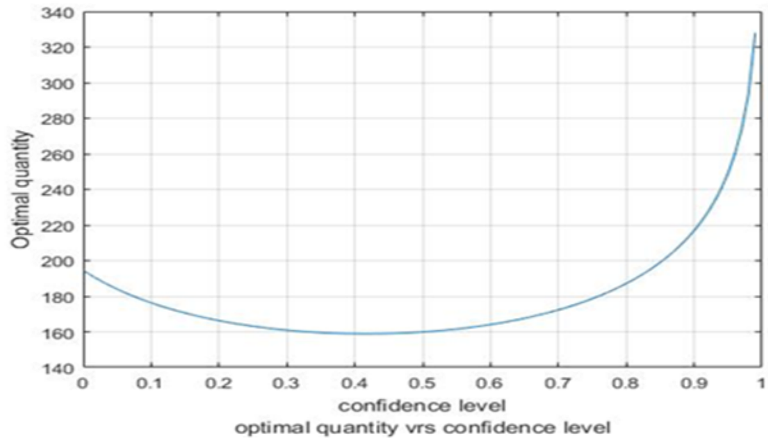

Fig.1: Optimal Quantity Graph For Illustration 2 14E(0.001)

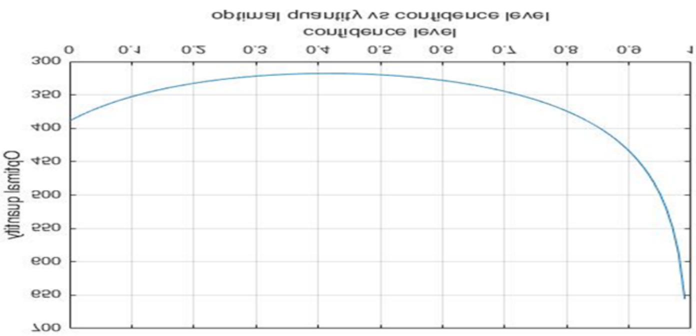

Fig.2: Optimal quantity graph for illustration.2 E(0.005)

The figures 3 and 4 below confirm that the risk-averse retailer's optimal quantinty decrease and increase in a market with exponential distribution when the confidence level increases from 0 to 0.9. For the market where the distribution function is normal as indicated in the following results in table 4 below are obtained for the illustration 2 above.

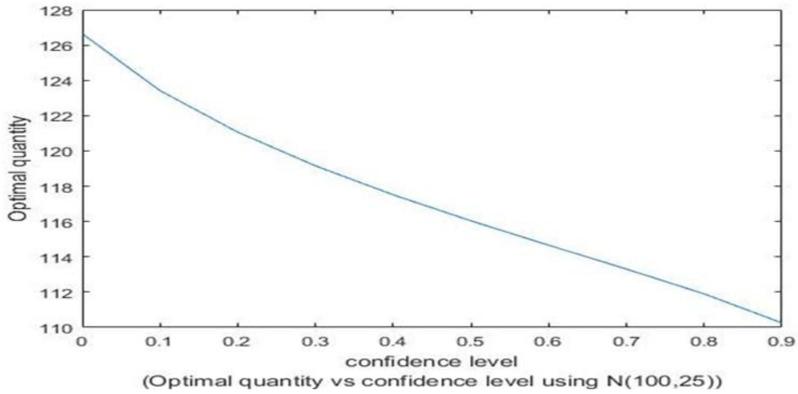

Fig.3.Shows The Risk-Averse Retailer's Optimal Quantinty Decrease

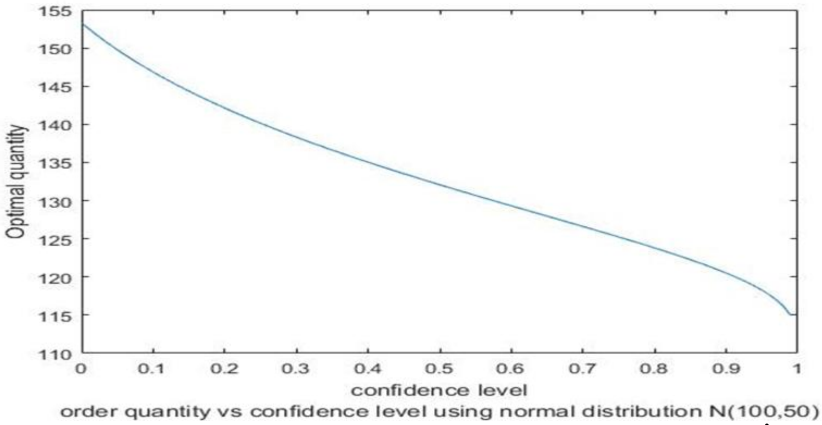

Fig 4. The Risk-Averse Retailer's Optimal Quantinty Decrease And İncrease

Illustration 3: Here we consider a risk-averse retailer dealing with perishable items that cost GH¢7 and has stock level dependent demand coefficient 0.5. We assume that the retailer sell the items at GH¢23.65 and salvages 0.8 of the remaining items at GH¢4.00. The demand for the product is stochastic and follows the normal distribution N(s,v^2). If the items deteriorate at the rate of 0.1 at the cost of GH¢2.00 and have a holding cost of GH¢0.29. The retailer dealing with this kind of goods in that market will also face a shortage cost per unit as GH¢2.00. Here the order quantity is calculated to reduce shortages to a minimum and also maximize the profit of the operation. This is calculated by the optimal order quantity for the conditional value at risk(CvaR) ;

$$q_{\alpha}^{*} = \frac{1}{2}\left[F^{-1}\left(\frac{(1-\alpha)(\mu+p)}{c_d\theta - s\varphi + H + c + \mu + p}\right) + F^{-1}\left(\frac{\mu+p+\alpha(c_{d\theta} - s\varphi + H + c)}{c_{d\theta} - s\varphi + H + c + \mu + p}\right)\right] \qquad (40)$$

D	α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N(100,50^2)	q_{α}^{*}	153	146	142	138	135	132	129	127	124	121
N(100,25^2)	q_{α}^{*}	127	123	121	119	118	116	115	113	112	110
N(1000,100^2)	q_{α}^{*}	1059	1043	1030	1019	1009	998	988	976	962	943

Table 5: Optimal order quantities of risk-averse retailer with Normal distribution for illustration

Table 5 provides the optimal order quantities of a risk-averse retailer in illustration 3. This table also shows that the risk-averse vendor's optimal quantity is invrsely related to the confidence level of a retailer when the market demand follows the normal distribution function.

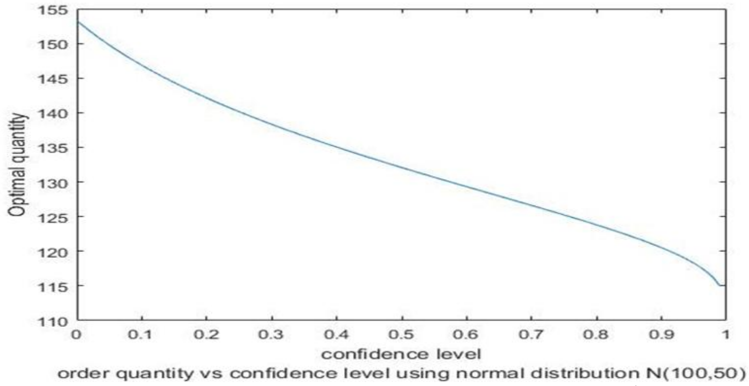

Fig.3: Quantity-Confidence Level Graph *N* (100,50) For İllustration 5.

Fig.4: Quantity-Confidence Level Graph *N* (100, 25) For İllustration 3

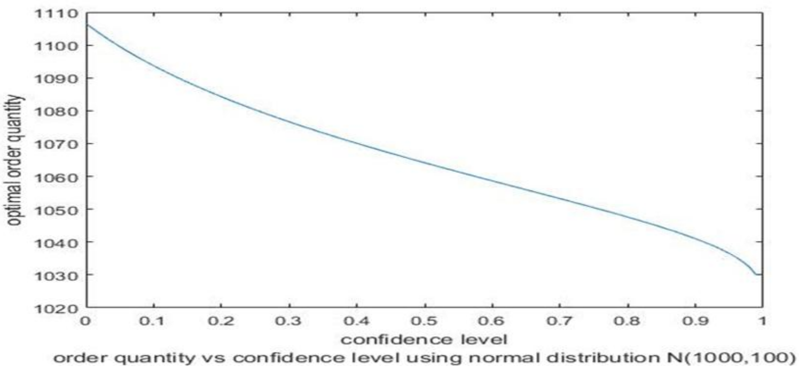

Fig.5: Quantity- Confidence Level Graph N (1000,100) For Illustration 3.

Fig.s 4, 5 and the table 5 above can be seen that the risk-averse retailers order quantity decreases as the confidence level increases. This means that in market where the demand for goods follows the normal distribution function the optimal profit optimization quantity is always greater than the quantity that optimizes the risk of operation. From this deduction it is obvious that the order quantity of a risk-neutral vendor is greater than that of the risk-averse vendor.

Sensitivity Analysis

Here we examine the effects of changes on the results of the proposed models to determining the stability of the models. When small change in the model parameter results in an unproportionally large change in the result the model is said to be unstable. On the other hand when change in the parameter leads to proportional change in the results of the model, then the model can be described as stable as discribed and summarised in sensitivity analysis of optimal solution of the price and time dependent loss function using the given parameters in table 6 below

Table.6: Sensitivity analysis for the loss function $\pi(p, T)$

Sensitivity analysis of optimal solution of the price and time dependent loss function				
Parameter		T	p (GH¢)	Profit (GH¢)
	0.7	0.4392	20.70	578
	0.8	0.4269	20.70	575
H	0.9	0.4369	20.72	571
	2	0.4346	20.72	571
	3	0.433	20.74	567
c_d	4	0.4337	20.75	566
	0.03	0.4407	20.70	585
	0.04	0.4418	20.62	593
θ	0.05	0.4316	20.73	567
	2	0.4801	20.76	605
	3	0.4751	20.35	655
S	4	0.4706	19.95	707

	0.6	0.4467	20.93	556
	0.7	0.4405	20.83	563
φ	0.8	0.4346	20.72	571
	0.3	0.3134	21.87	340
	0.4	0.4118	21.08	505
ω	0.5	0.3752	20.62	495
	3	0.5216	18.86	926
	4	0.477	19.80	733
C	5	0.4346	20.72	571

From the table 6 above it is the deduced that:

i. Holding cost of goods H increases with price and holding time but is inversely related to profit.. As the price increase causes low demand and consequently lead to a longer period of holding goods. Price increment has the potential of causing low demand which will lead to deterioration of goods and hence lower the profit. Therefore, holding cost relates negatively with the profit.
ii. Deterioration cost c_d is directly related to price but inversely related to profit of the business operation. As deterioration cost is high the retailer will sell at high price due to the high cost of deterioration.
iii. Deterioration rate (θ) increases with holding time and profit but inversely related to price.
iv. Salvage value is directly related to profit but inversely related to price and holding time.
v. Salvage ratio φ is directly related to profit but inversely related to price and replenishment cycle length.
vi. Cost C is directly related to price but inversely related to replenishment cycle length and profit

Table 7. Sensitivity analysis for the risk-neutral vendor model

Parameter	parameter value	optimal quantity	Profit(GH¢)
	2	441	2793.4
μ	3	449	2788.6
	4	457	2783.8
	0.03	444	2791.6
θ	0.04	443	2792.2
	0.05	441	2793.4
	1	445	2791
c_d	2	441	2793.4
	3	438	2795.2
	0.6	394	2821.6
φ	0.7	415	2809
	0.8	441	2793.4
	2	358	2843.2
S	3	394	2821.6
	4	441	2793.4
	0.7	448	2789.2
H	0.8	441	2793.4
	0.9	434	2797.6
	3	716	2628.4
C	4	529	2740.6
	5	441	2793.4
	17	423	2804.2
P	20	449	2788.6
	22	465	2779

Here, it was observe that:

i. Shortage cost is directly related to the optimal order quantity that is, the order quantity increases as the shortage cost rises. This is a result of the retailer's effort to minimize cost or maximize profit by reducing the unserviced demand to the minimum.
ii. As the deterioration rate increases the order quantity declines due to the retailer's efforts to reduce the number of items that perish in a trade cycle.
iii. Deterioration is inversely related to the optimal order quantity. Thus, when the cost of deterioration is high the retailer will order less.
iv. Salvage ratio is directly related to the optimal order quantity. The retailers order more because even if they are not able to sell all they can salvge large portion of the left over goods
v. Salvage price is also directly related with the order quantity thus, a positive change in the salvage price results in a positive change in the order quantity.
vi. Holding cost is inversely related to the optimal order quantity. Goods with high holding cost are ordered in lower quantities since a great potion of the retailer's revenue will be used for storage related expenses.

vii. Purchase cost is inversely related to the optimal order quantity. The retailer will not be able to buy large quantitie when the fee of purchase is high. On the other hand when purchase cost is low retailers can easily afford large quanties.

viii. Selling price is directly related to the retailer's optimal order quantity thus, the retailer orders more when the selling price is high. When the vendor's selling price increase the profit margin also increase and the vendor will order more for more profit. On the contrary the profit margins will be low and the business will not be attractive to vendors and hence order quantity of the item will fall.

Table 8 Sensitivity analysis on the risk-averse vendor model

Parameter	parameter value	optimal quantity	Profit(GH¢)
μ	2	117.1419	1033.4
	3	117.499	1040
	4	117.8388	1046.2
θ	0.03	117.2588	1035.6
	0.04	117.2001	1034.5
	0.05	117.1419	1033.4
c_d	2	117.1419	1033.4
	3	116.998	1030.8
	4	116.8566	1028.2
φ	0.6	115.0901	995.66
	0.7	116.0542	1013.4
	0.8	117.1419	1033.4
S	2	113.4359	965.22
	3	115.0901	995.66
	4	117.1419	1033.4
H	0.75	117.1419	1033.4
	0.8	115.9276	1011.1
	0.85	115.8028	1008.8
C	5	117.1419	1033.4
	7	112.7137	951.9321
	9	109.7915	898.16
P	14	115.0262	994.48
	19	117.1419	1033.4
	24	118.7696	1063.4
α	0.8	127.0437	1215.6
	0.85	125.3308	1184.1
	0.99	117.1419	1033.4

Thus, it was revealed that:

i. As cost related parameters increase the optimal order quantity of the risk-averse vendor decreases which also cause the profit to fall.

ii. As revenue related parameters such as salvage value, price and salvage ratio increase the optimal order quantity increases. As a result of that the profit also increases.

IV. Conclusion

The risk involved in carrying out business and the level of risk-aversion of a retailer has a significant effect on the retailer's inventory decisions. It is noted that the retailer is risk-neutral as they the retailer does not consider the risk levels when making inventory decisions but rather considers the possible returns and make decisions to minimize cost and maximize profit. A risk- neutral vendor with perishable items can optimize his or her operation using the loss function as given in illustrations 1and 2

It was observed that the loss function makes provision for the retailers with different price-dependent demand function and with a variety of perishable goods in a variety of markets whereby it provide the optimal solution procedure for retailers with perishable items but if the retailer is risk resistant this solution will be inefficient since it does not minimize the risk of the operation.

For the risk-averse vendor's optimal order quantity the retailer expected loss is formulated and optimized on the Conditional Value at Risk (CvaR). This showed that the optimal order quantity of risk-averse and risk-neutral retailers with perishable items in the same market is the same when the confidence level of the risk-averse retailer is zero.

This paper provides a solution procedure to determine the optimal cycle length and price for perishable items as the optimal order quantity is modeled measuring the risk using the CvaR. And it can be use in the inventory management of perishable items.

The sensitivity analysis in the paper shows that the models are stable since minor changes does not lead to huge changes in the functional results as the models gives no extra ordinary effect when there is slight change in the parameters. This means the models are stable. Risk-averse vendors can use these models to guide decisions in their business operations.

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