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A Survey On Inventory Model With Lost-Sale

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Abstract:
Classic inventory models commonly assume that surplus demand is backordered. However, research analysing customer behaviour in practice demonstrate that in many retail situations, unsatisfied demand is lost or replaced with an alternative item/location. Inventory systems with this lost-sales characteristic appear to be more challenging to analyse and resolve. Furthermore, to save costs, lost-sales inventory systems require different replenishment procedures than backorder systems. In this work, we present a survey on inventory models with lost sales. Furthermore, directions for future investigation are suggested.
Key Word: *Inventory model; Lost-sale.*

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I. Introduction

Inventory management [1] is a vital component of supply chain operations, ensuring that organisations have enough inventory to satisfy demand without overstocking or understocking. In practice, the quantity obtained may differ from the number ordered owing to worker strikes, inspection rejection, shipping damage, human counting errors, and so on. As a result, managers are frequently forced to make judgements in the face of ambiguous information. Inventory models [2] with lost-sale are those with shortages in the case where the quantity received is not certain, and the lead time, lost sales rate, and order processing cost are decision variables.

The global out-of-stock rate is around 7–8% as described in [3] and [4]. The client reaction to such stock-outs tends to be somewhat complex. According to Gruen et al., just 15% of customers who notice a stock out will wait for the item to return to the shelves, while the remaining 85% will either buy a different product (45%), visit another store (31%), or do not buy anything at all (9%). Similar figures are obtained by Sloot et.al, who conclude that 23% of customers will postpone their purchase in the event of excess demand. These findings indicate that the majority of the original demand is lost in many practical scenarios. However, most inventory models in the literature assume that excess demand is backordered, that is, customers wait for a new delivery to arrive. As the preceding studies demonstrate, such models are not typical of many real-world applications in a retail environment. Backorder models are more typically used in industrial settings.

Even though research into lost-sales inventory systems began about 1960, there are few applications that take into account lost sales. One of the main reasons is that inventory models with lost sales are more complex to analyze and compute than backorder models. In particular, the inventory level (inventory on hand minus backorders) cannot be negative in lost-sales models. Thus, lost-sales systems necessitate a distinct research strategy. The quantity of research papers and case studies on this topic has increased in recent years. Based on these observations and the trends investigated in practice, we conclude that there is an increasing interest and demand for study into lost-sales inventory models. Currently, none of the published overview works on inventory theory address lost-sales scenarios ([5], [6],[7], [8]). The purpose of this study is to provide a broad classification approach for the lost-sales inventory systems investigated in the literature.

The inventory model with lost sales assumes that when customer demand exceeds the available inventory, the unmet demand is lost rather than backordered. In practical terms, this means that customers who cannot immediately fulfill their needs from the business will turn to competitors or forego the purchase altogether. The key assumptions of the inventory model with lost sales include several important factors. It assumes finite inventory capacity, with limited storage space or specific replenishment constraints. Lost demand is another assumption, where customers who encounter stockouts abandon their purchases rather than wait for restocking. The model also accounts for demand uncertainty, recognizing that demand can fluctuate and is often represented using probabilistic distributions. Additionally, restocking occurs at regular or stochastic intervals, reflecting the variability in supply chain operations. These assumptions contrast with backordering models, where unmet demand is deferred rather than lost, making the lost sales model particularly relevant in highly competitive markets or perishable goods industries.

The inventory model with lost sales typically uses probabilistic demand and cost analysis to determine optimal inventory policies. Key parameters and costs considered in this model include holding costs (H), which represent the expense of storing unsold inventory; ordering costs (O), encompassing fixed or variable costs associated with replenishment; and lost sales costs (L), which account for revenue losses or penalties resulting from unmet demand. A common goal is to minimize the total cost, which is the sum of holding, ordering, and lost sales costs. Decision variables include the reorder point (R) and order quantity (Q). The optimization process balances the risk of stocking out against the costs of excess inventory.

II. Literature Review

The inventory dynamics can be modeled using stochastic demand functions (incorporating randomness to simulate real-world uncertainty) and service level constraints (ensuring that a certain percentage of demand is met). Inventory models with a backorder assumption have attracted the most interest in inventory research. This is mainly because order-up-to policies have been shown to be best for backorder models in periodic reviews by Karlin et.al [9]. This optimality conclusion has been extensively researched, resulting in various revisions and extensions ([10], [11], [12]). Various exact algorithms and approximation strategies have been devised to determine optimal or near-optimal values for the reorder and order-up-to levels ([13]).

Lost-sales inventory models are more difficult to analyze than backorder models since inventory cannot be negative. This necessitates a new research methodology. However, interest in lost-sales inventory models has increased in recent years. Lost sales happen when a customer is unable to purchase an item because it is not in stock. This could be the result of faulty demand predictions, supplier delays, or unanticipated demand spikes. Lost sales can result in missed opportunities, client unhappiness, and income loss.

In [14], Ben-Daya and Raouf proposed a continuous review inventory model in which lead time and order amount were used as decision factors while shortages were ignored. Later, Liang-Yuh et.al.[15] assumed that shortages are permitted, and expanded the Ben-Daya and Raouf model by including the stockout cost. Additionally, the effects of parameters were also studied.

The study of Bijvank et. al [16], was first of its kind to give a literature review on lost sales inventory models. They presented a general classification scheme for the lost-sales inventory systems that have been studied in the literature. They also compared information on lost-sales inventory models with backorder models. A detailed discussion on why optimal policies are difficult to find for lost-sales models, and how this can be dealt with, based on observations derived from the literature, has also been presented. Their survey includes continuous review lost sales models and periodic review lost sales models. The gap between theory and practice is also pointed out.

Bijvank and Johansen [17] modelled a period-review inventory system with lost sales with no restrictions on the lead time or the number of outstanding orders. Optimal policies and base-stock policies were studied. It was shown that restricted base-stock policies perform close to optimal. A new heuristic procedure finds base-stock levels with an optimality gap of 1%.

Saffari et. al. [18], considered an M/M/1 queueing system with inventory under the (r, Q) policy and with lost sales and general lead time. The stationary distributions of the joint queue length and on-hand inventory when lead times are random variables are derived and from stationary distributions long-run average performance measures and cost functions are formulated.

Woensel et. al. [19] joined inventory and handling into a single model for optimal decisions of inventory replenishment for a grocery retail store. The replenishment cost includes both fixed and variable components, dependent on the number of batches and units in the order including the shelf-stacking costs in retail stores. They used Markov Decision Processes to find the optimal policy. They assumed the period demand to be a known, nonnegative discrete random variable and the demand process is i.i.d. stationary. The lead time is fixed and shorter than the review period length. At the beginning of each review period, the inventory on-hand is observed and an order at is placed, expected to arrive after a fixed time units .The order is multiple of the fixed batch size .The stochastic demand is satisfied with current inventory ,if possible, and unsatisfied demand is lost. Then, the order placed at the beginning of the period arrives and afterwards stochastic demand continues to occur, up until the beginning of the next ordering moment. All demand occurring in this period not directly satisfied is again assumed to be lost. Results show that it is worthwhile to explicitly take handling costs into account when making inventory decisions

In [20], Baek et.al discussed an (s, S) production–inventory system with an attached Markovian service queue and customers are lost during stock-out periods. The production process is assumed to be a Poisson process. The c servers process customers that arrive in the system according to the poisson process and the service times are considered to be independent and identically distributed exponential random variables.

The supply chain is a portion of optimal analysis in operations management programs. Jarrett, Jeffrey. [21] considered seasonal variation in lost sales and analyzed it using time series in optimal planning programs for the supply chain. Priyan and Udayakumar [22] modelled a continuous review inventory system with controllable lead time and order processing cost, backorder price discount, with the assumption that the received quantity is uncertain. They studied a continuous review inventory model in which the buyer offers backorder price discount to his customers with outstanding orders during the shortage period to secure customer orders. The expected annual total cost per unit time is minimized by simultaneously optimizing the order quantity, order processing cost, backorder price discount, and lead time. An iterative procedure is developed to find the optimal solution. Numerical study shows that cost can be saved through backorder price discount.

Baek and Moon [23] discussed an M/M/1 queue with a production-inventory system and lost sales. They assumed that arrival is according to Poisson process and single server serves the customers in an exponentially distributed service manner. The stocks are replenished by either an external order under (r, Q)-policy, or an internal production. This is extension of the queueing system with inventory in which the stocks are delivered both by an outside supplier and an internal production. In [24], Baek et al. analyzed M/M/1 queuing model with an attached continuous-type inventory and lost sales. Arrival is according to a

poisson process and service time is exponentially distributed. Along with the queue, there is an internal finite storage for the inventory and each service requires an exponentially distributed random amount H of inventory from the storage. An outside supplier replenishes the inventory with a random lead time under an (s,Q) inventory control policy. The stationary joint probability distribution of queue length and inventory level in explicit product form for this queuing-inventory system is derived.

Bu et.al [25] presented an infinite-horizon lost-sales inventory model where the supply takes positive lead times and is a random function of the order quantity They studied a simple class of constant-order policies that place the same order in every period regardless of the system state. It is shown that this is asymptotically optimal with large lead times, and the optimality gap converges to zero exponentially fast in the lead time. Also, they constructed a simple heuristic COP and showed that it performs very close to the best COP. A numerical study is also done. In [26], Yue et. al. considered an M/M/1 queuing system with attached inventory under an (s, S) control policy. The server takes multiple vacations whenever the inventory is depleted. Stability condition is derived and stationary distribution in product form is obtained. Also, the conditional distributions of the inventory level when the server is on and operational, and when it is off due to a vacation, are derived.

III. Applications

The lost sales inventory model is particularly relevant in industries where unmet demand directly results in lost revenue. In the retail sector, especially in fast-moving consumer goods (FMCG), customers are unlikely to wait for restocking, making it crucial to maintain adequate stock levels. In the pharmaceutical industry, lost sales can have serious implications, not only for profitability but also for public health. Similarly, in the food and beverage industry, where products often have limited shelf lives, stockouts can lead customers to switch brands or stores. By leveraging this model, companies can optimize their inventory strategies to maintain a competitive edge and ensure customer satisfaction.

Inventory models with lost sales have diverse applications across industries where stockouts directly impact revenue and customer loyalty. In the retail sector, particularly in fast-moving consumer goods (FMCG), maintaining optimal inventory levels is critical since customers are unlikely to wait for restocking and may switch to competitors, leading to lost revenue and diminished brand loyalty. In the pharmaceutical industry, the stakes are even higher, as stockouts can result in significant public health implications and financial penalties. For the food and beverage sector, where products often have limited shelf lives, ensuring sufficient inventory prevents stockouts that could drive customers to switch brands or stores, negatively affecting market share. E-commerce platforms also rely on these models to maintain seamless operations, as online shoppers expect instant availability, with any delay potentially resulting in canceled orders or damaged brand reputation. Additionally, the model finds applications in seasonal industries like fashion, where missed opportunities to meet demand during peak periods can result in unsold inventory or lost competitive advantage. By applying inventory models with lost sales, businesses can better balance costs, improve customer satisfaction, and maintain a competitive edge in dynamic markets.

IV. Challenges And Limitations

Despite its utility, the inventory model with lost sales faces several challenges. Accurately predicting demand can be complex, especially in volatile markets where customer needs fluctuate unpredictably. Quantifying the costs of lost sales is often subjective, as it requires a nuanced understanding of customer behavior, including brand loyalty and purchasing patterns. Additionally, optimizing inventory levels under stochastic demand conditions can be computationally intensive, demanding advanced analytical tools. Furthermore, the model operates on the assumption that customers permanently abandon their purchase when stockouts occur, which may not always align with reality, as some customers might revisit the store later or opt for available substitutes.

Businesses employing the lost sales model can adopt various strategies to minimize the impact of stockouts. Demand sensing, which involves utilizing advanced analytics and AI, can significantly improve demand forecasting accuracy, helping businesses anticipate and prepare for fluctuations. Maintaining a safety stock acts as a buffer inventory to address unexpected surges in demand, reducing the likelihood of stockouts.

Additionally, implementing flexible replenishment strategies, such as adopting agile supply chains that enable more frequent or smaller restocking, ensures that inventory levels remain responsive to changing customer needs.

V. Scope For Further Research

In a lost-sales situation, order sizes do not have to increase the inventory position as much as in a backorder setting when the inventory position is low. Numerical investigations show that such policies with delays result in near-optimal costs for periodic review models. There is no comparison for continuous review systems. This is an intriguing topic to study in the future. Another intriguing area for future investigation is the impact of the aforementioned policy delays on the entire supply chain. When the ordering process is smoothed across time, the variation in demand for manufacturing and transportation decreases. There is little research on the impact of lost sales in a multi-echelon setting.

Modelling customer behaviour in the event of excess demand is another intriguing new component in inventory theory. Aside from backlogging, lost sales, and combinations of the two, a client can purchase a substitute goods or visit another store. Incorporating various customer reactions to stock outs would also impact replenishment policies. Yet another proposal for future research is to focus more on non-stationary circumstances. Even though steady-state circumstances are more analytically tractable, they lose relevance in real scenarios due to non-stationary demand and shorter product lifespans. As a result, the methodology for analyzing lost-sales inventory systems should evolve accordingly.

VI. Conclusion

The inventory model with lost sales is a vital tool for understanding and mitigating the impact of stockouts in supply chains. By balancing holding, ordering, and lost sales costs, businesses can craft strategies to maintain profitability and customer satisfaction. However, its effectiveness relies heavily on accurate demand forecasting, robust cost estimation, and adaptable supply chain practices. As markets grow increasingly dynamic, the role of inventory optimization models like the lost sales framework becomes ever more critical in sustaining competitive advantage. Based on this literature review on lost-sales inventory systems, we find that little is known about the ideal replenishment policy when excess demand is lost. The characteristics and numerical results obtained for optimal order amounts demonstrate that there is no framework for an easy-to-understand optimal replenishment policy that can be used in real-world applications. However, the most effective approximation policies presented in the literature involve some sort of delay in the ordered quantities. This is explicit in continuous review procedures, but implicit in periodic review systems with a maximum order size.

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