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Game Theoretical Perspectives On Dual-Channel Closed Loop Supply Chain With A Reward-Driven Remanufacturing Policy

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Abstract:
This paper evaluates the impact of reward driven remanufacturing policy and pricing schemes on the dual-channel supply chain competition. In the proposed model, a manufacturer sells products both through a retailer and directly to end users in the forward supply chain. In the reverse supply chain, the manufacturer collects used products to remanufacture them into new ones. Mathematical models for the supplier Stackelberg, retailer Stackelberg, Nash game, and centralized scenarios are developed to define pricing decisions and remanufacturing strategies. The optimality of all proposed models is analyzed theoretically.
Keywords: *Closed loop dual channel supply chain; Remanufacturing; Reward-driven policy; Stackelberg game; Nash Model*

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I. Introduction

In recent years, sustainable supply chain management, a critical aspect of operations management, has become achievable primarily through a well-structured supply chain network. As environmental regulations and concerns have intensified, many businesses are now driven to adopt green supply chains, achieved by integrating environmental and economic considerations into their traditional supply chain models. Traditional network designs either incorporate environmental considerations as constraints or objectives, or include additional collection and recycling centers to meet environmental goals. While a conventional forward supply chain moves products from upstream manufacturers to consumers, a reverse supply chain facilitates the return of used products from customers back to upstream partners for remanufacturing.

Extensive research on Closed-Loop Supply Chains (CLSCs) is available in works by Ostlin et al. (2008), Choi et al (2013), Khalili et al. (2015), Asghari et al. (2022), Luo et al. (2022), and Goli (2023). However, many studies highlight the typical assumption that customers dispose of used products without expecting rewards, with collection rates modeled based on investment levels. However, industries such as automotive, electronics, and furniture demonstrate reward-driven return policies, encouraging higher return volumes. Examples include car manufacturers offering high residual values, electronics brands like Staples and Hewlett Packard rewarding returns, and furniture shops incentivizing old product returns. Motivated by these practices, this study examines the performance of dual-channel CLSCs under reward-driven remanufacturing policies.

In today's competitive business landscape, driven by the expansion of diverse channel options, many companies have adopted multichannel distribution systems. Firms often establish multiple marketing channels to reach customers, including a direct channel where products are sold straight to the end user. Physical stores allow customers to engage directly with products, fostering trust in ways that technology-based channels cannot replicate. However, direct sales through online channels help manufacturers reduce costs, boost revenue, and tap into new market segments. As Chen et al (2012) noted, many customers prefer online purchases to save on transportation costs and time. Consequently, the performance of dual-channel supply chains under various constraints has gained significant attention from academics and industry leaders (Hua (2010), Kundu and Chakrabarti (2018), Pathak et al. (2022)).

This paper introduces a closed-loop dual-channel supply chain model where the manufacturer incentivizes end customers to return used products by offering rewards. Reflecting real-world conditions, we assume that the return rate can be modelled as a monotonically increasing function of the reward amount. To optimize the profit functions of both the manufacturer and the retailer, we utilize various game-theoretic approaches. This study makes two significant contributions to the literature. First, it sheds light on the coordination dynamics among members of a CLSC in a dual-channel supply chain. Second, it uniquely explores the characteristics of CLSCs under reward-driven return policies, distinguishing itself from prior research.

II. Model Development:

This paper examines a closed-loop dual-channel model in which a manufacturer produces a new product by manufacturing from raw materials at a cost of c_m per unit and remanufacturing used products at a cost of c_r per unit, where $c_r < c_m$. The manufacturer supplies the product to a retailer at a wholesale price (w) per unit, and the retailer sells the product to customers at a retail price of (p_r)per unit. Additionally, the manufacturer maintains a direct channel alongside the retail channel, selling the product directly to customers at a price of (p_m) per unit.

We assume a linear price dependent demand structure of direct channel and traditional retail channel as follows:

$$D_m = \rho a - \beta p_m + \delta p_r \quad (1)$$

$$D_r = (1-\rho)a - \beta p_r + \delta p_m \quad (2)$$

Here, $a\ (> 0)$ represents the total market potential, while $\rho\ (0 \le \rho \le 1)$ denotes the market share of the direct channel. Consequently, the demand in the direct and retail channels are ρa and $(1-\rho)a$, respectively. The parameters β and δ represent the price sensitivity and cross-price sensitivity of the channels. Since the demand for a channel is more strongly influenced by its own selling price than by the price of the other channel, we assume $\beta > \delta$.

The manufacturer collects used products directly from customers through a reward-driven return policy. The return rate of used products is assumed to be a function of the reward amount (r) provided to the customer for returning the products, denoted as $f(r)$, where $0 < f(r) < 1$. In this paper, consistent with real-world scenarios, we assume that the return rate behavior is approximated by a monotonically increasing function of r, expressed as

$f(r) = (1-\gamma^r)$ where $0 < \gamma < 1$

Hence, the profit functions of the manufacturer (π_m) and the retailer (π_r) are as follows

$$\pi_m = wD_r + p_m D_m - c_m(D_m + D_r) + (c_m - c_r - r)f(r)(D_m + D_r) \quad (3)$$

$$\pi_r = (p_r - w)D_r \quad (4)$$

Next, We optimize the above model under manufacturer Stackelberg (MS), retailer Stackelberg (RS), Nash model and centralized, structures.

***Manufacturer Stackelberg Mode (MS Model)l*:**

In a Manufacturer Stackelberg game, the manufacturer optimizes the direct channel price and wholesale price, taking into account the retailer's best response in setting the retail price.

Proposition1. In the, Manufacturer Stackelberg model, the optimal pricing strategy is given by

$$w^{MS^*} = \frac{[(1-\rho)\beta + \rho\delta]a}{2(\beta^2-\delta^2)} + \frac{1}{2}c_m - \frac{1}{2}(c_m - c_r - r)f(r) \quad (5)$$

$$p_m^{MS^*} = \frac{[\rho\beta + (1-\rho)\delta]a}{2(\beta^2-\delta^2)} + \frac{1}{2}c_m - \frac{1}{2}(c_m - c_r - r)f(r) \quad (6)$$

$$p_r^{MS^*} = \frac{[(3\beta^2-\delta^2)(1-\rho) + 2\rho\beta\delta]a}{4\beta(\beta^2-\delta^2)} + \frac{\beta+\delta}{4\beta}c_m - \frac{\beta+\delta}{4\beta}(c_m - c_r - r)f(r) \quad (7)$$

From Proposition 1, we observed that the retailer would buy the product from the manufacturer at wholesale price as long as $w^{MS^*} < p_m^{MS^*}$i.e. if $\rho > \frac{1}{2}$ otherwise, dual channel supply scenario does not exist.

Corollary 1:

(1) $\frac{\partial w^{MS^*}}{\partial c_m} = \frac{\partial p_m^{MS^*}}{\partial c_m} > \frac{\partial p_r^{MS^*}}{\partial c_m} > 0$

(2) $\frac{\partial w^{MS^*}}{\partial c_r} = \frac{\partial p_m^{MS^*}}{\partial c_r} > \frac{\partial p_r^{MS^*}}{\partial c_r} > 0$

Corollary 1 indicates that the manufacturer and the retailer will increase their whole sale price, direct selling price and retail price are increase as the cost associated with the manufacturing and remanufacturing increases. The costs c_m and c_r have greatest impact on whole sale price and direct selling price followed by retail price.

Next, to investigate the effect of reward amount on manufacturer's profit function, we obtain $\frac{\partial \pi_m(w^{MS^*}, p_m^{MS^*})}{\partial r} = \frac{1}{4\beta}[(c_m - c_r - r)\gamma^r \ln\gamma + 1 - \gamma^r][(3\beta^2 - \delta^2 - 2\beta\delta)[\gamma^r c_m + (1-\gamma^r)(c_r + r)] - [(1+\rho)\beta + (1-\rho)\delta]a]$. Since the first order partial derivate is involved in r, we cannot made any conclusion and moreover, the equation $\frac{\partial \pi_m(w^{MS^*}, p_m^{MS^*})}{\partial r} = 0$ does not yield explicit solution in r. So, we find r numerically, which optimizes $\pi_m(w^{MS^*}, p_m^{MS^*})$.

***Retailer Stackelberg Model (RS Model)*:**

In a Retailer Stackelberg game, the retailer optimizes the retail price, taking into account the manufacturer's best response in setting the wholesale price and the direct channel price. We set the retailer sales margin as $s_r = p_r - w$.

Proposition 2. In the, Retailer Stackelberg model, the optimal pricing strategy is given by

$$w^{RS^*} = \frac{(1-\rho)(\beta^2+\delta^2)+2\rho\beta\delta a}{4\beta(\beta^2-\delta^2)} + \frac{3\beta-\delta}{4\beta}c_m - \frac{3\beta-\delta}{4\beta}(c_m - c_r - r)f(r) \quad (8)$$
$$p_m^{RS^*} = p_m^{MS^*} = \frac{[\rho\beta+(1-\rho)\delta]a}{2(\beta^2-\delta^2)} + \frac{1}{2}c_m - \frac{1}{2}(c_m - c_r - r)f(r) \quad (9)$$
$$p_r^{RS^*} = p_r^{MS^*} = \frac{[(3\beta^2-\delta^2)(1-\rho)+2\rho\beta\delta]a}{4\beta(\beta^2-\delta^2)} + \frac{\beta+\delta}{4\beta}c_m - \frac{\beta+\delta}{4\beta}(c_m - c_r - r)f(r) \quad (10)$$

From Proposition 2, we observed that the retailer would buy the product from the manufacturer at wholesale price as long as $w^{MS^*} < p_m^{MS^*}$ i.e.
if $\rho > \frac{(\beta^2-\delta^2)[c_m(1-f(r))+(c_r+r)f(r)]+(\beta-\delta)a}{(3\beta-\delta)a}$ otherwise, dual channel supply chain scenario does not exist.

Corollary 2:
(1) $\frac{\partial w^{RS^*}}{\partial c_m} > \frac{\partial p_m^{RS^*}}{\partial c_m} > \frac{\partial p_r^{RS^*}}{\partial c_m} > 0$
(2) $\frac{\partial w^{RS^*}}{\partial c_r} > \frac{\partial p_m^{RS^*}}{\partial c_r} > \frac{\partial p_r^{RS^*}}{\partial c_r} > 0$

Corollary 2 indicates that when retailer acts as Stackleberg leader, the whole sale price, direct selling price and retail price are positively correlated with the cost associated with the manufacturing and remanufacturing increases. The costs c_m and c_r have greatest impact on the whole sale price, followed by direct selling price, whereas it has the least impact on the retail price.

Next, to investigate the effect of reward amount on manufacturer's profit function, we obtain $\frac{\partial \pi_m(w^{RS^*}, p_m^{RS^*})}{\partial r} = \frac{1}{4\beta}[(c_m - c_r - r)\gamma^r \ln\gamma + 1 - \gamma^r][(5\beta^2 - 3\delta^2 - 2\beta\delta)[\gamma^r c_m + (1-\gamma^r)(c_r + r)] - [(1+3\rho)\beta + 3(1-\rho)\delta]a]$. Since the first order partial derivate is involved in r, we cannot made any conclusion and moreover, the equation $\frac{\partial \pi_m(w^{RS^*}, p_m^{RS^*})}{\partial r} = 0$ does not yield explicit solution in r. So, we find r numerically, which optimizes $\pi_m(w^{RS^*}, p_m^{RS^*})$.

Nash Model

In this situation, the manufacturer and the retailer are equally powerful and optimize their own profits non-cooperatively and simultaneously. We set the retailer sales margin as $s_r = p_r - w$.

Proposition 3. In the, Nash model, the optimal pricing strategy is given by

$$w^{NS^*} = \frac{(1-\rho)(\beta^2+\delta^2)+3\rho\beta\delta a}{6\beta(\beta^2-\delta^2)} + \frac{4\beta-\delta}{6\beta}c_m - \frac{4\beta-\delta}{6\beta}(c_m - c_r - r)f(r) \quad (11)$$
$$p_m^{NS^*} = p_m^{MS^*} = \frac{[\rho\beta+(1-\rho)\delta]a}{2(\beta^2-\delta^2)} + \frac{1}{2}c_m - \frac{1}{2}(c_m - c_r - r)f(r) \quad (12)$$
$$p_r^{NS^*} = \frac{[(4\beta^2-\delta^2)(1-\rho)+3\rho\beta\delta]a}{6\beta(\beta^2-\delta^2)} + \frac{2\beta+\delta}{6\beta}c_m - \frac{2\beta+\delta}{6\beta}(c_m - c_r - r)f(r) \quad (13)$$

From Proposition 3, we observed that the retailer would buy the product from the manufacturer at wholesale price as long as $w^{NS^*} < p_m^{NS^*}$ i.e.

if $\rho > \frac{(2\beta-\delta)a+(\beta^2-\delta^2)[c_m(1-f(r))+(c_r+r)f(r)]}{(5\beta-\delta)a}$ otherwise, dual channel supply chain scenario does not exist.

Corollary 3:
(1) $\frac{\partial w^{NS^*}}{\partial c_m} > \frac{\partial p_m^{NS^*}}{\partial c_m} > \frac{\partial p_r^{NS^*}}{\partial c_m} > 0$
(2) $\frac{\partial w^{NS^*}}{\partial c_r} > \frac{\partial p_m^{NS^*}}{\partial c_r} > \frac{\partial p_r^{NS^*}}{\partial c_r} > 0$

Corollary 3 is similar to Corollary 2.

Next, to investigate the effect of reward amount on manufacturer's profit function, we obtain $\frac{\partial \pi_m(w^{NS^*}, p_m^{NS^*})}{\partial r} = \frac{1}{18\beta}[(c_m - c_r - r)\gamma^r \ln\gamma + 1 - \gamma^r][(13\beta^2 - 5\delta^2 - 8\beta\delta)[\gamma^r c_m + (1-\gamma^r)(c_r + r)] - [(4+5\rho)\beta + 5(1-\rho)\delta]a]$. Since the first order partial derivate is involved in r, we cannot made any conclusion and moreover, the equation $\frac{\partial \pi_m(w^{NS^*}, p_m^{NS^*})}{\partial r} = 0$ does not yield explicit solution in r. So, we find r numerically, which optimizes $\pi_m(w^{NS^*}, p_m^{NS^*})$.

Centralized Model:

In the centralized decision making, both the manufacturer and the retailer cooperate to achieve the maximum total profit of the dual channel supply chain. The profit function of the system under this scenario is

$$\pi^C = \pi_m + \pi_r = p_r D_r + p_m D_m - c_m(D_m + D_r) + (c_m - c_r - r)f(r)(D_m + D_r) \quad (14)$$

Proposition 4. In the, Centralized model, the optimal pricing strategy is given by

$$p_m^{C^*} = p_m^{MS^*} = \frac{[\rho\beta + (1-\rho)\delta]a}{2(\beta^2-\delta^2)} + \frac{1}{2}c_m - \frac{1}{2}(c_m - c_r - r)f(r) \quad (15)$$
$$p_r^{c^*} = \frac{[(1-\rho)\beta + \rho\delta]a}{2(\beta^2-\delta^2)} + \frac{1}{2}c_m - \frac{1}{2}(c_m - c_r - r)f(r) \quad (16)$$

Next, to investigate the effect of reward amount on manufacturer's profit function, we obtain $\frac{\partial \pi_c(p_m^{c^*}, p_r^{c^*})}{\partial r} = \frac{1}{2}[(c_m - c_r - r)\gamma^r \ln\gamma + 1 - \gamma^r][2(\beta-\delta)[\gamma^r c_m + (1-\gamma^r)(c_r + r)] - a]$. Since the first order partial derivate is involved in r, we cannot made any conclusion and moreover, the equation $\frac{\partial \pi_c(p_m^{c^*}, p_r^{c^*})}{\partial r} = 0$ does not yield explicit solution in r. So, we find r numerically, which optimizes $\pi_c(p_m^{c^*}, p_r^{c^*})$.

III. Numerical Analysis:

In this section, we compare the optimal profit of the channel members under different game theoretic perspectives through numerical analysis to illustrate the above theoretical results and obtain more clarity. We assume the values of the parameters in appropriate units are as follows: $a = 200$, $\beta = 0.7$, $\delta = 0.5$, $\rho = 0.7$, $c_m = 52$, $c_r = 10$, $\gamma = 0.5$.

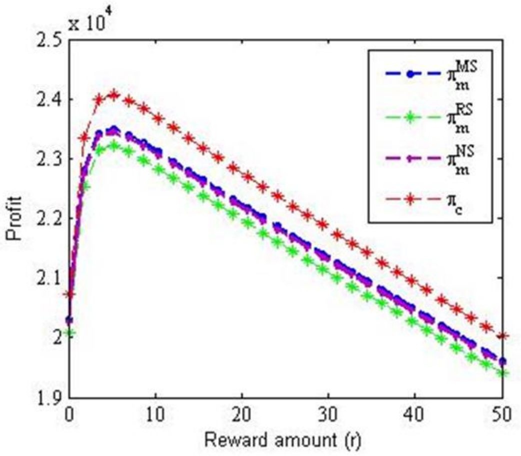

Figure 1: Impact of r on $\pi_m^{MS}, \pi_m^{RS}, \pi_m^{NS}$ and π_C

From the figure 1, we observe that the manufacturer's profit function in Manufacturer Stackelberg, Retailer Stackelberg and Nash model and total profit in Centralized model is stricty concave in r and Optimal solutions under different modes are shown in Table 1.

Table 1: Optimal solutions under different model

Model	w^*	p_r^*	p_m^*	r^*	π_r^*	π_m^*	Total Profit
MS	241.40	261.67	274.73	4.74	287.78	23503.70	
RS	221.12	261.67	274.73	4.74	575.57	23215.91	
Nash	227.88	254.91	274.73	4.74	511.61	23439.74	
Centralized		241.40	274.73	4.74			24079.26

IV. Conclusion:

In this paper proposed a close loop dual channel supply chain model, where manufacturer collect the used products from end customer by offering a reward. We employed different game theoretic approaches to optimize the manufacturer's and the retailer's profit functions. This study offers two primary contributions to the literature. First, it presents observations on the coordination among CLSC members within a dual-channel supply chain. Second, unlike previous studies, it examines the characteristics of CLSCs under reward-driven return policies.

Appendix

Proof of Proposition 1:

From Eq (4), we obtain the value of p_r^* by Solving $\frac{\partial \pi_r}{\partial p_r} = 0$ and the best response is given by $p_r^*(w, p_m) = \frac{(1-\rho)a + \beta w + \delta p_m}{2\beta}$ (A.1)

We also find that $\pi_r(p_r)$ is strictly concave in p_r, as $\frac{\partial^2 \pi_r}{\partial p_r^2} = -2\beta < 0$.

Substituting the value of p_r^* into Eq (3), we obtain the values of w^{MS^*} and $p_m^{MS^*}$ solving the first order conditions $\frac{\partial \pi_m}{\partial w} = 0$ and $\frac{\partial \pi_m}{\partial p_m} = 0$. Finally, putting the values of w^{MS^*} and $p_m^{MS^*}$ in Eq (A.1), we obtain $p_r^{RS^*}$

Now, $\frac{\partial^2 \pi_m}{\partial w^2} = -\beta < 0$ and the Hessian matrix of the profit function π_m is

$$H_1(w, p_m) = \begin{pmatrix} \frac{\partial^2 \pi_m}{\partial w^2} & \frac{\partial^2 \pi_m}{\partial w \partial p_m} \\ \frac{\partial^2 \pi_m}{\partial p_m \partial w} & \frac{\partial^2 \pi_m}{\partial p_m^2} \end{pmatrix} = \begin{pmatrix} -\beta & \delta \\ \delta & -\frac{2\beta^2 - \delta^2}{\beta} \end{pmatrix}$$

$|H_1| = 2(\beta^2 - \delta^2) > 0$. Hence the manufacturer's profit function is concave in w and p_m

Proof of Proposition 2:

Setting $p_r = s_r + w$ in Eq (3) and Eq (4) and from Eq (3) we obtain the value of w^* and p_m^* by solving $\frac{\partial \pi_m}{\partial w} = 0$ and $\frac{\partial \pi_m}{\partial p_m} = 0$ and the best responses are given by

$$w^*(s_r) = \frac{[(1-\rho)\beta + \rho\delta]a}{2(\beta^2 - \delta^2)} + \frac{1}{2}(c_m - s_r) - \frac{1}{2}(c_m - c_r - r)f(r) \quad \text{(A.2)}$$

$$p_m^*(s_r) = \frac{[\rho\beta + (1-\rho)\delta]a}{2(\beta^2 - \delta^2)} + \frac{1}{2}c_m - \frac{1}{2}(c_m - c_r - r)f(r) \quad \text{(A.3)}$$

We also find that, $\frac{\partial^2 \pi_m}{\partial w^2} = -2\beta < 0$ and the Hessian matrix of the profit function π_m is

$$H_2(w, p_m) = \begin{pmatrix} \frac{\partial^2 \pi_m}{\partial w^2} & \frac{\partial^2 \pi_m}{\partial w \partial p_m} \\ \frac{\partial^2 \pi_m}{\partial p_m \partial w} & \frac{\partial^2 \pi_m}{\partial p_m^2} \end{pmatrix} = \begin{pmatrix} -2\beta & 2\delta \\ 2\delta & -2\beta \end{pmatrix}$$

$|H_2| = 4(\beta^2 - \delta^2) > 0$. Hence the manufacturer's profit function is concave in w and p_m

Substituting the values of of w^* and p_m^* into Eq (4), we obtain the values of ${s_r}^{RS^*}$ solving the first order conditions $\frac{\partial \pi_r}{\partial s_r} = 0$.

$${s_r}^{RS^*} = \frac{(1-\rho)a - (\beta - \delta)\left[\left(1 - f(r)\right)c_m + (c_r + r)f(r)\right]}{2\beta} \quad \text{(A.4)}$$

Finally, putting the value of ${s_r}^{RS^*}$ into Eq (A.2) and Eq (A.3), we obtain values of w^{RS^*} and $p_m^{RS^*}$. The $p_r^{RS^*}$ is given by $p_r^{RS^*} = {s_r}^{RS^*} + w^{RS^*}$.

Now, $\pi_r(s_r)$ is strictly concave in s_r, as $\frac{\partial^2 \pi_r}{\partial s_r^2} = -\beta < 0$.

Proof of Proposition 3:

Setting $p_r = s_r + w$ in Eq (3) and Eq (4), we obtain the value of $w^{NS^*}, p_m^{NS^*}$ and $s_r^{NS^*}$ by solving $\frac{\partial \pi_m}{\partial w} = 0$, $\frac{\partial \pi_m}{\partial p_m} = 0$ and $\frac{\partial \pi_r}{\partial s_r} = 0$.

Finally, the value of $p_r^{NS^*}$ is given by $p_r^{NS^*} = {s_r}^{NS^*} + w^{NS^*}$.

Proof of Proposition 4:

From Eq (14), we obtain the values of $p_m^{C^*}$ and $p_r^{C^*}$ solving the first order conditions $\frac{\partial \pi^C}{\partial p_m} = 0$ and $\frac{\partial \pi^C}{\partial p_r} = 0$.

Now, $\frac{\partial^2 \pi^C}{\partial {p_m}^2} = -2\beta < 0$ and the Hessian matrix of the profit function π_m is

$$H_3(p_m, p_r) = \begin{pmatrix} \frac{\partial^2 \pi^C}{\partial {p_m}^2} & \frac{\partial^2 \pi^C}{\partial p_m \partial p_r} \\ \frac{\partial^2 \pi^C}{\partial p_r \partial p_m} & \frac{\partial^2 \pi^C}{\partial {p_r}^2} \end{pmatrix} = \begin{pmatrix} -2\beta & 2\delta \\ 2\delta & -2\beta \end{pmatrix}$$

$|H_3| = 4(\beta^2 - \delta^2) > 0$. Hence the manufacturer's profit function is concave in p_m, p_r

Proof of Corollary 1:

From Proposition 1, we have

$\frac{\partial p_r^{MS^*}}{\partial c_m} = \frac{(\beta+\delta)}{4\beta} \left(1 - f(r)\right)$; $\frac{\partial p_r^{MS^*}}{\partial c_r} = \frac{(\beta+\delta)}{4\beta} f(r)$;

$\frac{\partial w^{MS^*}}{\partial c_m} = \frac{\partial p_m^{MS^*}}{\partial c_m} = \frac{1}{2}\left(1 - f(r)\right)$ and $\frac{\partial w^{MS^*}}{\partial c_r} = \frac{\partial p_m^{MS^*}}{\partial c_r} = \frac{1}{2} f(r)$

Now, $\frac{\partial w^{MS^*}}{\partial c_m} - \frac{\partial p_r^{MS^*}}{\partial c_m} = \frac{3\beta - \delta}{4\beta}\left(1 - f(r)\right) > 0$ and $\frac{\partial w^{MS^*}}{\partial c_r} - \frac{\partial p_r^{MS^*}}{\partial c_r} = \frac{\beta - \delta}{4\beta} f(r) > 0$

Proof of Corollary 2:

From Proposition 2, we have

$\frac{\partial p_r^{RS^*}}{\partial c_m} = \frac{(\beta+\delta)}{4\beta} \left(1 - f(r)\right)$;$\frac{\partial p_r^{RS^*}}{\partial c_r} = \frac{(\beta+\delta)}{4\beta} f(r)$;$\frac{\partial w^{RS^*}}{\partial c_m} = \frac{3\beta - \delta}{4\beta} \left(1 - f(r)\right)$;$\frac{\partial w^{RS^*}}{\partial c_r} = \frac{3\beta - \delta}{4\beta} f(r)$

$\frac{\partial p_m^{RS^*}}{\partial c_m} = \left(1 - f(r)\right)$ and $\frac{\partial p_m^{RS^*}}{\partial c_r} = \frac{1}{2} f(r)$

Now, $\frac{\partial w^{RS^*}}{\partial c_m} - \frac{\partial p_m^{RS^*}}{\partial c_m} = \frac{\beta - \delta}{4\beta}\left(1 - f(r)\right) > 0$; $\frac{\partial w^{RS^*}}{\partial c_r} - \frac{\partial p_m^{RS^*}}{\partial c_r} = \frac{\beta - \delta}{4\beta} f(r) > 0$

$\frac{\partial p_m^{RS^*}}{\partial c_m} - \frac{\partial p_r^{RS^*}}{\partial c_m} = \frac{3\beta - \delta}{4\beta}\left(1 - f(r)\right) > 0$ and $\frac{\partial p_m^{RS^*}}{\partial c_r} - \frac{\partial p_r^{RS^*}}{\partial c_r} = \frac{\beta - \delta}{4\beta} f(r) > 0$

Proof of Corollary 4

The proof is similar to Corollary 3.

References

[1] Hua, G., S. Wang, And T. C. E. Cheng. 2010. "Price And Lead Time Decisions In Dual-Channel Supply Chains." European Journal Of Operational Research 205 (1): 113–126

[2] Pathak, U., Kant, R., & Shankar, R. (2022). Modelling Closed-Loop Dual-Channel Supply Chain: A Game-Theoretic Approach To Maximize The Profit. Cleaner Logistics And Supply Chain, 4, 100064.

[3] Kundu, S., & Chakrabarti, T. (2018). Joint Optimal Decisions On Pricing And Local Advertising Policy Of A Socially Responsible Dual-Channel Supply Chain. American Journal Of Mathematical And Management Sciences, 37(2), 117-143.

[4] Chen, J., H. Zhang, Andy. Sun. 2012. "Implementing Coordination Contracts In A Manufacturer Stackelberg Dual-Channel Supply Chain." Omega 40 (5): 571–583.

[5] Ostlin, J., Sundin, E., & Björkman, M. (2008). Importance Of Closed-Loop Supply Chain Relationships For Product Remanufacturing. International Journal Of Production Economics, 115(2), 336-348.

[6] Choi, T., Y. Li, And L. Xu. (2013). "Channel Leadership, Performance And Coordination Inclosed Loop Supply Chains." International Journal Of Production Economics 146: 371–380.

[7] Khalili, K., M. Tavana, And M. Najmodin. 2015. "Reverse Logistics And Supply Chains: A Structural Equation Modeling Investigation." International Journal Of Industrial Engineering: Theory, Applications And Practice 22 (3): 354–368.

[8] Asghari, M., Afshari, H., Mirzapour Al-E-Hashem, S. M. J., Fathollahi-Fard, A. M., & Dulebenets, M. A. (2022). Pricing And Advertising Decisions In A Direct-Sales Closed-Loop Supply Chain. Computers & Industrial Engineering, 171, 108439.

[9] Luo, R., Zhou, L., Song, Y., & Fan, T. (2022). Evaluating The Impact Of Carbon Tax Policy On Manufacturing And Remanufacturing Decisions In A Closed-Loop Supply Chain. International Journal Of Production Economics, 245, 108408.

[10] Goli, A. (2023). Integration Of Blockchain-Enabled Closed-Loop Supply Chain And Robust Product Portfolio Design. Computers & Industrial Engineering, 179, 109211.