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Application Of Fuzzy Goal Programming In Agriculture: A Chance Constrained Approach

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Abstract
This work considered a real life problem of agricultural Production. It applied the Fuzzy Goal Programming technique to solve a chance constrained crop production problem. The objectives are considered as bi-criteria crisp functions, while the coefficient of the constraint matrix are considered as triangular fuzzy numbers. The right-hand side parameter of the system constraint is a gumbel random variable and also a right sided fuzzy number. The chance constrained fuzzy programming problem is transformed by the general chance constrained methodology, and the membership function of the fuzzy parameters used to obtain an equivalent deterministic model. The deterministic fuzzy goal programming problem was solved and the bounds of the fuzzy goals determined and utilized to obtain the membership function of the kth objective. The fuzzy goal programming model was constructed by the weighted goal programming approach. The proposed approach was applied to analyze the activities of small holder farmers in Ebonyi State, Nigeria. Analysis using LINGO 14.0 software showed an under achievement of the mean cultivated area in the presence of an optimal crop yield, implying a satisficing solution to the decision makers' goal. It also indicates that the fuzzy goal programming technique can efficiently solve a chance constrained programming problem.

Keywords: *Fuzzy goal programming, triangular fuzzy numbers, chance constrained programming, gumbel random variables, membership function, bi-criteria optimization.*

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I. Introduction

The major role of agriculture is to provide adequate food for an increasing population. Agriculture therefore, is the sure way to unravel the puzzle of food security. The availability of food is the first dimension of food security. To figure out how much food that is available, it is expected that we calculate how much food that is produced. Improved food production increases the livelihood of rural dwellers, and indeed small holder farmers thereby reducing poverty. The work (Akinsuyi,2011) reports that small holder farmers are the backbone of Nigeria's agricultural sector. The Nigerian government through the value chain policy has presented small holder farmers with opportunities to maximize food production. Ebonyi State Nigeria ranks high among states with large number of small holder farmers. It is well known for it's bulk cultivation of rice, yam, and cassava among other staple foods. The small holder farmers encounter a lot of problems which include but not limited to environmental factors in the crop production process. The decision making process of crop production is not an easy task. Most times, the decision maker encounters conflicting, and non commensurable objectives which must be achieved simultaneously. Mathematical programming plays a vital role in explaining real world problems and finding solutions (Arbaiy,2012).

Chance constrained programming is a special field of mathematical programming that deals with a situation, where some or all of the parameters of a problem are described by probabilistic variables rather than deterministic quantities. It was first introduced by chance and cooper(1959). Chance constrained programming is a means of describing constraints in mathematical models according to their probability levels of attainment. Several researchers have done extensive work in the area of chance constrained programming. The work (Mohanty et al, 2019) presented multi objective chance constrained programming problems involving some continuous random parameters. They provided numerical illustrations to prove the efficiency of the solution approach. (Biswas and Modak,2012) solved a multi objective chance constrained programming problem involving exponential fuzzy random variables by the fuzzy goal programming technique. They solved an example to illustrate the proposed technique and compared solutions. (Mmahi et al,2023) developed a multi objective chance constrained fuzzy programming model involving gumbel random variables and solved by the fuzzy goal programming technique.

Results indicate that a chance constrained problem can be sufficiently modeled by the gumbel distribution and that the fuzzy goal programming technique is an efficient tool for satisficing a multi objective chance constrained problem. In reality, decision making processes are multiple objective in nature, a special case of two objective functions is termed ”Bi-Criteria Optimization” the goal programming technique converts the multiple objective problems into a single goal that will yield an efficient solution.

Goal programming as a multiple objective mathematical model proffers solution to conflicting objectives. The solution may not be optimal with respect to all the conflicting objectives but it provides a fair compromise solution by increasing the satisfaction in the system (Mmahi et al,2023). Several researchers have studied goal programming, the work (Orumie and Ebong,2014) presents survey of current methods for solving linear goal programming models. The goal programming model formulation assumes that the decision maker is able to determine the goals precisely whereas many functions reflect imprecise parameters. In principle, every element of a set contains some degree of imprecision. It is necessary that we consider the degree of membership of each element to a given set.

Fuzzy set theory which was first introduced by Zadeh(1965) deals with uncertainty due to imprecision and vagueness. (Malik et al,2023) demonstrated the applicability of fuzzy goal programming technique to model an apple cultivation planning problem and obtained optimum solution to the prob- lem. In this work, we considered the simultaneous occurrence of fuzziness and randomness in the agricultural production of rice, yam, and cassava in Ebonyi state, Nigeria while evaluating the effect of hydrological extreme event of rainfall on the production of the selected crop varieties. It is assumed that the right-hand side of the constraint equation is a random variable following the gumbel distribution while the coefficients of the constraint matrix are triangular fuzzy numbers.

II. Fuzzy Set

In this section, we recall some basic notations in fuzzy set theory,

Definition 2.1 (Fuzzy set) *If X is a collection of objects denoted generically by x, then a fuzzy set*
$\tilde{A}$ in X is defined to be a set of ordered pairs : membership function for the fuzzy set.
$\tilde{A} = \{(x, \mu_{\tilde{A}}(x) : x \in X\}$. *Here, $\mu_{\tilde{A}}(x)$ is called the*

The range of the membership function is [0, 1]. The value 0 is used to represent complete non mem- bership while 1 is used to represent complete membership. Values in between are used to represent degrees of membership.

Definition 2.2 (Fuzzy number) *A fuzzy set $\tilde{A}$ defined by the universal set of real numbers* R *is said to be a fuzzy number if its membership function has the following characteristics.*
i. $\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0, 1]$ *is continuous,*
ii. $\mu_{\tilde{A}}(x)$, *for all* $x \in (-\infty, a] \cup [d, +\infty)$,
iii. $\mu_{\tilde{A}}(x)$ *is strictly increasing on* $[a, b]$ *and strictly decreasing on* $[c, d]$,
iv. $\mu_{\tilde{A}}(x) = 1$ *for* $x \in [a, b]$ *where* $a \leq b \leq c \leq d$.

III. Mathematical Formulation

Chance Constrained Fuzzy Programming (CCFP)Model

The multi objective CCFP model is given as

$$\text{Opt } Z_k = \sum_{j=1}^{q} C_{kj}x_j, \qquad k = 1, 2, \cdots, K;$$

Subject to

$$P\left[\sum_{j=1}^{q} \tilde{a}_{ij}x_j \leq \tilde{b}_i\right] \geq 1 - \gamma_i \quad i = 1, 2, \cdots, p;$$

$$x_j \geq 0, \quad j = 1, 2, \cdots, q.$$

Here, C_{kj} are the cost coefficients, b_i are gumbel random variables, and a_{ij} are triangular fuzzy numbers. $\gamma_i (0 \leq \gamma_i \leq 1)$ $i = 1, 2, \cdots, p$ are real numbers representing the tolerance level of the parameters, and x_j are the decision variables.

Membership Function

In a decision making environment,the aspiration levels may not be define precisely, the satisfaction derived depends on the level of achievement of the goal target. Sometimes, the goal target can only be achieved to some degree thereby discrediting the black and white scenerio of achievement and non- achievement. In such cases, the gray area of the level of achievement is considered. The mode α_i of the gumbel random variable b_i is assumed to be a right sided fuzzy number with tolerance limit ρ_i and membership function represented as

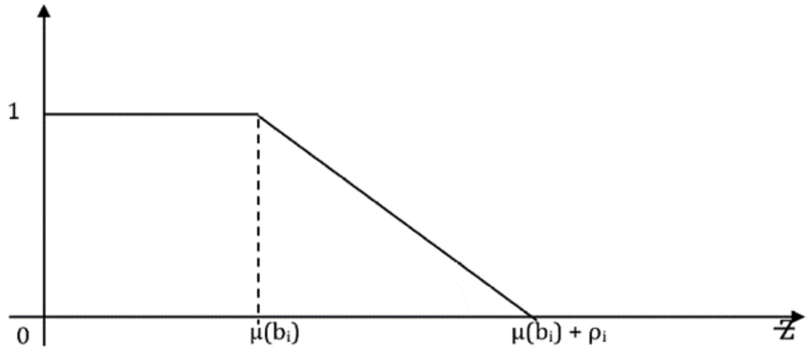

Figure 1: Right sided fuzzy number

The constraint coefficients a_{ij} are considered as triangular fuzzy numbers of the form A=(a^L , a_{ij}, a^R)

IV. Fuzzy Goal Programming (Fgp) Model Construction

The decision maker's goal is to achieve the aspiration level to the best possible extent by increasing the level of satisfaction of each fuzzy goal in the system. The algebraic formulation of the weighted fuzzy goal programming model is given as

The resultant model is solved to find a satisfying solution to the bi-criteria chance constrained fuzzygoal programming problem.

V. Problem Formulation

The proposed technique is applied to a real life problem of agricultural production. The study aims at analyzing the effect of hydrological extreme event of rainfall on some selected crop varieties namely; rice, yam and cassava cultivated by small holder farmers in Ebonyi state, Nigeria over an eight year period (2016-2023). Data from the Ebonyi state value chain programme of the Federal government of Nigeria was utilized to obtain the mean cultivated land area and the total crop yield. Rainfall data obtained from the Nigerian Meteorological agency was utilized to obtain the amount of rainfall in the state within the period of time. The data set is represented in Table 1, Table 2, and Table 3. The solution approach to the problem is given in the following steps.

- **Step i**:Consider a crop production problem, select rice, yam, and cassava cultivation.
- **step ii**:Consider the amount of rainfall as a constraint to crop production.
- **step iii**: Choose a suitable mathematical model for this problem (Chance Constrained Fuzzy Programming CCFP model).
- **step iv**: Collect secondary data from designated agencies.
- **step v**: Compute the data for annual mean cultivated area, annual crop yield and annual rainfall.
- **step vi**: Tabulate the data.
- **step vii**: Analyze the data with a suitable computer software(LINGO 14.0).
- **step viii**: Check the optimal solution and designate as goal target.

VI. Conclusions

In this case study, we have considered the simultaneous occurrence of fuzzy and chance constrained pa- rameters in agricultural production.The fuzzy goal programming technique was utilized to incorporate all the desired objectives into a single achievement function. The proposed technique has minimized the mean cultivated land area within the state, which is a satisfying solution to the goals of the small holder farmers in the state. The proposed methodology can be adopted to address various other agricultural production challenges. It can be extended for non-linear chance constrained problems. However, it is concluded that the described methodology will add a new dimension to solving problems in the agri- cultural sector.

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