On Fuzzy Baire -Dominated Spaces

G. Thangaraj

Dept of Mathematics, Thiruvalluvar University
Serkkadu, Vellore -632 115, Tamilnadu, India.

N. Raji

Dept of Mathematics, K. M. G. College of Arts and Science ,
Gudiyattam, TamilNadu, India.

Abstract

A new class of fuzzy topological spaces, namely fuzzy Baire -dominated spaces, is introduced and studied in this paper. It is established that fuzzy Baire dominated and fuzzy extraresolvable spaces are fuzzy hyperconnected spaces, and fuzzy extremally disconnected spaces. The conditions under which fuzzy Baire -dominated and fuzzy submaximal spaces become fuzzy first category spaces are also obtained. It is established that fuzzy Baire - dominated and fuzzy hyperconnected spaces are fuzzy first category spaces. A condition under which fuzzy weakly Baire - dominated spaces become fuzzy Baire - dominated spaces is also obtained in this paper.

Keywords : *Fuzzy nowhere dense set, fuzzy first category set, fuzzy residual set, fuzzy Baire set, fuzzy first category space, fuzzy submaximal space, fuzzy semiregular space, fuzzy extremally disconnected space.*

I. Introduction

Any application of Mathematical notions depends firmly on how one introduces basic ideas that may yield various theories in different directions. If the basic idea is appropriately introduced, then not only the existing theories stand but also the possibility of emerging new theories increases. On these lines, the notion of fuzzy sets as a new approach for modelling uncertainties was introduced by **L. A. Zadeh** [23] in 1965. Based on Zadeh's concept, **C. L. Chang** [4] developed the theory of fuzzy topological in 1968. From then on, quite a number of studies have been conducted in general theoretical areas and in various application sides.

The notion of Baire sets in classical topology was introduced and studied by **Andrzej Szymanski** [1].The concept of fuzzy Baire sets in fuzzy topological spaces was introduced and studied by **G. Thangaraj** and **R.Palani**[12] in terms of fuzzy open sets and fuzzy residual sets. In classical topology, **Haruto Ohta** and **Ken-Ichi Tamano** [6] introduced the concept of Baire - dominationed spaces and weakly Baire - dominationed spaces. Motivated on these lines, the notions of fuzzy Baire -dominationed spaces and weakly Baire -dominationed spaces are introduced in terms of fuzzy Baire sets. It is observed in [22] that fuzzy extraresolvable spaces are not fuzzy hyperconnected spaces.An answer to a question, when fuzzy extraresolvable spaces become fuzzy hyperconnected spaces ?, is obtained in this paper. The conditions for fuzzy Baire - dominated and fuzzy submaximal spaces to become fuzzy first category spaces are also obtained.It is observed in [11] that fuzzy Baire spaces are fuzzy second category spaces. An answer to a question, which fuzzy topological spaces are fuzzy first category spaces?, is also obtained in this paper. It is found that Baire - dominated spaces are fuzzy weakly Baire - dominated spaces. A condition under which fuzzy weakly Baire - dominated spaces become fuzzy Baire - dominated spaces , is also obtained in this paper.

II. Preliminaries

Several basic notions and results used in the sequel, are given for making the exposition self - contained. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to C.L.Chang (1968). Let X be a non-empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a function from X into I. The fuzzy set is defined as (, for all and the fuzzy set is defined as (, for all .

Definition 2.1[4] : A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions :

(a). ∈ T and ∈ T

(b). If A, B∈T, then A B ∈ T,

(c). If ∈T for each i∈J, then ∈T.

T is called a fuzzy topology for X, and the pair (X,T) is a fuzzy topological space or fts for short. Members of T are called fuzzy open sets of X and their complements are called fuzzy closed sets in X.

Definition 2.2 [4] : Let (X,T) be a fuzzy topological space and be any fuzzy set defined on X. The fuzzy interior , the fuzzy closure and the complement of λ are defined respectively as follows :

(i). int () { / , T } ;

(ii). cl () { / , 1 T } ;

(iii). () 1 () , for all

For a family of fuzzy sets in the union and intersection , are defined respectively as

(iv). ;

(v)..

Lemma 2.1 [2] : For a fuzzy set of a fuzzy topological space X,

(i). 1 cl () int (1) and (ii). 1 int (cl (1

Definition 2.3 : A fuzzy set λ in X is called a

(i). fuzzy -set in (X,T) if where for and

fuzzy -set in (X,T) if where 1 for [3].

(ii). fuzzy regular- open set in (X,T) if λ int cl (λ) and

fuzzy regular -closed set in (X,T) if λ cl int (λ) [2].

(iii).fuzzy semi - open set in (X,T) if and

fuzzy semi - closed set in (X,T) if [2].

(iv).fuzzy dense set in (X,T) if there exists no fuzzy closed set μ in (X,T) such that λ μ 1. That is, cl (λ)in (X,T) [9] .

(v). fuzzy nowhere dense set in (X,T) if there exists no non-zero fuzzy open set μ in (X,T) such that μ cl (λ). That is, int cl (λ)0, in (X,T) [9].

(vi). fuzzy first category set in (X,T) if , where are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category [9] .

(vii). fuzzy residual set in (X,T) if is a fuzzy first category set in (X,T) [11].

(viii).fuzzy Baire set in (X,T) if , where is a fuzzy open set and is a fuzzy residual set in X [12].

(ix). fuzzy pseudo-open set in (X,T) if $\lambda \vee$, where is a fuzzy open set and is a fuzzy first category set in (X,T) [13].

Definition 2.4 : A fuzzy topological space (X,T) is called a

(i). fuzzy submaximal space if for each fuzzy set λ in X such that cl (λ) 1, λ T [3].

(ii). fuzzy globally disconnected space if each fuzzy semi-open set is fuzzy open in (X,T) [14].

(iii). fuzzy Baire space if , where are fuzzy nowhere dense sets in (X,T) [11].

(iv). fuzzy second category space if , where are fuzzy nowhere dense sets in (X,T) [11].

(v). fuzzy first category space if 1, where (s are fuzzy nowhere dense sets in (X,T)[11].

(vi). fuzzy extraresolvable space if whenever are fuzzy dense sets in (X,T), then is a fuzzy nowhere dense set in (X,T) [22].

(vii). fuzzy semiregular space if and only if the collection of all fuzzy regular open sets of X forms a base for fuzzy topology τX [2].

(viii).fuzzy Brown space if for any two non-zero fuzzy sets λ and μ in (X,T), cl(, in (X,T) [19].

(ix).fuzzy extremally disconnected space if the closure of every fuzzy open set of (X,T) is fuzzy open in (X,T) [5].

(x). fuzzy open hereditarily irresolvable space if int cl (λ) ≠ 0, for any non-zero fuzzy set λ in (X,T), then int (λ) 0, in (X,T) [10].

(xi).fuzzy hyperconnected space if every non -null fuzzy open subset of (X,T) is fuzzy dense

in ([**8**].

Theorem 2.1 [**16**]: If a fuzzy set λ is a fuzzy Baire set in a fuzzy submaximal space (X,T), then 1λ is a fuzzy -set in (X,T).

Theorem 2.2 [**16**] :If a fuzzy set λ is a fuzzy Baire set in a fuzzy submaximal space (X,T), then λ is a fuzzy -set in (X,T).

Theorem 2.3 [**16**] :If λ is a fuzzy Baire set in a fuzzy globally disconnected space (X,T), then 1λ is a fuzzy -set in (X,T).

Theorem 2.4 [**16**]: If λ is a fuzzy Baire set in a fuzzy second category (but not fuzzy Baire) space (X,T), then there exists a fuzzy pseudo-open set in (X,T) such that 1.

Theorem 2.5 [**17**]:If λ is a fuzzy Baire set in a fuzzy extraresolvable space (X,T), then int (λ) 0, in (X,T).

Theorem 2.6 [**21**] : If a fuzzy topological space (X,T) is a fuzzy hyperconnected space, then (X,T) is not a fuzzy semiregular space.

Theorem 2.7 [**19**] :If a fuzzy topological space (X,T) is a fuzzy submaximal space, then (X,T) is a fuzzy Brown space.

Theorem 2.8 [**20**] : If 1, where (s are fuzzy sets defined on X such that int( ) 0, in a fuzzy Brown space, then (X,T) is a fuzzy first category space.

Theorem 2.9 [**18**] : If int (λ) for a fuzzy set λ defined on X in a fuzzy submaximal space (X, T), then λ is a fuzzy nowhere dense set in (X,T).

Theorem 2.10 [**22**] :If a fuzzy topological space (X, T) is a fuzzy extraresolvable space, then (X, T) is not a fuzzy hyperconnected space.

Theorem 2.11[**15**] : If a fuzzy topological space (X, T) is a fuzzy hyperconnected space, then (X, T) is not a fuzzy open hereditarily irresolvable space.

Theorem 2.12[**7**] : In any fuzzy topological space (X, τ), the following conditions are equivalent :
(i) (X,τ) is fuzzy hyperconnected space.
(ii). Every fuzzy subset of X is either fuzzy dense or fuzzy nowhere dense set there in.

III. Baire – Dominated Spaces

Definition 3.1 :A fuzzy topological space (X,T) is called a fuzzy Baire - dominated space if for each collection { } (i1 to) of fuzzy closed sets with ……. and 0, there exists a collection} (i1 to) of fuzzy Baire sets in (X,T)with for each i , and 0, in (X,T).

Proposition 3.1 : If) 1, where { }(I1 to) is a collection of fuzzy open sets with …in a fuzzy Baire - dominated space (X,T), then there exists a collection { } (i1 to) of fuzzy Baire sets in (X,T)with 1 for each i , and 1, in (X,T).

Proof : Let { } (i 1 to) bea collection of fuzzy open sets with … and) 1, in (X,T). Then, () 's are fuzzy closed sets with …..and) 110, in (X,T). Since (X,T) is a fuzzy Baire - dominated space, for the collection of fuzzy closed sets { }, there exists a collection { } (i1 to) of fuzzy Baire sets in (X,T) with1for each I and 0 in (X,T). Now 1, implies that 1 and 10 in (X,T).

Proposition 3.2 : If) 1 , where {}(i1 to) is a collection offuzzy open sets with … in a fuzzy Baire – dominated space (X,T), then 1, where are fuzzy closed sets and fuzzy first category sets in (X,T) and .

Proof : Let { } (i 1 to) be a collection of fuzzy open sets with … and) 1, in (X,T). Since(X,T) is a fuzzy Baire - dominated space, by Proposition **3.1,**there exists a collection { } (i1 to) of fuzzy Baire sets in (X,T) , with 1 for each i, and 1. Since { }'s are fuzzy Baire sets in (X,T) , ( ), where are fuzzy open sets and fuzzy residual sets in (X,T) and∨(). Let and . Then, ()'s are fuzzy closed sets and ( )'s are fuzzy first category sets in (X,T) and1and.

Proposition 3.3 : If { } (i 1 to) is a collection of fuzzy closed with ……. and 0, in a fuzzy Baire - dominated space (X,T), then there exists a collection {} (i1 to) of fuzzy Baire sets in (X,T) such that and in (X,T).

Proof : Let { } (i 1 to) be a collection of fuzzy closed sets with ……and 0, in (X,T). Since(X,T) is a fuzzy Baire - dominated space, there exists a collection { } (i1 to) of fuzzy Baire sets in (X,T), withfor each i and 0, in (X,T). Now0, implies that 1 Since , 1 1in (X,T). Also11. Then, 1. This implies that 1and Hence, it follows that and in (X,T).

IV. Baire - Dominated Spaces and Other Fuzzy Topological Spaces

Proposition 4.1 : If) 1, where { } (i1 to) is a collection of fuzzy open sets with …in a fuzzy Baire - dominated and fuzzysubmaximal space (X,T), then there exists a collection { } (i1 to of fuzzy -sets in (X,T) withfor each i , and 1, in (X,T).
Proof : Let { } (i 1 to) be a collection of fuzzy open sets with   … and) 1, in (X,T). Since (X,T) is a fuzzy Baire - dominated space, by Proposition **3.1,** there exists a collection { } (i1 to) of fuzzy Baire sets in (X,T) with 1 for each i, and 1, in (X,T).Also since (X,T) is a fuzzy submaximal space, by Theorem **2.1,** for the fuzzy Baire sets ()'s in (X,T) , (1are fuzzy -sets in (X,T). Let 1. Thus, there exists a collection { } (i 1 to of fuzzy -sets in (X,T) with for each i , and   1, in (X,T).

Proposition 4.2 : If) 1, where { } (i 1 to) is a collection of fuzzy open sets with …in a fuzzy Baire - dominated and fuzzy globally disconnected space (X,T), then there exists a collection { } (i1 to of fuzzy −sets in (X,T), with for each i , and 1, in (X,T).
Proof : Th e proof follows from Proposition **3.1** and Theorem **2.3.**

Proposition 4.3 : If for each collection { } (i 1 to) of fuzzy closed sets with   ……. and 0, in a fuzzy Baire - dominated and fuzzy second category (but not fuzzy Baire) space (X,T), there exists a collection { } (i 1 to) of fuzzy pseudo-open sets in (X,T) with for each i , and) 1, in (X,T).
Proof : Let { } (i 1 to) be a collection of fuzzy closed sets with   ……. and 0, in (X,T) . Since (X,T) is a fuzzy Baire - dominated space, there exists a collection { } (i1 to) of fuzzy Baire sets in (X,T) , with for each i , and 0, in (X,T). Also since (X,T) is a fuzzy second category (but not fuzzy Baire) space, for the fuzzy Baire sets {} 's in (X,T), by Theorem **2.4,** there exist fuzzy pseudo-open sets in (X,T) such that 1and thus 1.Then, in (X,T). Now 1, implies that and 1. Thus, 1, in (X,T).

Proposition 4.4 : If for each collection { } (i 1 to) of fuzzy closed sets with   ……. and 0, in a fuzzy Baire - dominated and fuzzy submaximal space (X,T), then there exists a collection { } (i1 to of fuzzy −sets in (X,T), with in (X,T).
Proof : Let { } (i 1 to) be a collection of fuzzy closed sets with   ……. and 0, in (X,T) . Since (X,T) is a fuzzy Baire - dominated space, there exists a collection { } (i1 to) of fuzzy Baire sets in (X,T) , with for each i and   0, in (X,T). Also since (X,T) is a fuzzy submaximal space, by Theorem **2.2,** the fuzzy Baire sets {} 's are fuzzy -sets in (X,T). Hence, there exists a collection { } (i 1 to of fuzzy −sets in (X,T), with , in (X,T).

   It is established in [22] that fuzzy extraresolvable spaces are not fuzzy hyperconnected spaces. The following proposition gives   a condition for fuzzy extra-resolvable spaces to become fuzzy hyperconnected   spaces.

Proposition 4.5 : If a fuzzy topological space (X,T) is a fuzzy Baire - dominated and fuzzy extraresolvable space, then (X,T) is a fuzzy hyperconnected space.
Proof : Let { } (i 1 to) be a collection of fuzzy open sets with   … and)1, in(X,T). Since (X,T) is a fuzzy Baire - dominated space, by Proposition **3.1,** there exists a collection { } (i1 to) of fuzzy Baire sets in (X,T) with 1, for each i, and 1.Now 1, implies that 1 and int (1). Since   (X,T) is the fuzzy extraresolvable space, by Theorem **2.5**, for the fuzzy Baire sets {} 's, int() 0, in (X,T). Then, int (1 0 and then, by Lemma **2.1,** 1 cl () 0 Thus, cl () 1, in (X,T) and the fuzzy open sets{ } 's are fuzzy dense sets in (X,T). Hence (X,T) is a fuzzy hyperconnected space.

Proposition 4.6: If a fuzzy topological space (X,T) is a fuzzy Baire - dominated and fuzzy extraresolvable space, then (X,T) is a fuzzy extremally disconnected space.
Proof : The proof follows from Proposition **4.5** and Theorem **2.10.**

Proposition 4.7 : If a fuzzy topological space (X,T) is a fuzzy Baire - dominated and fuzzy extraresolvable space , then (X,T) is not a fuzzy semiregular space.
Proof : The proof follows from Proposition **4.5** and Theorem **2.6.**

Proposition 4.8 : If a fuzzy topological space (X,T) is a fuzzy Baire - dominated and fuzzy extraresolvable space, then (X,T) is not a fuzzy open hereditarily irresolvable space.
Proof : The proof follows from Proposition **4.5** and Theorem **2.11.**

     The following proposition gives a condition for fuzzy Baire - dominated and fuzzy submaximal spaces to become fuzzy first category spaces
.
Proposition 4.9 : If fuzzy Baire – dominated and fuzzy submaximal space (X,T) inwhich each fuzzy -set is having zero interior, then (X,T) is a fuzzy first category space.
Proof : Let { } (i 1 to) be a collection of fuzzy open sets with … and) 1, in (X,T). Since (X,T) is a fuzzy Baire - dominated and fuzzy submaximal space , by Proposition **4.1,** there exists a collection { } (i1 to of fuzzy -sets in (X,T),withfor each i and 1, in (X,T). By hypothesis, int() 0, in (X,T).Since (X,T) is a fuzzy submaximal space, by Theorem **2.7**, (X,T) is a fuzzy Brown space. Thus, 1, where (s are fuzzy sets defined on X such that int () 0 in the fuzzy Brown space (X,T). Then, by Theorem **2.8**, (X,T) is a fuzzy first category space.

Corollary 4.1: If fuzzy Baire -dominated and fuzzy submaximal space (X,T) in which each fuzzy-set is fuzzy dense, then (X,T) is not a fuzzy second category space.
Proof : Let { } (i 1 to) be a collection of fuzzy open sets with … and) 1, in (X,T). Since (X,T) is a fuzzy Baire - dominated and fuzzy submaximal space , by Proposition **4.1,** there exists a collection { } (i1 to of fuzzy -sets in (X,T), with for each i and 1.Now, (1)'s are fuzzy -sets in (X,T) and by hypothesis, cl (1) 1 and then 1) 1 [by Lemma **2.1]**, implies that int() 0, in (X,T). Since (X,T) is a fuzzy submaximal space, by Theorem **2.9**, ( )'s are fuzzy nowhere dense sets n (X,T).Thus, 1, where (s are fuzzy nowhere dense sets in (X,T), implies that (X,T) is a fuzzy first category space and hence (X,T) is not a fuzzy second category space.

Corollary 4.2 : If a fuzzy topological space (X,T) is a fuzzy Baire - dominated and fuzzy submaximal space (X,T) in which each fuzzy -set is fuzzy dense, then (X,T) is not a fuzzy Baire space.
Proof : Let { } (i 1 to) be a collection of fuzzy open sets with … and) 1, in (X,T). Since (X,T) is a fuzzy Baire - dominated and fuzzy submaximal space, by Proposition **4.1,** there exists a collection { } (i1 to of fuzzy –sets in (X,T), with for each i and 1. Now, (1)'s are fuzzy -sets in (X,T) and by hypothesis, cl (1) 1 .Then , by Lemma **2.1**, 1) 1 and this implies that int() 0, in (X,T). Since (X,T) is a fuzzy submaximal spac by Theorem **2.9**, ( )'s are fuzzy nowhere dense sets in (X, T). Thus, 1, where (s are fuzzy nowhere dense sets in (X,T) and then int () int(1) Hence (X,T) is not a fuzzy Baire space.

     The following proposition shows that fuzzy Baire - dominated and fuzzy hyperconnected spaces are fuzzy first category spaces.

Proposition 4.10: If a fuzzy topological space (X,T) is a fuzzy Baire - dominated, and fuzzy hyperconnected space, then (X,T) is a fuzzy first category space.
Proof : Let { } (i1 to) be a collection of fuzzy open sets with … and) 1, in (X,T). Since (X,T) is a fuzzy Baire - dominated space, by Proposition **3.2,** 1, where are fuzzy closed sets and fuzzy first category sets in (X,T) and . Since (X,T) is a fuzzy hyperconnected space, by Theorem **2.13**, are fuzzy nowhere dense sets in (X,T). Then , where ()'s are fuzzy nowhere dense sets in (X,T) and thus ()'s fuzzy first category sets in (X,T).Hence 1, implies that 1 , where ()'s are fuzzy nowhere dense sets in (X,T). Hence (X,T) is a fuzzy first category space.

Proposition 4.11 : If 0, for a collection { } (i 1 to) of fuzzy sets with ……in which intcl() 0 in a fuzzy Baire - dominated and fuzzy nodec space (X,T), then there exists a collection { } (i 1 to) of fuzzy Baire sets in (X,T) , with for each i , and 0, in (X,T).
Proof : Let { } (i1 to) be a collection of fuzzy sets with ……and 0, in (X,T) . Then, intcl ()intcl ()intcl ()intcl ()….. and intcl () 0, implies that intcl () 0, for each i .That is, { } 'sare fuzzy nowhere dense sets in (X,T). Since (X,T) is a fuzzy nodec space,{ } 's are fuzzy closed sets in (X,T). Thus { } (i 1 to) is a collection of fuzzy closed sets with ….. and 0, in (X,T) . Since (X,T) is a fuzzy Baire - dominated space, there exists a collection { } (i1 to) of fuzzy Baire sets with for each i , and 0, in (X,T).

V. Weakly Baire -Dominated Spaces
VI.

Definition 5.1 :A fuzzy topological space (X,T) is   called a fuzzy weakly Baire - dominated space if for each collection { } (i 1 to) of fuzzy regularclosed sets with   ……. and   0, there exists a collection {} (i1 to) of fuzzy Baire sets in (X,T) with for each i , and 0, in (X,T).

Proposition 5.1: If a fuzzy topological space (X,T) is   a   fuzzy   Baire -dominated   space, then (X,T) is   a fuzzy   weakly Baire - dominated space.
Proof: Let { } (i 1 to) be a collection   of fuzzy regular closed sets with   ……. and   0, in (X,T).Since fuzzy regular closed sets are fuzzy closed   sets in fuzzy topological spaces , { } (i 1 to) is a collection of fuzzy closed sets in (X,T). Since (X,T) is a fuzzy Baire - dominated   space, there exists a collection { } (i1 to) of fuzzy Baire sets in (X,T) withfor each i and 0, in (X,T).Thus, for the   collection { }   (i 1 to) of fuzzy regular   closed sets with   ……. and   0, the existence of a collection { } (i1 to) of fuzzy Baire sets in (X,T) with, for each i, and 0, in (X,T), shows that (X,T) is   a fuzzy weakly Baire - dominated space.

The following proposition gives   a condition underwhich fuzzy weakly Baire - dominated spaces   become   fuzzy   Baire - dominated spaces.

Proposition 5.2 : If each fuzzy closed set is a fuzzy semi-open set in   a fuzzy weakly Baire - dominated space (X,T), then (X,T) is a fuzzy   Baire - dominated space.
Proof : Let { } (i 1 to)   be a collection   of fuzzy closed sets   with …and   0, in (X,T). By hypothesis,   the fuzzy closed sets () 's are fuzzy semi-open sets in (X,T) and   then   cl int), in (X,T). Now, cl int)   cl)   cl int). This implies that   cl int) and thus { }'s are   fuzzy regular closed sets in (X,T). Since (X,T) is a fuzzy weakly Baire - dominated   space, there exists a collection { }   (i1 to) of fuzzy Baire sets in (X,T) with for each i and 0, in (X,T).Thus, for the collection { } (i 1 to) of fuzzy closed sets with   … and 0, the existence of a collection {} (i1 to) of fuzzy Baire   sets in   (X,T) with ,for each   i and 0, shows that (X,T) is   a fuzzy Baire- dominated space.

VII.   Conclusion

In this paper, the   notions of fuzzy   Baire -dominationed   spaces and weakly Baire- dominationed   spaces are introduced and studied. It is established that fuzzy Baire –dominated and fuzzy extraresolvable spaces are fuzzy hyperconnected spaces, fuzzy extremally disconnected spaces, but are neither   fuzzy open hereditarily irresolvable spaces , nor fuzzy semiregular spaces. The conditions under which fuzzy Baire -dominated and fuzzy submaximal spaces become fuzzy first category spaces are also obtained. It is established that fuzzy Baire - dominated and fuzzy hyperconnected spaces are fuzzy first category spaces. It is found that Baire - dominated   spaces are fuzzy weakly Baire - dominated spaces. A condition under which fuzzy weakly Baire - dominated spaces become   fuzzy Baire - dominated spaces , is also obtained in this paper.

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