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# Adjacency Energy Of Sum - Eccentricity Divided By Diameter And Product – Eccentricity Divided By Diameter Of Graphs

M.Mutharasi, Dr.M. Deva Saroja
[1]*research Scholar (Register Number: 19221172092014)*
[2]*research Supervisor, Assistant Professor,*
[1,2] *Pg And Research Department Of Mathematics,*
*Rani Anna Government College For Women, Tirunelveli - 627 008. (India)*
*Affiliated To Manonmaniam Sundaranar University, Abishekapatti,*
*Tirunelveli - 627 012.*

***Abstract***
*In this paper, we introduce the concepts sum - eccentricity divided by diameter of graph G, it is denoted by* $\left(\frac{SE}{diam}\right)(G)$ *and product - eccentricity divided by diameter of graph G, it is denoted by* $(\frac{PE}{diam})(G)$. *We find the adjacency energy of sum - eccentricity divided by diameter and product - eccentricity divided by diameter of some classes of graphs.*

***Keywords:*** *sum - eccentricity, product - eccentricity, diameter, spectrum and energy.*

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## I. Introduction

Let G be a finite and undirected simple graph with m vertices named by $v_1, v_2, \cdots, v_m$. Then the adjacency matrix A(G) of the graph G is a square matrix of order m, whose $(i,j)^{th}$entry is equal to 1 if the vertices $v_i$ and $v_j$ are adjacent and equal to zero otherwise. The characteristic polynomial of the adjacency matrix, ie., $det(\eta I_m - A(G))$, where I is the unit matrix of order m, is said to be the characteristic polynomial of the graph G and will be denoted by $P(G,\eta)$. The eigenvalues of a graph G are defined as the eigenvalues of its adjacency matrix A(G), and so they are just the roots of the equation $P(G,\eta) = 0$. Since A(G) is a real symmetric matrix, its eigenvalues are all real, denoting them by $\eta_1, \eta_2, \cdots, \eta_m$, and as a whole, they are called the spectrum of G. In 1970, I.Gutman introduced the concept of the energy of G. [5]

Let $e(v_i)$ denote the eccentricity of the vertex $v_i$, for $i = 1,2,\cdots,m$. For vertices $v_i,\ v_j \in V(G)$, the distance $d(v_i, v_j)$ is defined as the length of the shortest path between $v_i$ and $v_j$ in G [13]. The eccentricity of a vertex is the maximum distance from it to any other vertex. $e(v_i) = \max_{v_i \in V(G)} d(v_i, v_j)$.

The diameter of a graph G, denoted by diam(G), is the maximum eccentricity of any vertex in the graph or the greatest distance between any pair of vertices. [8]

## II. Preliminary

**Lemma 2.1 [2]**

Let M, N, P and Q be matrices with M invertible. Then we have $\begin{vmatrix} M & N \\ P & Q \end{vmatrix} = |M||Q - PM^{-1}N|$

**Lemma 2.2 [2]**

Let M, N, P and Q be matrices. Let $S = \begin{pmatrix} M & N \\ P & Q \end{pmatrix}$ if M and P commutes. Then $|S| = |MQ - PN|$.

**Lemma 2.3 [3]**

If $A(K_p)$ is the adjacency matrix of $K_p$, then $A^2(K_P) = (p-2)A(K_p) + (p-1)I_p$.

**Definition 2.4 [3]**

Let $K_{2p}$ be a complete graph with vertices $2p, p = 1,2,\dots,n$. We delete the edge joining the vertices $i$ and $p+i, 1 \le i \le p$. The resulting graph $D_1(K_{2p})$ has the order $2p$ and has $2p(p-1)$ edges. Further it is regular of degree $2p-2$.

**Definition 2.5 [3]**

Consider the complete graph $K_{2p}$ with $2p$ vertices. We split the vertices into two equal parts and delete the edges between that spilted parts. We obtain a disconnected graph such a graph is of order $2p$ and has $p(p-1)$ edges. Further it is regular of degree $p-1$. We denote it by $D_2(K_{2p})$.

**Definition 2.6 [3]**

Consider the complete graph $K_{2p}$ with $2p$ vertices. We split the vertices into two equal parts such that the vertices 1 to $p$ in one part and $p+1$ to $2p$ in the other part. Now delete the edges between the vertices in the same parts also edges joining $i$ and $p+i$, $1 \leq i \leq p$. The resulting graph is of order $2p$ and has $p(p-1)$ edges. Further it is regular of degree $p-1$. We denote it by $D_3(K_{2p})$.

**Definition 2.7 [3]**

Consider a pair of complete graphs $K_p$ with vertex set $\{v_i, i = 1,2,3, \dots p\}$ and $\{u_j, j = 1,2,3, \dots p\}$. We obtain a graph joining $v_i$ to $u_i$, for $i = 1,2,3, \dots p$. Such a graph is of order $2p$ and $p^2$ edges. Further it is regular of degree p. We denote it by $J\left({K_p}^p\right)$.

**Definition 2.8 [11]**

$K_{1,1,n}$ is a graph obtained by attaching root of a star $K_{1,n}$ at one end of $P_2$ and other end of $P_2$ is joined with each pendant vertex of $K_{1,n}$.

**Definition 2.9 [12]**

A globe graph $Gl_n$ is a graph obtained from two isolated vertex are joined by n paths of length 2.

## III. Main Result

**Adjacency energy of sum - eccentricity divided by diameter of graphs**

Let $G = (V, X)$ be a connected simple graph with $|V| = m$ vertices and $|E| = q$ edges. Let $e(v_i), e(v_j)$ be the eccentricity of the vertices $v_i, v_j$ respectively, for all $i, j = 1,2, \cdots, m$. Then the adjacency matrix of sum eccentricity divided by diameter of the graph is defined as

$$se_{ij} = \begin{cases} \dfrac{e(v_i) + e(v_j)}{diam\ G}, & if\ v_i\ adjacent\ to\ v_j \\ 0, & otherwise \end{cases}$$

The adjacency matrix of sum - eccentricity divided by diameter is a symmetric matrix with eigenvalues as $\eta_1 \geq \eta_2 \geq \cdots \geq \eta_m$. The characteristic polynomial of $(\frac{SE}{diam})(G)$ is given by $\left|\eta I - (\frac{SE}{diam})(G)\right|$. The adjacency energy of sum - eccentricity divided by diameter of the graph G is defined as the sum of the absolute values of $\eta_i$, $i = 1,2, \cdots, m$. $E\left[(\frac{SE}{diam})(G)\right] = \sum_{i=1}^{m} |\eta_i|$.

**Adjacency energy of sum - eccentricity divided by diameter of standard graphs**

**Theorem 3.1.1**

Let $K_m$ be a complete graph. Then $E\left[(\frac{SE}{diam})(K_m)\right] = 4(m-1)$, where $m \geq 2$.

**Proof:**

Let $K_m$ be the complete graph with m vertices. Then the adjacency matrix of sum - eccentricity divided by diameter of $K_m$ is,

$$(\frac{SE}{diam})(K_m) = \begin{bmatrix} 0 & 2 & 2 & \cdots & 2 \\ 2 & 0 & 2 & \cdots & 2 \\ 2 & 2 & 0 & \cdots & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & 2 & \cdots & 0 \end{bmatrix}$$

and its characteristic polynomial is,

$$P\left((\frac{SE}{diam})(K_m), \eta\right) = \left(\eta - 2(m-1)\right)(\eta + 2)^{m-1}$$

Hence $S_p\left[(\frac{SE}{diam})(K_m)\right] = \begin{pmatrix} 2(m-1) & -2 \\ 1 & m-1 \end{pmatrix}$

and $E\left[(\frac{SE}{diam})(K_m)\right] = 4(m-1)$.

**Theorem 3.1.2**
Let $K_{1,m}$ be a star graph. Then $E\left[\frac{SE}{diam}(K_{1,m})\right] = 3\sqrt{m}$, where $m \geq 2$.
**Proof:**
Let $K_{1,m}$ be the star graph with $m+1$ vertices. Then the adjacency matrix of sum - eccentricity divided by diameter of $K_{1,m}$ is,

$$(\frac{SE}{diam})(K_{1,m}) = \begin{bmatrix} 0 & \frac{3}{2} & \frac{3}{2} & \cdots & \frac{3}{2} \\ \frac{3}{2} & 0 & 0 & \cdots & 0 \\ \frac{3}{2} & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{3}{2} & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Therefore, $P\left((\frac{SE}{diam})(K_{1,m}), \eta\right) = \left(\eta^2 - \frac{9}{4}m\right)(\eta)^{m-1}$
Hence $S_p\left[(\frac{SE}{diam})(K_{1,m})\right] = \begin{pmatrix} -\frac{3}{2}\sqrt{m} & \frac{3}{2}\sqrt{m} & 0 \\ 1 & 1 & m-1 \end{pmatrix}$
and $E\left[(\frac{SE}{diam})(K_{1,m})\right] = 3\sqrt{m}$.

**Theorem 3.1.3**
Let $K_{m,m}$ be a complete bipartite graph. Then $E\left[(\frac{SE}{diam})(K_{m,m})\right] = 4m$, where $m \geq 1$.
**Proof:**
Let $K_{m,m}$ be the complete graph with $2m$ vertices. Then the adjacency matrix of sum - eccentricity divided by diameter of $K_{m,m}$ is,

$$(\frac{SE}{diam})(K_{m,m}) = \begin{bmatrix} 0 & 2J \\ 2J & 0 \end{bmatrix}, \text{ where } J = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}.$$

Therefore, $P\left((\frac{SE}{diam})(K_{m,m}), \eta\right) = (\eta^2 - 4m^2)(\eta)^{2m-2}$.
Hence $S_p\left[(\frac{SE}{diam})(K_{m,m})\right] = \begin{pmatrix} -2m & 2m & 0 \\ 1 & 1 & 2m-2 \end{pmatrix}$
and $E\left[(\frac{SE}{diam})(K_{m,m})\right] = 4m$.

**Adjacency energy of sum - eccentricity divided by diameter of some regular graphs obtained from complete graph**
**Theorem 3.2.1**
Let $D_1(K_{2m})$ be the edge deleting graph 1 of $K_{2m}$. Then $E\left[(\frac{SE}{diam})(D_1(K_{2m}))\right] = 8(m-1)$, where $m \geq 2$.
**Proof:**
Let $D_1(K_{2m})$ be the edge deleting graph 1 of $K_{2m}$ with order 2m, $m = 2,3,\cdots,n$ and $2m(m-1)$ edges. Then the adjacency matrix sum - eccentricity divided by diameter of $D_1(K_{2m})$ is,

$$(\frac{SE}{diam})(D_1(K_{2m})) = \begin{bmatrix} 2A(K_m) & 2A(K_m) \\ 2A(K_m) & 2A(K_m) \end{bmatrix}.$$

Therefore, $P\left((\frac{SE}{diam})(D_1(K_{2m})), \eta\right) = \begin{vmatrix} \eta I_m - 2A(K_m) & -2A(K_m) \\ -2A(K_m) & \eta I_m - 2A(K_m) \end{vmatrix}$
$= |(\eta I_m - 2A(K_m))^2 - (2A(K_m))^2|$ (by lemma 2.2)
$= |(\eta^2 I_m - 2\eta(2A(K_m))|$
$= (2\eta)^m \left|\frac{\eta^2}{2\eta} I_m - 2A(K_m)\right|$
$= (2\eta)^m \left(\frac{\eta}{2} - 2(m-1)\right)(\frac{\eta}{2} + 2)^{m-1}$
$= (\eta)^m (\eta - 4(m-1))(\eta + 4)^{m-1}$
Hence $S_p[(\frac{SE}{diam})(D_1(K_{2m}))] = \begin{pmatrix} 0 & -4 & 4(m-1) \\ m & m-1 & 1 \end{pmatrix}$
and $E\left[\frac{SE}{diam}(D_1(K_{2m}))\right] = 8(m-1)$.

**Theorem 3.2.2**
Let $D_3(K_{2m})$ be the edge deleting graph 3 of $K_{2m}$. Then $E\left[\frac{SE}{diam}(D_3(K_{2m}))\right] = 8(m-1)$, where $m \geq 3$.

**Proof:**
Let $D_3(K_{2m})$ be the edge deleting graph 3 of $K_{2m}$ with order 2m, $m = 3,4,\cdots,n$ and $m(m-1)$ edges. Then adjacency matrix of sum - eccentricity divided by diameter of $D_3(K_{2m})$ is,
$(\frac{SE}{diam})(D_3(K_{2m})) = \begin{bmatrix} 0 & 2A(K_m) \\ 2A(K_m) & 0 \end{bmatrix}$.
Therefore, $P((\frac{SE}{diam})(D_3(K_{2m})),\eta) = \begin{vmatrix} \eta I_m & -2A(K_m) \\ -2A(K_m) & \eta I_m \end{vmatrix}$
$= |\eta I_m| \left|\eta I_m - \frac{(2A(K_m))^2}{\eta}\right|$ (by lemma 2.1)
$= \eta^m \left|\eta I_m - \frac{4(m-2)A(K_m)+4(m-1)I_m}{\eta}\right|$ (by lemma 2.3)
$= |\eta^2 I_m - 4(m-2)A(K_m) - 4(m-1)I_m|$
$= (m-2)^m \left|\left(\frac{\eta^2-4(m-1)}{m-2}\right) I_m - 4A(K_m)\right|$
$=(m-2)^m \,(\frac{\eta^2-4(m-1)}{m-2} - 4(m-1))(\frac{\eta^2-4(m-1)}{m-2} + 4\,)^{m-1}$
$= (\eta^2 - 4(m-1)^2)(\eta^2-4)^{m-1}$
Hence $S_p[(\frac{SE}{diam})\big(D_3(K_{2m})\big)] = \begin{pmatrix} -2(m-1) & 2(m-1) & -2 & 2 \\ 1 & 1 & m-1 & m-1 \end{pmatrix}$
and $E\left[(\frac{SE}{diam})(D_3(K_{2m}))\right] = 8(m-1)$.

**Theorem 3.2.3**
Let $J({K_m}^m)$ be the join of complete graph. Then $E\left[(\frac{SE}{diam})(J({K_m}^m))\right] = 8(m-1)$, where $m \geq 3$.
**Proof:**
Let $J({K_m}^m)$ be the join of complete graph order $2m$ and $m^2$ edges. Then adjacency matrix of sum - eccentricity divided by diameter of $J({K_m}^m)$ is,
$(\frac{SE}{diam})(J({K_m}^m)) = \begin{bmatrix} 2A(K_m) & 2I_m \\ 2I_m & 2A(K_m) \end{bmatrix}$.
Therefore, $P\left((\frac{SE}{diam})(J({K_m}^m)),\eta\right) = \begin{vmatrix} \eta I_m - 2A(K_m) & -2I_m \\ -2I_m & \eta I_m - 2A(K_m) \end{vmatrix}$
$= (\eta I_m - 2A(K_m))^2 - (2I_m)^2$
$= ((\eta-2)I_m - 2A(K_m))((\eta+2)I_m - 2A(K_m))$
$= \big((\eta-2)I_m - 2(m-1)\big)\,\big((\eta-2)I_m + 2\big)^{m-1}$
$\big((\eta+2)I_m - 2(m-1)\big)\,((\eta+2)I_m + 2)^{m-1}$
$= \eta^{m-1}(\eta-2m)(\eta-2(m-2))(\eta+4)^{m-1}$
Hence $S_p[(\frac{SE}{diam})\big(J({K_m}^m)\big)] = \begin{pmatrix} 0 & -4 & 2(m-2) & 2m \\ m-1 & m-1 & 1 & 1 \end{pmatrix}$
and $E\left[(\frac{SE}{diam})(J({K_m}^m))\right] = 8(m-1)$.

**Adjacency energy of sum - eccentricity divided by diameter of complement of some regular graphs obtained by complete graph.**
In [4] the complement graphs of $D_1(K_{2m})$, $D_2(K_{2m})$, $D_3(K_{2m})$ and $J({K_m}^m)$ are denoted by $\overline{D_1(K_{2m})}$, $\overline{D_2(K_{2m})}$,$\overline{D_3(K_{2m})}$ and $\overline{J({K_m}^m)}$ . $\bar{A} = J - I - A$ where $\bar{A}$ is the adjacency matrix of complement graph.
**Theorem 3.3.1**
Let $\overline{D_2(K_{2m})}$ be the complement of edge deleting graph 2 of $K_{2m}$. Then $E\left[(\frac{SE}{diam})\big(\overline{D_2(K_{2m})}\big)\right] = 4m$, where $m \geq 2$.
**Proof:**
Let $\overline{D_2(K_{2m})}$ be the complement of edge deleting graph 2 of $K_{2m}$. Then the adjacency matrix of sum - eccentricity divided by diameter of $\overline{D_2(K_{2m})}$ is,
$(\frac{SE}{diam})\,\overline{\big(D_2(K_{2m})\big)} = \begin{pmatrix} 0 & 2J \\ 2J & 0 \end{pmatrix}$, where $J = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$.
Therefore, $P\left((\frac{SE}{diam})\,\overline{\big(D_2(K_{2m})\big)},\eta\right) = \eta^{2m-2}(\eta-2m)(\eta+2m)$
Hence $S_p\left[(\frac{SE}{diam})\big(\overline{D_2(K_{2m})}\big)\right] = \begin{pmatrix} 2m & -2m & 0 \\ 1 & 1 & 2m-2 \end{pmatrix}$
and $E\left[(\frac{SE}{diam})\big(\overline{D_2(K_{2m})}\big)\right] = 4m$.

**Theorem 3.3.2**
Let $\overline{D_3(K_{2m})}$ be the complement of edge deleting graph 3 of $K_{2m}$. Then $E\left[(\frac{SE}{diam})\left(\overline{D_3(K_{2m})}\right)\right] = 8(m-1)$ , where $m \geq 2$.
**Proof:**
Let $\overline{D_3(K_{2m})}$ be the complement of edge deleting graph 3 of $K_{2m}$. Then the adjacency matrix of sum - eccentricity divided by diameter of $\overline{D_3(K_{2m})}$ is,

$$(\frac{SE}{diam})\overline{\left(D_3(K_{2m})\right)} = \begin{pmatrix} 2A(K_m) & 2I_m \\ 2I_m & 2A(K_m) \end{pmatrix}$$

$= (\frac{SE}{diam})(J(K_m{}^m))$ (by theorem (3.2.3))
Since $E\left[\left(\frac{SE}{diam}\right)\left(J(K_m{}^m)\right)\right] = 8(m-1)$, we get
$E[(\frac{SE}{diam})\left(\overline{D_3(K_{2m})}\right)] = 8(m-1)$.
**Theorem 3.3.3**
Let $\overline{J(K_m{}^m)}$ be the complement of join of complete graph. Then $E[(\frac{SE}{diam})\left(\overline{J(K_m{}^m)}\right)] = 8(m-1)$ , where $m \geq 3$.
**Proof:**
Let $\overline{J(K_m{}^m)}$ be the complement of join of complete graph. Then the adjacency matrix of sum - eccentricity divided by diameter of $\overline{J(K_m{}^m)}$ is,

$$(\frac{SE}{diam})\overline{\left(J(K_m{}^m)\right)} = \begin{pmatrix} 0 & 2A(K_m) \\ 2A(K_m) & 0 \end{pmatrix}$$

$= (\frac{SE}{diam})(D_3(K_{2m}))$ (by theorem 3.2.2)
Since $E[(\frac{SE}{diam})\left(D_3(K_{2m})\right)] = 8(m-1)$, we get
$E[(\frac{SE}{diam})\left(\overline{J(K_m{}^m)}\right)] = 8(m-1)$.

**Adjacency energy of sum - eccentricity divided by diameter of some irregular graphs**
**Theorem 3.4.1**
Let $F_m$ be a friendship graph. Then $E\left[\frac{SE}{diam}(F_m)\right] = 2(2m-1) + \frac{1}{2}(2 \pm \sqrt{18m+4})$, where $m \geq 2$.
**Proof:**
The adjacency matrix of sum - eccentricity divided by diameter of the friendship graph $F_m$ with $2m+1$ vertices is,

$$(\frac{SE}{diam})(F_m) = \begin{bmatrix} 0 & \frac{3}{2} & \frac{3}{2} & \cdots & \frac{3}{2} & \frac{3}{2} \\ \frac{3}{2} & 0 & 2 & \cdots & 0 & 0 \\ \frac{3}{2} & 2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{3}{2} & 0 & 0 & \cdots & 0 & 2 \\ \frac{3}{2} & 0 & 0 & \cdots & 2 & 0 \end{bmatrix}.$$

Therefore, $P\left((\frac{SE}{diam})(F_m), \eta\right) = \left(\eta^2 - 2\eta - \frac{9}{2}m\right)(\eta-2)^{m-1}(\eta+2)^m$.
Hence $S_p\left[(\frac{SE}{diam})(F_m)\right] = \begin{pmatrix} \frac{2-\sqrt{18m+4}}{2} & \frac{2+\sqrt{18m+4}}{2} & 2 & -2 \\ 1 & 1 & m-1 & m \end{pmatrix}$.
and $E\left[(\frac{SE}{diam})(F_m)\right] = 2(2m-1) + \frac{1}{2}(2 \pm \sqrt{18m+4})$ .

**Theorem 3.4.2**
Let $Gl_m$ be a globe graph. Then $E\left[(\frac{SE}{diam})(Gl_m)\right] = 4\sqrt{2m}$ , where $m \geq 2$.
**Proof:**
The adjacency matrix of sum - eccentricity divided by diameter of the globe graph $Gl_m$ with $m+2$ vertices is,

$$(\frac{SE}{diam})(Gl_m) = \begin{bmatrix} 0 & 0 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 0 & 2 & 2 & \cdots & 2 & 2 \\ 2 & 2 & 0 & 0 & \cdots & 0 & 0 \\ 2 & 2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & 0 & 0 & \cdots & 0 & 0 \\ 2 & 2 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Therefore, $P\left((\frac{SE}{diam})(Gl_m), \eta\right) = (\eta^2 - 8m)(\eta)^m$.

Hence $S_p\left[(\frac{SE}{diam})(Gl_m)\right] = \begin{pmatrix} -2\sqrt{2m} & 2\sqrt{2m} & 0 \\ 1 & 1 & m \end{pmatrix}$

and $E\left[(\frac{SE}{diam})(Gl_m)\right] = 4\sqrt{2m}$ .

**Theorem 3.4.3**

Let $K_{1,1,m}$ be a graph. Then $E\left[(\frac{SE}{diam})(K_{1,1,m})\right] = 1 + \frac{1}{2}(1 \pm \sqrt{18m+1})$, where $m \geq 1$.

**Proof:**

The adjacency matrix of sum - eccentricity divided by diameter of a graph $K_{1,1,m}$ with $m+2$ vertices is,

$$(\frac{SE}{diam})(K_{1,1,m}) = \begin{bmatrix} 0 & 1 & 3/2 & 3/2 & \cdots & 3/2 & 3/2 \\ 1 & 0 & 3/2 & 3/2 & \cdots & 3/2 & 3/2 \\ 3/2 & 3/2 & 0 & 0 & \cdots & 0 & 0 \\ 3/2 & 3/2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 3/2 & 3/2 & 0 & 0 & \cdots & 0 & 0 \\ 3/2 & 3/2 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Therefore, $P\left((\frac{SE}{diam})(K_{1,1,m}), \eta\right) = (\eta)^{m-1}(\eta+1)(2\eta^2 - 2\eta - 9m)$

Hence $S_p\left[(\frac{SE}{diam})(K_{1,1,m})\right] = \begin{pmatrix} \frac{1}{2}(1-\sqrt{18m+1}) & \frac{1}{2}(1+\sqrt{18m+1}) & -1 & 0 \\ 1 & 1 & 1 & m-1 \end{pmatrix}$

and $E\left[(\frac{SE}{diam})(K_{1,1,m})\right] = 1 + \frac{1}{2}(1 \pm \sqrt{18m+1})$ .

**Theorem 3.4.4**

Let $B_{m,m}$ be a bistar graph. Then $E\left[(\frac{SE}{diam})(B_{m,m})\right] = \frac{1}{3}(\pm 2 \pm \sqrt{25m+4})$, where $m \geq 1$.

**Proof:**

The adjacency matrix of sum - eccentricity divided by diameter a bistar graph $B_{m,m}$ with $2m+2$ vertices is,

$$(\frac{SE}{diam})(B_{m,m}) = \begin{bmatrix} 0 & 5/3 & \cdots & 5/3 & 4/3 & 0 & \cdots & 0 \\ 5/3 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 5/3 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 4/3 & 0 & \cdots & 0 & 0 & 5/3 & \cdots & 5/3 \\ 0 & 0 & \cdots & 0 & 0 & 5/3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 5/3 & \cdots & 0 \end{bmatrix}.$$

Therefore,

$P\left((\frac{SE}{diam})(B_{m,m}), \eta\right) = (\eta)^{2m-2}(9\eta^2 - 12\eta - 25m)(9\eta^2 + 12\eta - 25m)$.

Hence $S_p\left[(\frac{SE}{diam})(B_{m,m})\right] =$

$$\begin{pmatrix} \frac{1}{3}(-2-\sqrt{25m+4}) & \frac{1}{3}(2+\sqrt{25m+4}) & \frac{1}{3}(2-\sqrt{25m+4}) & \frac{1}{3}(\sqrt{25m+4}-2) & 0 \\ 1 & 1 & 1 & 1 & 2m-2 \end{pmatrix}$$

and $E\left[(\frac{SE}{diam})(B_{m,m})\right] = \frac{1}{3}(\pm 2 \pm \sqrt{25m+4})$.

**Theorem 3.4.5**

Let $B^2{}_{m,m}$ be a square bistar graph. Then $E\left[(\frac{SE}{diam})(B^2{}_{m,m})\right] = 1 + \frac{1}{2}(1 \pm \sqrt{36m+1})$ .

**Proof:**

The adjacency matrix of sum - eccentricity divided by diameter a square bistar graph $B^2{}_{m,m}$ with $2m+2$ vertices is,

$$(\frac{SE}{diam})\left(B^2{}_{m,m}\right) = \begin{bmatrix} 0 & 1 & 3/2 & 3/2 & \cdots & 3/2 & 3/2 \\ 1 & 0 & 3/2 & 3/2 & \cdots & 3/2 & 3/2 \\ 3/2 & 3/2 & 0 & 0 & \cdots & 0 & 0 \\ 3/2 & 3/2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 3/2 & 3/2 & 0 & 0 & \cdots & 0 & 0 \\ 3/2 & 3/2 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Therefore, $P\left((\frac{SE}{diam})\left(B^2{}_{m,m}\right), \eta\right) = (\eta)^{2m-1}(\eta+1)(\eta^2-\eta-9m)$

Hence $S_p\left[(\frac{SE}{diam})\left(B^2{}_{m,m}\right)\right] = \begin{pmatrix} \frac{1}{2}(1-\sqrt{36m+1}) & \frac{1}{2}(1+\sqrt{36m+1}) & -1 & 0 \\ 1 & 1 & 1 & 2m-1 \end{pmatrix}.$

and $E\left[(\frac{SE}{diam})\left(B^2{}_{m,m}\right)\right] = 1 + \frac{1}{2}(1 \pm \sqrt{36m+1}).$

## IV. Adjacency Energy Of Product - Eccentricity Divided By Diameter Of Graphs

**Definition:**

Let $e(v_i), e(v_j)$ be the eccentricity of the vertices $v_i, v_j$ respectively, for all $i, j = 1,2,\cdots,m$. Then the adjacency matrix of the product - eccentricity by diameter, is defined as

$$pe_{ij} = \begin{cases} \dfrac{e(v_i)e(v_j)}{diam\ G}, & if\ v_i\ adjacent\ to\ v_j \\ 0, & otherwise \end{cases}$$

The adjacency matrix of product - eccentricity divided by diameter is a symmetric matrix with eigenvalues as $\eta_1 \geq \eta_2 \geq \cdots \geq \eta_m$. The characteristic polynomial of $(\frac{PE}{diam})(G)$ is given by $\left|\eta I - (\frac{PE}{diam})(G)\right|$. The adjacency energy of product - eccentricity divided by diameter the graph G is defined as the sum of the absolute values of $\eta_i$, $i = 1,2,\cdots,m$. $E\left[(\frac{PE}{diam})(G)\right] = \sum_{i=1}^{m}|\eta_i|$.

**Adjacency energy of product − eccentricity divided by diameter of some standard graphs**

**Theorem 4.1.1**

Let $K_m$ be a complete graph. Then $E\left[(\frac{PE}{diam})(K_m)\right] = 2(m-1)$, where $m \geq 2$.

**Proof:**

The adjacency matrix of their product - eccentricity divided by diameter of the complete graph $K_m$ with m vertices is,

$$(\frac{PE}{diam})(K_m) = \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ 1 & 1 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 0 \end{bmatrix} = A(K_m)$$

Since $E(K_m) = 2(m-1)$, we get

$E\left[(\frac{PE}{diam})(K_m)\right] = 2(m-1)$.

**Theorem 4.1.2**

Let $K_{1,m}$ be a star graph. Then $E\left[(\frac{PE}{diam})\left(K_{1,m}\right)\right] = 2\sqrt{m}$, where $m \geq 1$.

**Proof:**

The adjacency matrix of product - eccentricity divided by diameter of the star graph $K_{1,m}$ with $m+1$ vertices is,

$$(\frac{PE}{diam})\left(K_{1,m}\right) = \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} = A(K_{1,m})$$

Since $E\left(K_{1,m}\right) = 2\sqrt{m}$, we get

$E\left[(\frac{PE}{diam})(K_m)\right] = 2\sqrt{m}$.

**Theorem 4.1.3**

Let $K_{m,m}$ be a complete bipartite graph. Then $E\left[\frac{PE}{diam}\left(K_{m,m}\right)\right] = 4m$.

**Proof:**

The adjacency matrix of product- eccentricity divided by diameter of the complete bipartite graph $K_{m,m}$ with 2m vertices is,

$(\frac{PE}{diam})(K_{m,m}) = \begin{bmatrix} 0 & 2J \\ 2J & 0 \end{bmatrix}$, where $J = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$.

$= (\frac{SE}{diam})(K_{m,m})$

Since $E\left[(\frac{SE}{diam})(K_{m,m})\right] = 4m$, we get

$E\left[(\frac{PE}{diam})(K_{m,m})\right] = 4m.$

**Adjacency energy of product - eccentricity divided by diameter of some regular graphs obtained from complete graph**

**Theorem 4.2.1**

Let $D_1(K_{2m})$ be the edge deleting graph 1 of $K_{2m}$. Then $E\left[(\frac{PE}{diam})(D_1(K_{2m}))\right] = 8(m-1)$, where $m \geq 2$.

**Proof:**

Let $D_1(K_{2m})$ be the edge deleting graph 1 of $K_{2m}$ order 2m, $m = 2,3,\cdots,n$ and $2m(m-1)$ edges. Then the adjacency matrix of product - eccentricity divided by diameter is,

$(\frac{PE}{diam})(D_1(K_{2m})) = \begin{bmatrix} 2A(K_m) & 2A(K_m) \\ 2A(K_m) & 2A(K_m) \end{bmatrix}$.

$= (\frac{SE}{diam})(D_1(K_{2m}))$

Since $E\left[(\frac{SE}{diam})(D_1(K_{2m}))\right] = 8(m-1)$, we get

$E\left[(\frac{PE}{diam})(D_1(K_{2m}))\right] = 8(m-1).$

**Theorem 4.2.2**

Let $D_3(K_{2m})$ be the edge deleting graph 3 of $K_{2m}$. Then $E\left[(\frac{PE}{diam})(D_3(K_{2m}))\right] = 12(m-1)$, where $m \geq 3$.

**Proof:**

Let $D_3(K_{2m})$ be the edge deleting graph 3 of $K_{2m}$ order 2m, $m = 3,4,\cdots,n$ and $m(m-1)$ edges. Then the adjacency matrix of product - eccentricity divided by diameter is,

$(\frac{PE}{diam})(D_3(K_{2m})) = \begin{bmatrix} 0 & 3A(K_m) \\ 3A(K_m) & 0 \end{bmatrix}$.

Therefore, $P((\frac{PE}{diam})(D_3(K_{2m})),\eta) = \begin{vmatrix} \eta I_m & -3A(K_m) \\ -3A(K_m) & \eta I_m \end{vmatrix}$

$= |\eta I_m| \left|\eta I_m - \frac{(3A(K_m))^2}{\eta}\right|$ (by lemma 2.1)

$= \eta^m \left|\eta I_m - \frac{9(m-2)A(K_m)+9(m-1)I_m}{\eta}\right|$ (by lemma 2.3)

$= |\eta^2 I_m - 9(m-2)A(K_m) - 9(m-1)I_m|$

$= (m-2)^m \left|\left(\frac{\eta^2-9(m-1)}{m-2}\right) I_m - 9A(K_m)\right|$

$=(m-2)^m (\frac{\eta^2-9(m-1)}{m-2} - 9(m-1))(\frac{\eta^2-9(m-1)}{m-2} + 9)^{m-1}$

$= (\eta^2 - 9(m-1)^2)(\eta^2 - 9)^{m-1}$

Hence $S_p[(\frac{PE}{diam})(D_3(K_{2m}))] = \begin{pmatrix} -3(m-1) & 3(m-1) & -3 & 3 \\ 1 & 1 & m-1 & m-1 \end{pmatrix}$

and $E\left[(\frac{PE}{diam})(D_3(K_{2m}))\right] = 12(m-1)$.

**Theorem 4.2.3**

Let $J(K_m{}^m)$ be the join of complete graph. Then $E\left[(\frac{PE}{diam})(J(K_m{}^m))\right] = 8(m-1)$, where $m \geq 3$.

**Proof:**

Let $J(K_m{}^m)$ be the join of complete graph order $2m$ and $m^2$ edges. Then the adjacency matrix of product - eccentricity divided by diameter is,

$(\frac{PE}{diam})(J(K_m{}^m)) = \begin{bmatrix} 2A(K_m) & 2I_m \\ 2I_m & 2A(K_m) \end{bmatrix}$.

$= (\frac{SE}{diam})(J(K_m{}^m))$

Since $E\left[(\frac{SE}{diam})(J(K_m{}^m))\right] = 8(m-1)$, we get

$E\left[\frac{PE}{diam}(J(K_m{}^m))\right] = 8(m-1).$

**Adjacency energy of product - eccentricity divided by diameter of the complement of some regular graphs obtained by complete graph.**

**Theorem 4.3.1**

Let $\overline{D_2(K_{2m})}$ be the complement of edge deleting graph 2 of $K_{2m}$. Then $E\left[(\frac{PE}{diam})(\overline{D_2(K_{2m})})\right] = 4m$ , where $m \geq 2$.

**Proof:**

Let $\overline{D_2(K_{2m})}$ be the complement of edge deleting graph 2 of $K_{2m}$. Since $A(D_2(K_{2m})) = \begin{pmatrix} A(K_m) & 0 \\ 0 & A(K_m) \end{pmatrix}$, we get the adjacency matrix of product - eccentricity divided by diameter is,

$(\frac{PE}{diam})\ \overline{(D_2(K_{2m}))} = \begin{pmatrix} 0 & 2J \\ 2J & 0 \end{pmatrix}$, where $J = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$.

$= (\frac{SE}{diam})\ \overline{(D_2(K_{2m}))}$

Also since $E\left[(\frac{SE}{diam})(\overline{D_2(K_{2m})})\right] = 4m$, we get

$E\left[(\frac{PE}{diam})(\overline{D_2(K_{2m})})\right] = 4m.$

**Theorem 4.3.2**

Let $\overline{D_3(K_{2m})}$ be the complement of edge deleting graph 3 of $K_{2m}$. Then $E\left[(\frac{PE}{diam})(\overline{D_3(K_{2m})})\right] = 8(m-1)$ , where $m \geq 2$.

**Proof:**

Let $\overline{D_3(K_{2m})}$ be the complement of edge deleting graph 2 of $K_{2m}$. Since $(\frac{PE}{diam})(D_3(K_{2m})) = \begin{pmatrix} 0 & 3A(K_m) \\ 3A(K_m) & 0 \end{pmatrix}$, we get the adjacency matrix of product - eccentricity divided by diameter is,

$$(\frac{PE}{diam})\overline{(D_3(K_{2m}))} = \begin{pmatrix} 2A(K_m) & 2I_m \\ 2I_m & 2A(K_m) \end{pmatrix}$$

$= (\frac{PE}{diam})(J({K_m}^m))$ (by theorem (4.2.3))

Also since $E[(\frac{PE}{diam})(J({K_m}^m))] = 8(m-1)$, we get

$E[(\frac{PE}{diam})(\overline{D_3(K_{2m})})] = 8(m-1).$

**Theorem 4.3.3**

Let $\overline{J({K_m}^m)}$ be the complement of join of complete graph. Then $E\ [(\frac{PE}{diam})(\overline{J({K_m}^m)})] = 12(m-1)$ , where $m \geq 3$.

**Proof:**

Let $\overline{J({K_m}^m)}$ be the complement of join of pair of complete graph. Since $(\frac{PE}{diam})(J({K_m}^m)) = \begin{pmatrix} 2A(K_m) & 2I_m \\ 2I_m & 2A(K_m) \end{pmatrix}$, we get the adjacency matrix of product - eccentricity divided by diameter is,

$$(\frac{PE}{diam})\overline{(J({K_m}^m))} = \begin{pmatrix} 0 & 3A(K_m) \\ 3A(K_m) & 0 \end{pmatrix}$$

$= (\frac{PE}{diam})(D_3(K_{2m}))$ (by theorem 4.2.2)

Also since $E[(\frac{PE}{diam})(D_3(K_{2m}))] = 12(m-1)$, we get

$E[(\frac{PE}{diam})(\overline{J({K_m}^m)})] = 12(m-1).$

**Adjacency energy product - eccentricity divided by diameter of some irregular graphs**

**Theorem 4.4.1**

Let $F_m$ be a friendship graph. Then $E\left[(\frac{PE}{diam})(F_m)\right] = 2(2m-1) + \left(1 \pm \sqrt{2m+1}\right)$, where $m \geq 2$.

**Proof:**

Let $F_m$ be a friendship graph with $2m+1$ vertices. Then the adjacency matrix of product - eccentricity divided by diameter is,

$$(\frac{PE}{diam})(F_m) = \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 2 & \cdots & 0 & 0 \\ 1 & 2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & 0 & 2 \\ 1 & 0 & 0 & \cdots & 2 & 0 \end{bmatrix}.$$

Therefore, $P\left((\frac{PE}{diam})(F_m), \eta\right) = (\eta^2 - 2\eta - 2m)(\eta - 2)^{m-1}(\eta + 2)^m$

Hence $S_p\left[(\frac{PE}{diam})(F_m)\right] = \begin{pmatrix} 1+\sqrt{2m+1} & 1-\sqrt{2m+1} & 2 & -2 \\ 1 & 1 & m-1 & m \end{pmatrix}$

and $E\left[(\frac{PE}{diam})(F_m)\right] = 2(2m-1) + (1 \pm \sqrt{2m+1})$ .

**Theorem 4.4.2**

Let $Gl_m$ be a globe graph. Then $E\left[(\frac{PE}{diam})(Gl_m)\right] = 4\sqrt{2m}$.

**Proof:**

Let $Gl_m$ be a globe graph with $m+2$ vertices. Then the adjacency matrix of product - eccentricity divided by diameter is,

$$(\frac{PE}{diam})(Gl_m) = \begin{bmatrix} 0 & 0 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 0 & 2 & 2 & \cdots & 2 & 2 \\ 2 & 2 & 0 & 0 & \cdots & 0 & 0 \\ 2 & 2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & 0 & 0 & \cdots & 0 & 0 \\ 2 & 2 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

$= (\frac{SE}{diam})(Gl_m)$.

Also since $E\left[(\frac{SE}{diam})(Gl_m)\right] = 4\sqrt{2m}$, we get

$E\left[(\frac{PE}{diam})(Gl_m)\right] = 4\sqrt{2m}$.

**Theorem 4.4.3**

Let $K_{1,1,m}$ be a graph. Then $E\left[(\frac{PE}{diam})(K_{1,1,m})\right] = \frac{1}{2} + \frac{1}{4}(1 \pm \sqrt{32m+1})$ .

**Proof:**

Let $K_{1,1,m}$ be a graph with $m+2$ vertices. Then the adjacency matrix of product - eccentricity divided by diameter is,

$$(\frac{PE}{diam})(K_{1,1,m}) = \begin{bmatrix} 0 & 1/2 & 1 & 1 & \cdots & 1 & 1 \\ 1/2 & 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Therefore, $P\left((\frac{PE}{diam})(K_{1,1,m}), \eta\right) = (\eta)^{m-1}(2\eta + 1)(2\eta^2 - \eta - 4m)$.

Hence, $S_p\left[(\frac{PE}{diam})(K_{1,1,m})\right] =$

$$\begin{pmatrix} \frac{1}{4}(1-\sqrt{32m+1}) & \frac{1}{4}(1+\sqrt{32m+1}) & -\frac{1}{2} & 0 \\ 1 & 1 & 1 & m-1 \end{pmatrix}$$

and $E\left[(\frac{PE}{diam})(K_{1,1,m})\right] = \frac{1}{2} + \frac{1}{4}(1 \pm \sqrt{32m+1})$.

**Theorem 4.4.4**

Let $B_{m,m}$ be a bistar graph. Then $E\left[(\frac{PE}{diam})(B_{m,m})\right] = \frac{2}{3}(\pm 1 \pm \sqrt{9m+1})$ .

**Proof:**

Let $B_{m,m}$ be a bistar graph with $2m+2$ vertices. Then the adjacency matrix of product - eccentricity divided by diameter is,

$$(\frac{PE}{diam})(B_{m,m}) = \begin{bmatrix} 0 & 2 & \cdots & 2 & 4/3 & 0 & \cdots & 0 \\ 2 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 2 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 4/3 & 0 & \cdots & 0 & 0 & 2 & \cdots & 2 \\ 0 & 0 & \cdots & 0 & 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 2 & \cdots & 0 \end{bmatrix}.$$

Therefore,

$$P\left((\frac{PE}{diam})(B_{m,m}), \eta\right) = (\eta)^{2m-2}(3\eta^2 - 4\eta - 12m)(3\eta^2 + 4\eta - 12m)$$

Hence $S_p\left[(\frac{PE}{diam})(B_{m,m})\right] =$

$$\begin{pmatrix} \frac{2}{3}(-1-\sqrt{9m+1}) & \frac{2}{3}(1+\sqrt{9m+1}) & \frac{2}{3}(1-\sqrt{9m+1}) & \frac{2}{3}(\sqrt{9m+1}-1) & 0 \\ 1 & 1 & 1 & 1 & 2m-2 \end{pmatrix}$$

and $E\left[(\frac{PE}{diam})(B_{m,m})\right] = \frac{2}{3}(\pm 1 \pm \sqrt{9m+1})$ .

**Theorem 4.4.5**

Let $B^2{}_{m,m}$ be a square bistar graph. Then $E\left[(\frac{PE}{diam})(B^2{}_{m,m})\right] = \frac{1}{2} + \frac{1}{4}(1 \pm \sqrt{64m+1})$

**Proof:**

Let $B^2{}_{m,m}$ be a square bistar graph with $2m+2$ vertices. Then the adjacency matrix of product - eccentricity divided by diameter is,

$$(\frac{PE}{diam})(B^2{}_{m,m}) = \begin{bmatrix} 0 & 1/2 & 1 & 1 & \cdots & 1 & 1 \\ 1/2 & 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Therefore, $P\left((\frac{PE}{diam})(B^2{}_{m,m}), \eta\right) = (\eta)^{2m-1}(2\eta+1)(2\eta^2 - \eta - 8m)$.

Hence $S_p\left[(\frac{PE}{diam})(B^2{}_{m,m})\right] =$

$$\begin{pmatrix} \frac{1}{4}(1-\sqrt{64m+1}) & \frac{1}{4}(1+\sqrt{64m+1}) & -\frac{1}{2} & 0 \\ 1 & 1 & 1 & 2m-1 \end{pmatrix}$$

and $E\left[(\frac{PE}{diam})(B^2{}_{m,m})\right] = \frac{1}{2} + \frac{1}{4}(1 \pm \sqrt{64m+1})$ .

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