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Analysis Of the Ecological System Of the Bay Of Bengal Using Single And Two Species Models

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Abstract
One of the most ecological applications of the differential equations system is the predator-prey problem. Predator-prey model is effective and often used in the environmental science field because it allows researchers both to observe the dynamic of animal populations and make predictions as to how they will develop over time. In this paper, we will discuss about small fish and shark population of the Bay of Bengal by using the Malthusian model, logistic model, and predator-prey model. We also discuss the steady-state and stability analysis. In order to check the system's stability, corresponding eigenvalues are discussed for a set of equilibrium points. Finally, we will establish a numerical model to verify the analytical results.

Keywords: *Predator-prey model, steady-state, stability analysis, equilibrium points, numerical model.*

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I. Introduction

The principle of exponential growth for human population was first propounded by Malthus an English clergyman and political economist in the first edition of his book entitled an essay on the principle of population published in 1798. Belgian mathematician Verhulst introduced the logistic equation as a model for human population growth in 1838. He mentions this as a logistic growth. A Mathematical Model Applied to Understand the Dynamical to insufficient census data he was incapable of testing the validity of this model. A biologist Humberto D'Ancona did a statistical study on the numbers of each species sold at three main Italian ports in 1920. By his study of these fish species from 1914-1923, he came to a dramatic conclusion. He supposes that this predator-prey relevance between the sharks, rays, and fish was in their regular states outside of human interaction, namely before and after the war. An American mathematical biologist Lotka discovered and improved many of the same conclusions and models as Volterra around the same time. Finally, Vito Volterra wrote down a simple pair of differential equations to describe this system, which are known as predator-prey models. Some of the prey-predator models were discussed by Wangersky, 1978, Varma, 1977, Freedman, 1980, and Narayan, 2006. A population model with time delay was proposed by Kapur, in 1980. Volterra formulated a distributed time delay model for prey-predator ecological models. Kapur, 1980 discussed the solution in the closed form for that model. Perko, 2000 described the differential equation and dynamical system. Beretta and Takeuchi, 1987 described the global stability of single-species diffusion Volterra models with continuous time delays. Also, in the last decades, many researchers described a stage-structured predator-prey model with distributed maturation delay and harvesting (Kapur, J. N. 1980) and the Dynamical Behavior of Discrete Prey-Predator system with Scavenger (Raj, M. R., A.G. M. Selvam and R. Janagaraj. 2013).

A predator is an organism that eats another organism. The prey is the organism that the predator eats. Some examples of predator and prey are lion and zebra, bear and fish, and fox and rabbit. The words "predator" and "prey" are almost always used to mean only animals that eat animals, but the same concept also applies to plants: Bear and berry, rabbit and lettuce, grasshopper, and leaf. Predator and prey evolve together. The prey is part of the predator's environment and the predator dies if it does not get food, so it evolves whatever is necessary in order to eat the prey: speed, stealth, camouflage (to hide while approaching the prey), a good sense of smell, sight, or hearing (to find the prey), immunity to the prey's poison, poison (to kill the prey) the right kind of mouthparts or digestive system, etc. Likewise, the predator is part of the prey's environment and the prey dies if it is eaten by the predator, so it evolves whatever is necessary to avoid being eaten: speed, camouflage (to hide from the predator), a good sense of smell, sight or hearing (to detect the predator), thorns, poison (to spray when approached or bitten), etc. Mathematical modeling of the exploitation of biological resources is still a very interesting field of research. Mathematical modeling is frequently an evolving process. Regulated mathematical exploration can often be conducted to better understand bio-economic models. The unfastened imbalance in turn leads to the essential exchange [25]. The eventual model may or may not be free of any important imbalance but the exploration of the eventual model can thus be expected to publish significant and nontrivial features of the system. In the last decades, self-interest has been rising resolutely in the scheming and perusing of mathematical models of population interactions. In this paper, we will consider an environmental model containing two related populations-a prey population, such as fish, and a predator population, such as sharks.

The Bay of Bengal, the largest bay in the world forms the northeastern part of the Indian Ocean. Roughly triangular, it is bordered mostly by India and Sri Lanka to the west, Bangladesh to the north, and Myanmar to the east. The Bay of Bengal occupies an area of 2172000 square kilometers (839000 sq. mi). Some large rivers the Ganges and its tributaries such as the Padma and Hooghly, and the Brahmaputra and its tributaries such as the Irrawaddy River, Godavari, Mahanandi, Krishna, and Knavery flow into the Bay of Bengal, Among the important parts are Chennai, Chittagong, Kolkata, Mongla, Paradip, Tuticorin, Visakhapatnam, and Yangon. Many major rivers of the Indian subcontinent flow west to east before draining into the Bay of Bengal. The Ganges is the northernmost of these. Its main channel enters and flows through Bangladesh, where it is known as the Padme River, before joining the Meghna River. There are many fishes in the Bay of Bengal such as goal fish, fish, shrimp fish, cardinal fish, rays, sharks, spike fish, etc.

In this paper, we are going to discuss the ecological balance of the Bay of Bengal for sharks and small fish. First of all, we will discuss the single-species Malthusian model and the Logistic model. After these, we will discuss two species of predator-prey model. We will also discuss the stability analysis of the critical point and also will discuss the numerical simulation to verify the analytic result.

II. Analysis Of the Ecological System Of the Bay Of Bengal

On Behalf of One Species Model

First, we will discuss the small fish and shark population of the Bay of Bengal according to the Malthusian model and logistic model.

On Behalf of One Species Malthusian Model

In this study, we will first take a species of the Bay of Bengal, say spotted fish. Suppose $x = \phi(t)$ be the population of fish at time t. The variation of the population is the rate of change of x is proportional to the current value of x, i.e.

$$\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = ax \qquad (2.1.1)$$

where a is the proportionality constant, known as the growth rate of spotted fish and a is positive i.e. $a > 0$, then the population of fish is growing. On the other hand, if a is negative $.e.$ $a < 0$, then the population of the fish is decreasing. This formula was first published by the British economist **Thomas Malthus** in 1798 (Malthus, 1798)

Solving equation (2.1.1) with the initial condition when $t = 0$ then $x(0) = x_0$ we can write

$$\therefore x = x(t) = x_0 e^{at} \qquad (2.1.2)$$

Thus the Malthus model with $a > 0$ shows that the population of spotted fish will grow exponentially for all time. We can show this in Figure 2.1.

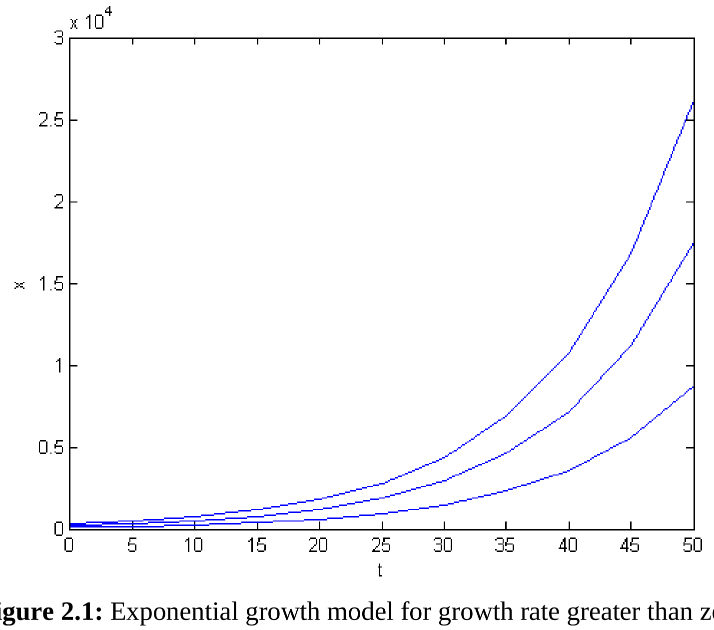

Figure 2.1: Exponential growth model for growth rate greater than zero.

From Figure 2.1, it is clear that for different initial populations and positive growth rate, the population will grow exponentially upward. On the other hand, if the growth rate is negative then the population will decrease exponentially. Now we will verify this model for the small fish population and shark population of the Bay of Bengal.

Analysis of the small fish population of the Bay of Bengal. From the census of small fish in the Bay of Bengal we have Table 2.1.

Source: HALDER, G.C. 2010. National Plan of Action for Small Fisheries in Bangladesh. pp. 75-89. In: Hussain, M.G. and Hog, M.E. (eds), Sustainable Management of Fisheries Resources of the Bay of Bengal. Support to BOBLME project, Bangladesh Fisheries Institute, Bangladesh. 122 p.

Table 2.1: Census of small fish in Bay of Bengal

Year	Population
1975	550000000
1985	650000000
1995	680000000
2005	670000000
2015	700000000

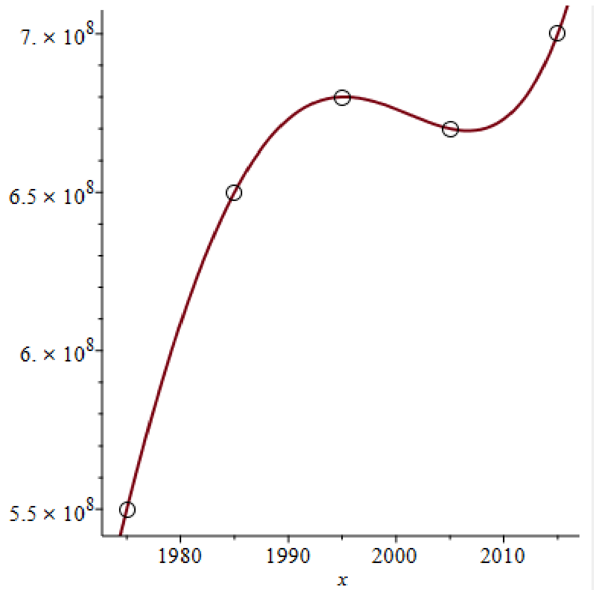

Figure 2.2: The real population graph of small fish from the census data in Table 2.1

In Figure 2.2, It is clear that the number of fish is not constant because sometimes it is increasing and sometimes it is decreasing.

Again, from the curve-fitting graph, we can get a better idea of the small fish population corresponding year.

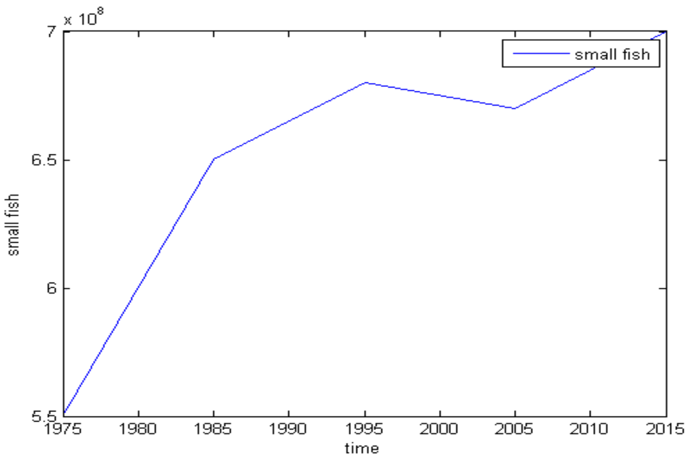

Figure 2.3: Curve fitting for the small fish population

In Figure 2.3, we have just shown the curve-fitted population graph of small fish in the Bay of Bangle. Here the population of the small fish is given in y-axis with corresponding years in x-axis. From Table 2.1 we see that in the year 1975, the small fish population of the Bay of Bengal was 5500000000 which is implemented in Figure 2.1. Again, in 1985, the small fish population was 65000000 which has also been implemented in Figure 2.1. Thus, all the populations to the corresponding year are implemented in the graph. If select a year from Figure 2.1, we can have the approximate population of small fish to the corresponding year.

An exponential graph for the small fish population of the Bay of Bengal.

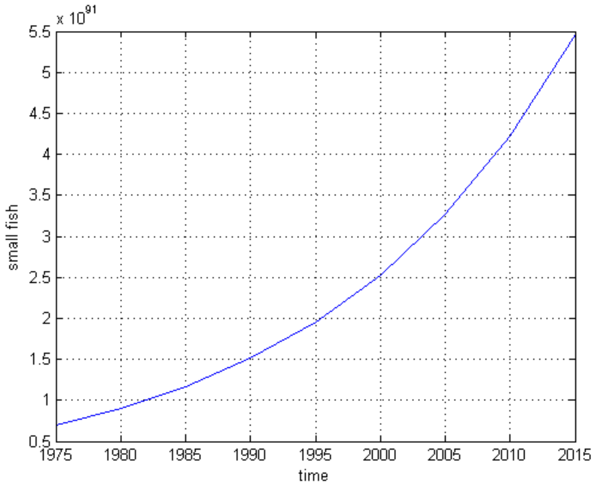

Figure 2.4: Exponential graph of small fish population of Bay of Bengal.

From Figure 2.2, Figure 2.3, and Figure 2.4 we may reach in a decision that the small fish population of the Bay of Bengal follows the Malthusian model. And it is also clear that the population of small fish is increasing day by day.

An analysis of the shark population of the Bay of Bengal. From the census of sharks in Bay of Bengal

Table 2.2: Census of Sharks in Bay of Bengal

Year	Population
1975	223000
1985	192250
1995	153400
2005	140000
2015	125400

Source: HALDER, G.C. 2010. National Plan of Action for Shark Fisheries in Bangladesh. pp. 75-89. In: Hussain, M.G. and Hog, M.E.(eds), Sustainable Management of Fisheries Resources of the Bay of Bengal. Support to BOBLME project, Bangladesh Fisheries Institute, Bangladesh. 122 p.

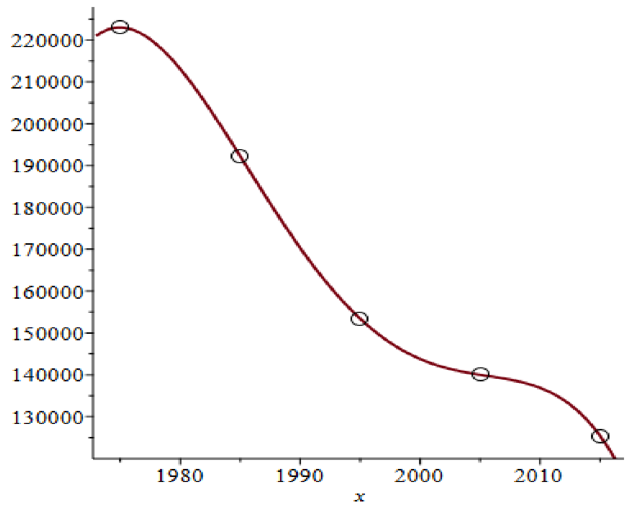

Figure 2.5: The real population graph of shark.

In Figure 2.5, we have just shown the real population graph of sharks in the Bay of Bengal. The population of the shark is given in y-axis with corresponding years in x-axis. From the Table 2.2, the year of 1975 the shark population of Bay of Bengal was 223000 which has implemented in the Figure 2.5. Again, in 1985, the shark population was 192250 which has also implemented in the Figure 2.5 and thus, from the Figure 2.5, we can get an idea of the number of sharks corresponding to year.
From the curve fitting of shark population, we can have a better idea of the current shark population of the graph.

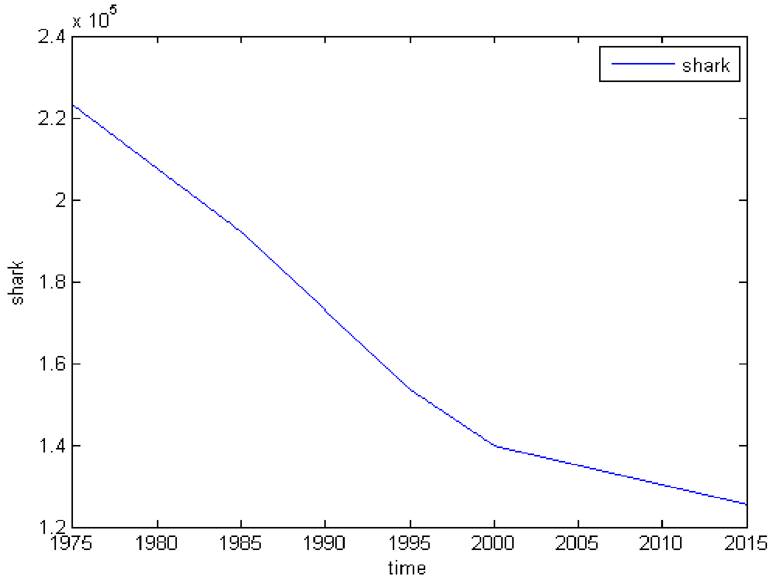

Figure 2.6: Curve fitting for the shark population

In Figure 2.6, we have just shown the curve-fitted population graph of sharks in the Bay of Bengal. The population of the shark is given in y-axis with corresponding years in x-axis. From Table 2.2 the year of 1975 the shark population of the Bay of Bengal was 223000 which is implemented in Figure 2.6. Again, in 1985, the shark population was 192250 which has also been implemented in Figure 2.6 And thus, all the populations to the corresponding year are implemented in the graph. If we select a year from Figure 2.6, we can have the approximate population of sharks to the corresponding year.

An exponential graph of the shark population of the Bay of Bengal.

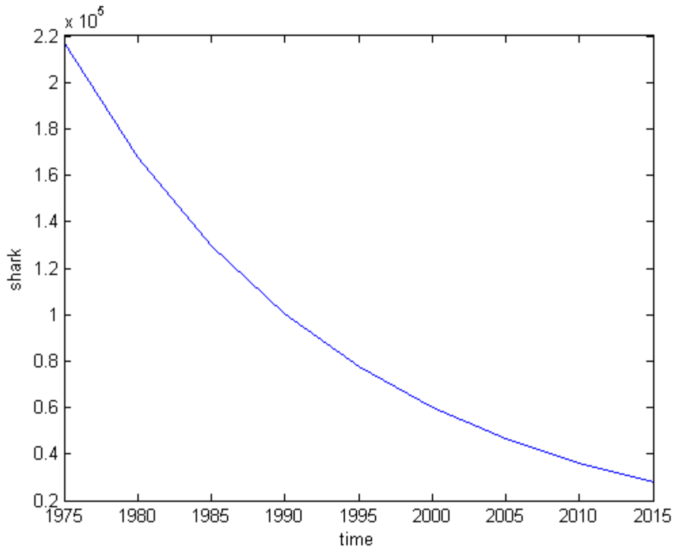

Figure 2.7: Exponential graph of shark population of Bay of Bengal.

From Figure 2.5, Figure 2.6, and Figure 2.7 we may reach in a decision that the shark population of the Bay of Bengal follows the Malthusian model, not fully but for a long time. And it is also clear that the population of sharks is decreasing day by day.

On Behalf of One Species Logistic Model

In reality, this model is unrealistic because environments impose limitations on population growth. A more accurate model postulates that the relative growth rate $\frac{x'}{x}$ decreases when x approaches the carrying capacity k of the environment. The corresponding equation is the so-called logistic differential equation:

$$\frac{dx}{dt} = ax\left(1 - \frac{x}{k}\right) \qquad (2.1.3)$$

The above equation is also known as **Verhulst's equation** because the Belgian Mathematician **P. F. Verhulst** introduced the equation (2.1.3) as a model for human population growth in 1838.

Solving equation (2.1.3) ∴ $x = \frac{k}{1+Ae^{-at}}$ where $A = \frac{K-X_0}{X_0}$

Here k is a carrying capacity and X_0 is an initial population.

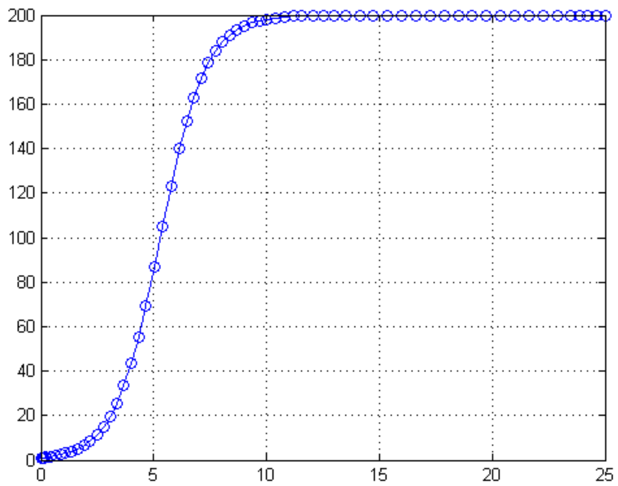

Figure 2.8: Logistic growth model, where Carrying capacity 200

From Figure 2.8, we see that when the time is positively infinite i.e., $t \to \infty$ then the population always approaches to the carrying capacity k. So, from this Figure, it is clear that a population cannot grow infinitely.

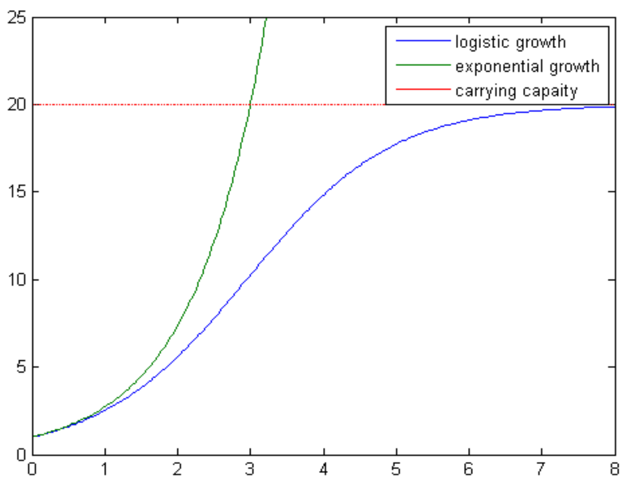

Figure 2.9: Comparable graph of logistic growth and exponential growth model

From Figure 2.9, we see the difference between the Malthusian model and the logistic model. From the graph, it is clear that the logistic model is better than the logistic model to describe a population.
Draw a Logistic Growth model for a small fish population

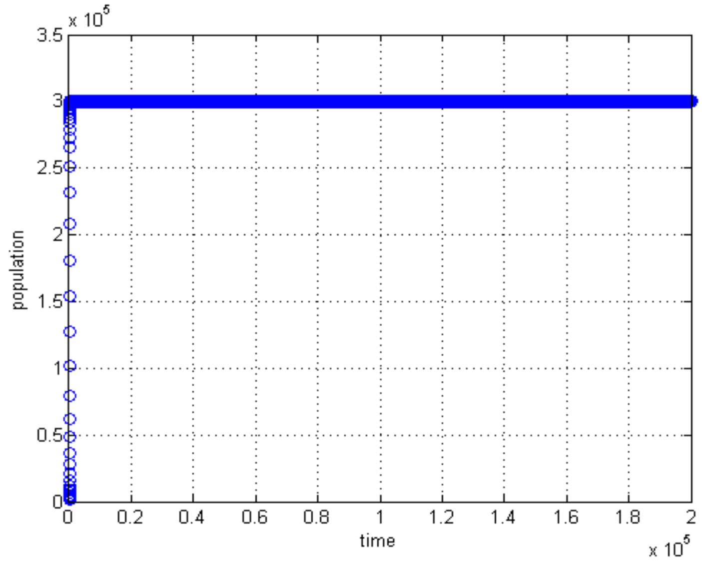

Figure 2.10: Logistic growth model for small fish, where Carrying capacity 3500000

From Figure 2.10, we see that with the increase of time, the population is increasing. When the time is positively infinite i.e., $t \to \infty$ then the population always approaches the carrying capacity $k = 350000$. So, from this Figure, it is clear that the small population cannot grow infinitely which is suitable for nature.

Draw a Logistic Growth model for the shark population

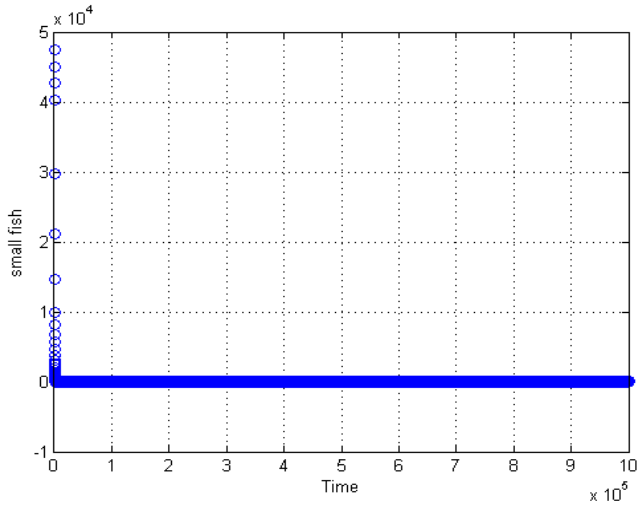

Figure 2.11: Logistic growth model for shark, where Carrying capacity 120000

From Figure 2.11, we see that with the increase of time, the population is decreasing. When the time is positively infinite i.e., $t \to \infty$ then the population always approaches the carrying capacity $k = 120000$. So, from this Figure, it is clear that the shark population cannot decline infinitely which is suitable for nature.

On Behalf of Two Species Model
Predator-Prey Model: Fish and Sharks

Predator-prey models are developed by focusing on primary population variables and basing the models on the assumption that other less impacting variables do not exist. Predation models focus on factors such as the 'natural' growth rate, or birth rate, and the carrying capacity of the environment in which the population resides. Once these variables are established, the major population-decreasing variables are added, namely in this study the predation, or kill rate. To keep models simplified, assumptions must be made that would be unrealistic in most natural predator-prey situations. Particularly, the following assumptions will be made in this paper

(i) The predator is completely dependent on the prey as the only food source.
(ii) The prey species has an unlimited food supply
(iii) There is no threat to the prey besides the predator species being studied

The predator-prey model is

$$\left.\begin{aligned}\frac{dx}{dt} &= ax - bxy\\ \frac{dy}{dt} &= -dx + cxy\end{aligned}\right] \qquad (2.2.1)$$

where $a, b, c, d > 0$

Here, $x(t)$ and $y(t)$ represent the number of prey and predators.

$\frac{dx}{dt}$ = Growth rate of prey (small fish) population.

$\frac{dy}{dt}$ = Growth rate of predator (shark) population.

a = Reproduction rate of prey.

b = Proportional to the number of prey that a predator can eat.

c = Amount of energy that a prey supplies to the consuming predator.

d = Death rate of predators.

Lotka-Volterra model

Let $x(t)$ denotes the number of a certain species of small fish(eaten by sharks) in a specific region of the Bay of Bengal and $y(t)$ denote the number of sharks in the same area. Then

$$\left.\begin{aligned}\frac{dx}{dt} &= ax - bxy\\ \frac{dy}{dt} &= -dx + cxy\end{aligned}\right], \text{ where } x(0) = x_0\,, y(0) = y_0 \qquad (2.2.2)$$

This system of equations is called the Lotka-Volterra model.

Note that in the absence of the predator (when $y = 0$), the prey population grows exponentially. If the prey is absent (when $x = 0$), the predator population would decay exponentially to zero due to starvation.

Now the solution of the system (2.2.2) becomes

$$lny - y + clnx - x = C$$

where C corresponds to initial conditions.

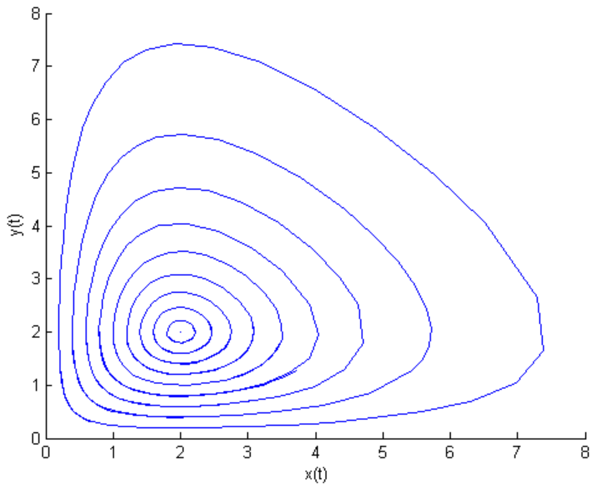

Figure 2.12: Closed phase plane trajectories for the Lotka-Volterra system.

Now the solution of the given differential equation describes a curve in the xy-plane as *t*, and this curve is known as trajectory. If the trajectory through (0,0) consists entirely of the point is an equilibrium point or a critical point.

i.e. at (0,0), the system (2.2.2) becomes

$0 = \frac{dx}{dt} = ax - bxy \Rightarrow (a - by)x = 0$

$\therefore x = 0$ or $a - by = 0 \Rightarrow y = \frac{a}{b}$
and $0 = \frac{dy}{dt} = -cx + dxy \Rightarrow (-c + dx)y = 0$
$\therefore y = 0$ or $-c + dx = 0 \Longrightarrow x = \frac{c}{d}$
Therefore the critical points are $(0,0)$ and $\left(\frac{c}{d}, \frac{a}{b}\right)$
The Jacobean of this system

$$J(x,y) = \begin{pmatrix} a - by & -bx \\ dy & -c + dx \end{pmatrix}$$

At $(0,0)$, the linearized system has a coefficient matrix

$$J(x,y) = \begin{pmatrix} a & -b \\ d & -c \end{pmatrix}$$

The eigenvalues are a and $-c$. Hence it is an unstable saddle point.
At $\left(\frac{c}{d}, \frac{a}{b}\right)$, the linearized system has coefficient matrix

$$J(x,y) = \begin{pmatrix} 0 & -\frac{bc}{d} \\ \frac{ad}{b} & 0 \end{pmatrix}$$

The characteristic equation is

$$J - \lambda I = 0$$

$$\begin{pmatrix} 0 & -\frac{bc}{d} \\ \frac{ad}{b} & 0 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} -\lambda & -\frac{bc}{d} \\ \frac{ad}{b} & -\lambda \end{pmatrix} = 0$$

$$\Rightarrow \lambda^2 + ac = 0$$

$$\therefore \lambda = \pm\sqrt{ac}\, i$$

Hence the required eigenvalues are $\pm\sqrt{ac}\, i$. So it is a marginally stable centre.

Variations of the basic Lotka-Volterra model

One obvious shortcoming of the basic predator-prey system is that the population of the prey species would grow unbounded, exponentially, in the absence of predators. There is an easy solution to this unrealistic behavior. We'll just replace the exponential growth term in the first equation by the two-term logistic growth expansion:

$$\frac{dx}{dt} = (ax - rx^2) - bxy \qquad x(0) = x_0$$
$$\frac{dy}{dt} = -cy + dxy \qquad y(0) = y_0$$

The functions $x(t)$ and $y(t)$ represent the populations of prey and predator at the time t, respectively. Also, the numbers x_0 , y_0 represent the initial sizes of prey and predator, respectively.

Here the birth rate of the prey population is negatively influenced by the natural death rate of prey which is shown by the term of $-rx^2$, where r is a non-negative constant and rx^2 is the natural death rate of prey.

Therefore, in the absence of predators, the first equation becomes the logistic equation. The prey population would instead stabilize at the environmental carrying capacity given by the logistic equation.

The above differential equation can be written as

$$\frac{dx}{dt} = x(a - rx - by)$$
$$\frac{dy}{dt} = y(-c + dx)$$

Now, we are going to solve the following equations at the equilibrium point or critical point $(0,0)$.
i.e., at $(0,0)$, $\frac{dx}{dt} = 0$ when $x = 0$ *or* $a - rx - by = 0 \Rightarrow x = \frac{a-by}{r}$

$\frac{dy}{dt} = 0$ when $y = 0$ or $-c + dx = 0 \Rightarrow x = \frac{c}{d}$
Therefore, one of our critical points is (0,0).
For $x = \frac{a-by}{r}$ when $y = 0$ then $x = \frac{a-b.0}{r} = \frac{a}{r}$
Thus, one of our critical points is $\left(\frac{a}{r}, 0\right)$
For $x = \frac{a-by}{r}$ when $x = \frac{c}{d}$
Then $\frac{c}{d} = \frac{a-by}{r} \Rightarrow y = \frac{ad-cr}{bd}$
Thus, one of our critical points is $\left(\frac{c}{d}, \frac{ad-cr}{bd}\right)$.
Now, to study the stability of the equilibrium points we first need to find the Jacobean matrix which is:

$$J(x,y) = \begin{pmatrix} a - 2rx - by & -bx \\ dy & -c + dx \end{pmatrix}$$

To study the stability at $(0,0)$

$$J(0,0) = \begin{vmatrix} a - \lambda & 0 \\ 0 & -c - \lambda \end{vmatrix}$$

$\therefore \lambda = a, \lambda = -c$
This is semi-stable because one eigenvalue is negative and one is positive
To study the stability at $\left(\frac{a}{r}, 0\right)$

$$J\left(\frac{a}{r}, 0\right) = \begin{pmatrix} -a - \lambda & -\frac{ab}{r} \\ 0 & -c + \frac{ad}{r} - \lambda \end{pmatrix} = (-a - \lambda)\left(-c + \frac{ad}{r} - \lambda\right)$$

$$\therefore \lambda = -a, \quad \lambda = \frac{-cr + ad}{r}$$

This is stable if $\lambda = \frac{-cr + ad}{r} < 0$ (i.e. $ad < cr$).
and unstable if $\lambda = \frac{-cr + ad}{r} > 0$ $(i.e.\ ad > cr)$.
To study the stability at $\left(\frac{c}{d}, \frac{ad-cr}{bd}\right)$

$$J\left(\frac{c}{d}, \frac{ad - cr}{bd}\right) = \begin{pmatrix} a - 2r\frac{c}{d} - b\frac{ad - cr}{bd} - \lambda & \frac{-bc}{d} \\ \frac{ad - cr}{b} & -\lambda \end{pmatrix}$$

$$= \begin{pmatrix} -r\frac{c}{d} - \lambda & \frac{-bc}{d} \\ \frac{ad - cr}{b} & -\lambda \end{pmatrix}$$

$$= \frac{\lambda cr + \lambda^2 d - c^2 r + cad}{d}$$

$\therefore \lambda = \frac{-cr + \sqrt{c^2r^2 + 4dc^2r - 4cad^2}}{2d}$, $\lambda = \frac{-cr - \sqrt{c^2r^2 + 4dc^2r - 4cad^2}}{2d}$
If we simplify a little more,
$\therefore \lambda = \frac{-cr + \sqrt{c^2r^2 + 4dc^2r - 4cad^2}}{2d}$, $\lambda = -\frac{1}{2}\frac{cr - i\sqrt{-c^2r^2 - 4dc^2r + 4cad^2}}{d}$
$\therefore \lambda = \frac{-cr - \sqrt{c^2r^2 + 4dc^2r - 4cad^2}}{2d}$, $\lambda = -\frac{1}{2}\frac{cr + i\sqrt{-c^2r^2 - 4dc^2r + 4cad^2}}{d}$
This point is stable because both of the real parts are negative. The imaginary numbers imply that it will be periodic.

Checking The Variation of Eigenvalues in Different Cases
Case 1: Stability of prey$(ad > cr)$

In this case, $\frac{a}{r}$ is the stable point for the prey population in a predator-free world, and $\frac{c}{d}$ the term of is the critical point for the prey population living with predators. This case holds true without initial conditions until $x_0 > 0$ and $y_0 > 0$.

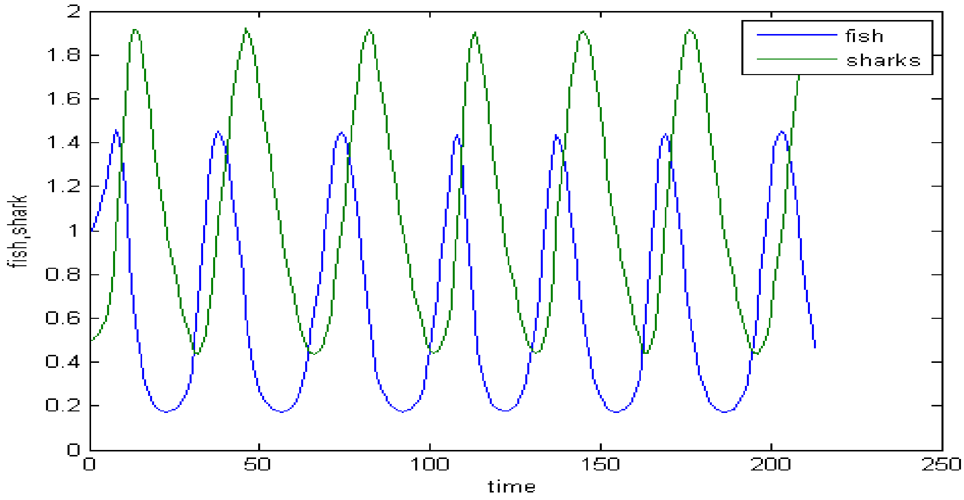

Figure 2.13: Variation of shark and fish against time where
$u(x,y) = x(6-2x-4y)$, $v(x,y) = y(-3+5x), x(0) = 1 , y(0) = .5$

Figure 2.13, shows the critical point for the fish population living with sharks. Because when sharks are increasing at the same time fish are decreasing.

Case 2: Stability of prey$(ad < cr)$

For the second case, when $\frac{a}{r}$ is less than $\frac{c}{d}$, we arrive at an interesting conclusion. When $\frac{a}{r} < \frac{c}{d}$, then for the predator equation, the critical point becomes a negative number. So $y = \frac{-cr+ad}{bd}$ results in a negative number. So, in the second case, the predator population will die out regardless of the initial conditions. Therefore, the solution would be converged to the predator-free stable point.

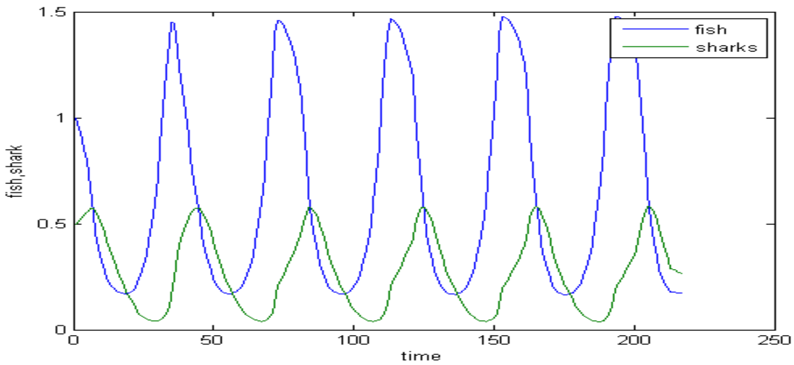

Figure 2.14: Variation of shark and fish against time where
$u(x,y) = x(2-6x-4y)$, $v(x,y) = y(-3+5x), x(0) = 1, y(0) = .5$

Therefore, Figure 2.14 shows that the shark population will die out regardless of the initial conditions. So the solution would converge to the shark-free stable point.

Case 3: When $(a = b = 1)$

For the third case, we pay attention to the question: What will happen if all of the constants are the same? A simple glance at the equations tells us that this would be similar to the second case: We get $\frac{a}{r} = \frac{c}{d} = 1$, yet the critical point would be $(1,0)$ which is on the y-axis and identical to the stable point for the prey population in a predator-free world. So here the predator dies out again.

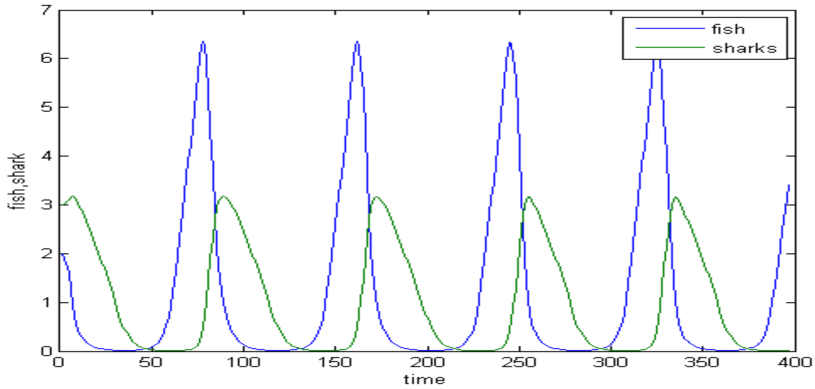

Figure 2.15: Variation of shark and fish against time where
$u(x, y) = x(1 - 1x - 1y)$, $v(x, y) = y(-1 + 1x)$, $x(0) = 2, y(0) = 3$

Figure 2.15, shows that the critical point would be $(1,0)$ which is on the y-axis and identical to the stable point for the prey population in a predator-free world. So here the sharks die out again.

Case 4: Stability of both species$(r = 0)$

Let, $r = 0$ which implies it is the model in the simplest form of the prey-predator model. The new equations look like this:

$$\frac{dx}{dt} = x(a - by)$$
$$\frac{dy}{dt} = y(-c + dx)$$

So the critical point becomes $(\frac{c}{d}, \frac{a}{d})$ and we get an ellipse around the above critical point. The shape and size depend on the constants and initial conditions. So, both the prey and predator populations increase and decrease in the above cyclic pattern.

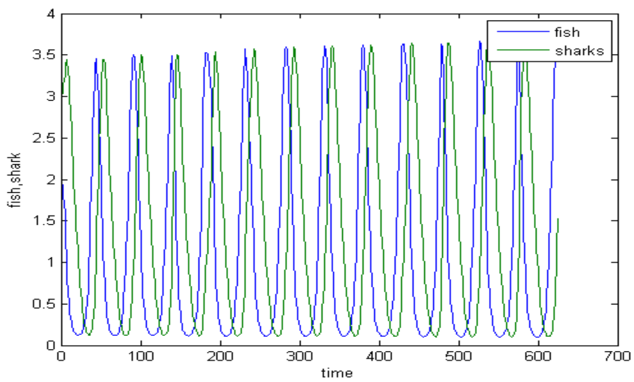

Figure 2.16: Variation of shark and fish against time where
$u(x, y) = x(2 - 0x - 1y)$, $v(x, y) = y(-1 + 1x)$, $x(0) = 2, y(0) = 3$

Figure 2.16, shows that both the fish and shark populations wax and wane in a cyclic pattern with the sharks lagging behind the fish.

Numerical Simulation

Besides analytic findings, numerical simulation is also important, because simulations can be used to validate the analytic findings. For various choices of the parameters of the model, we have performed the simulations using MATLAB. It is observed that they are in good agreement with our analytic findings.

To point out the effects of various parameters on the predator-prey model, the growth rate of prey a, death rate of predator c and interaction rate of predator d and prey c are computed. The variation of prey population along the y-axis and the variation of predator population along the *x*-axis are shown in Figures 2.17 - 2.20

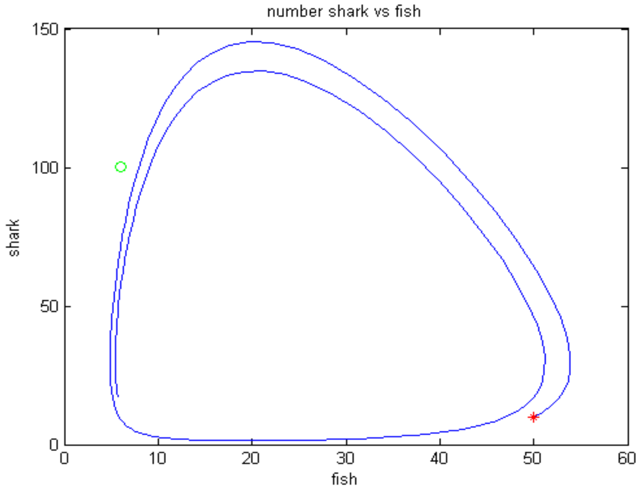

Figure 2.17: Variation of shark against small fish for $x(0) = 50, y(0) = 10,$
$a = .03, b = .001, c = .1, d = .005$

Figure 2.17, displays that the trajectory in the phase space is cyclic inside the equilibrium state, this means that this equilibrium point is unstable for such type of parameter.

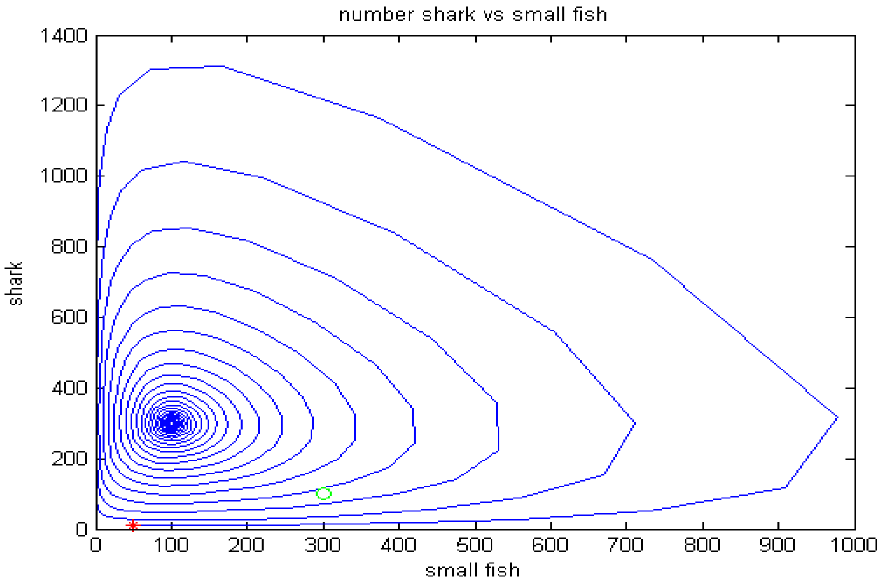

Figure 2.18: Variation of shark against small fish for $x(0) = 50, y(0) = 10,$
$a = .3, b = .001, c = .1, d = .001$

Figure 2.18, displays that the trajectory in the phase space is cyclic and it goes away from the equilibrium. This means that this equilibrium point is unstable for such type of parameter.

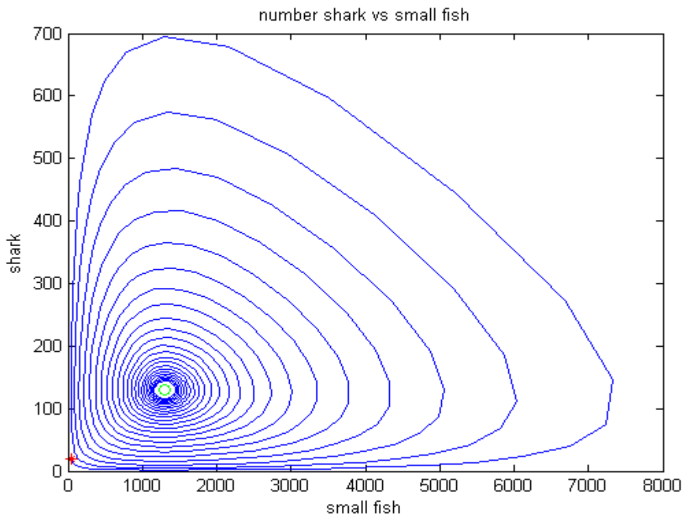

Figure 2.19: Variation of shark against small fish for $x(0) = 50, y(0) = 20$, $a = 1.3, b =..01, c = 1.3, d = .001$

Figure 2.19, displays that the trajectory in the phase space is cyclic and it is returned to the equilibrium directly. This means that this equilibrium point is stable for such type of parameter.

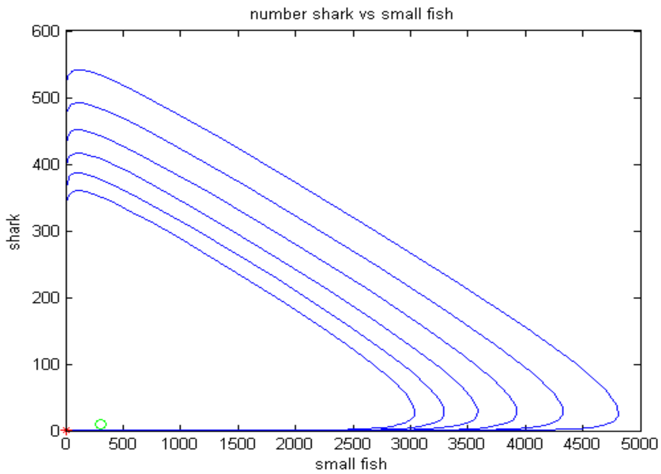

Figure 2.20: Variation of shark against small fish for $x(0) = 1 \times 10^{-3}$, $y(0) = 1 \times 10^{-4}, a = .3, b = .01, c = .1, d = .001$

Figure 2.20, shows that when the initial conditions close to (0,0) the trajectory converge to the first equilibrium. This confirms the unstable property of the (0,0) point. So the equilibrium point is unstable.

III. Conclusion

The Lotka-Volterra Predator-Prey Model is a rudimentary model of the complex ecology of this world. It assumes just one prey for the predator and vice versa. It also assumes there is a influences like disease, changing conditions, pollution, and so on. However, the model can be expanded to include other variables, and we have the Lotka-Volterra predator-prey model, which models help to describe the ecological balance in the Bay of Bengal.

In this paper, we consider the dynamical behavior of the predator-prey model then we establish a modified Lotka-Volterra predator-prey model then we can polish the equations by adding more variables and getting a better content of the ecology. We also analyze the stability of the predator-prey model. Finally, it is graphically shown that the analytic result uses different parameter values.

This model is an excellent tool to teach the principals involved in ecology and to show some rather counter-initiative results. It also shows a special relationship between biology and mathematics. This model is similar to the models of orbits with spirals, contours, and curves.

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