IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-0661,p-ISSN: 2278-8727, Volume 20, Issue 1, Ser. 3 (Jan. – Feb. 2024), PP 47-59
www.iosrjournals.org

Ion acoustic solitary waves of plasmas with relativistic thermal ions and degenerated electrons, positrons for weakly relativistic regime

Mst.Ayrin Aktar
Assistant Professor, Department of Mathematics, Begum Rokeya University, Rangpur
Email: ayrin.amath.ru@gmail.com
Md. Mashiar Rahman
Assistant Professor,Department of Mathematics, Begum Rokeya University, Rangpur
Email: moshiur.math@brur.ac.bd
Mosammat Fahmida Afroz Bhuiyan,
Assistant professor,Department of Mathematics,Siddheswari College,Dhaka, Bangladesh.
Email: sumifahmidaafroz@gmail.com

Abstract:
This research work investigates the nonlinear dissemination of electrostatic ion acoustic solitary waves (IASWs) for weakly relativistic regime by considering the thermal ions speed is comparable with the speed of light and degenerates electrons, positrons. It is assumed that the densities of electrons and positrons follow the Fermi law. The Korteweg-de Varies (KdV) equation is derived to analyze the solation solution for both cases of ultra-relativistic and non-relativistic degenerated electrons and positrons by using the well known reductive perturbation technique. It is the angsts method which is employed to carry out the integration of this equation. The amplitude, width and phase velocity of solutions, and electrostatic IASWs together with the influence of unperturbed Fermi type positron to electron density ratio, Fermi type electron to positron temperature ratio, ion to electron temperature ratio and relativistic stream factor are described (graphically) to study the small amplitude localized in weakly relativistic of IASWs for an un-magnetized three component plasma system consisting of Fermi type electrons and positrons in space plasma as well as some astrophysical compact objects.
Keywords: *Electrostatic propagations; Solution; Fermi-type electrons and positrons; The angsts method*

Date of Submission: 04-02-2024 Date of Acceptance: 14-02-2024

I. Introduction

The three component un-magnetized or magnetized plasmas for relativistic regimes are one of the most enigmatic constructs in the present day of physics. They draw physicist's attention as the paradigmatic objects to test possible plasma theories of relativistic regimes. Although still there is no convincing observational data in favor of conclusively proving the existence of electron-positron-ions plasmas in relativistic regimes, there exist certainly sufficient evidences that make the study of such objects and the effects on their environment a matter of great importance to space physics as well as some astrophysics compact objects. It is therefore of interest to formulate plasma physics problems in the context of relativity, when the particle velocities are compared with light speed. It can be applied in observing of many issues of space physics as well as astrophysical environments such as plasma sheet boundary layer of earth's magnetosphere [1], in laser-plasma interaction [2, 3], the polar regions of neutron stars [4], in pulsar magnetospheres [5], active galactic nuclei [6], at the center of Milky Way galaxy [7], in primordial universe and in the process of expanding universe [5, 8, 9] etc. In many previous literatures [10-18], researchers have described many issues concerning the electron-positron-ion plasmas. Most of these studies are used to analyze the basic properties of linear and nonlinear physical phenomena to the three or two component plasmas system for non-relativistic regime. Furthermore, the electrostatic effects of ion acoustic (IA), dust ion acoustic (DIA), electron acoustic (EI) waves, etc for relativistic regime can no longer be invisible. Thus, the basic properties of ion acoustic waves or electron acoustic waves for relativistic regime have appeared as one of the most interesting research areas for the scholars day by day.

It is observable that a very few works has been carried out for investigating the basic properties of nonlinear waves for a fully ionized collision less e-p-i plasma system for relativistic regimes. For instance, Shah et al. [19] have analyzed the electrostatic compressive and rare active shocks and solation relativistic plasmas occurring in polar regions of pulsar, Gill et al. [20] have described the influences of the plasma parameters on the propagation of ion acoustic solation in weakly relativistic magnetized electron-positron-ion plasma, Trebuchet and Debarring [21] have described the ion-acoustic solitary waves in fully relativistic ion-election-positron plasma, Masood and Rizvi [22] have investigated two dimensional electrostatic shock waves in e-p-i plasmas and Saeed et al. [23] have investigated the solution propagation in relativistic three component plasma system which consists of Boltzmann electron, positron and relativistic thermal ions. Therefore, the theoretical and numerical study of nonlinear structures, namely solitary structure, shock structure, double layers associated with IA, DIA, EI waves for relativistic regimes have received a great interest in space, astrophysical and laboratory plasmas.

Recently, quantum plasmas have drawn reasonable attention of researcher scholars, due to their significant applications in astrophysical environments [24], in high intense laser solid density experiments [25], in ultra cold plasmas [26], in micro-plasmas [27] and in micro electronics devices [28]. Chandrasekhar [29] has been developed the mathematical expression in the general form $P = \frac{\pi m_e^4 c^5}{3h^3}[r(2r^2-3)\sqrt{1+r^2} + 3\sinh^{-1} r]$ for electron degeneracy pressure in a fully degenerate Fermi gas, where h , c and $r = p_{Fe}/mc$ are the Plank constant, speed of light and normalized relativity parameter respectively. Then the electron number densities can be defined as $n_e = (8\pi m_e^3 c^3 / 3h^3)$. It is notated that $P = (1/20)(3/\pi)^{2/3}(h^2/m_e)n_e^{5/3}$ as $r \to 0$ and $P = \frac{1}{8}(\frac{3}{\pi})^{1/3} hcn_e^{4/3}$ as $r \to \infty$. Therefore, the ratio of Fermi type electron and positron temperature for a three dimensional non-relativistic zero temperature with degenerate electrons can be defined as $\sigma_F = \alpha^{-2/3}$, whereas $\sigma_F = \alpha^{-1/3}$ for ultra-relativistic Fermi-gas [30]. There is amount of literature [31-38], where the authors have described in concerning many issues for plasmas by considering degenerates electrons and positrons. Zebra et al. [31] have investigated a collision less e-p-i quantum plasma with ultra-relativistic degenerate electrons and positrons and studied the existence regions for ion solitary pulses. Roy et al. [32] thought out an e-p-i plasma and meticulously examined the principal features of solitary waves and double layers. Noah [33] have studied the nonlinear propagation of PA waves in an e-p plasma with an electron beam and scrutinized that the maximum amplitude of the wave decreases as the positron temperature increases and the region of PA waves spreads as the positron temperature increases. Zohar et al. [34] have assumed dense plasma containing non-relativistic degenerate cold ion and both non-relativistic and ultra-relativistic degenerate electron fluids and studied the fundamental features of electrostatic shock structures. In contrary to the classical plasma, it is illustrious that quantum plasma (containing electrons, positrons and ions) exhibits the properties of high-plasma particle number densities and low-temperatures [35]. Mehdipoor and Neirameh [36] have investigated the nonlinear propagation of IA waves in an ideal plasmas containing degenerate electrons. More recently, Hossen et al. [37, 38] have considered relativistic quantum plasma and investigated the profiles of solitary, shock and double layer structures. Since, the astrophysical quantum plasmas can be confined by stationary heavy ions. So, the effect of the heavy ions has to be taken into account, especially for astrophysical observations (such as white dwarfs, neutron stars, black holes, etc), where the degenerate plasma pressure and heavy ions play an important role in the formation and stability of the existing waves. All of the authors did not consider the effect of heavy ions, where the ions speed is comparable to speed of light and their different charging situations which can significantly modify the propagation of solitary structures. As far as we know, no theoretical and numerical investigation has been made on study the IAWs for weakly relativistic regimes in a three component plasma system by considering relativistic thermal ions and degenerate electrons and positrons. Therefore, the main motivation of our research work to study the basic features of IASWs by deriving the KdV equation in a dense plasma containing electrons, positions for both cases of non-relativistic as well as ultra-relativistic Fermi gas and relativistic thermal ions.

This paper is organized as follows: In Section 2, we summarize the proposed governing fluids equations in a fully ionized un-magnetized collision less plasma system for weakly relativistic regime. The derivation and analytical solution to the KdV equation is given in Section 3. We investigate the nonlinear propagation of IASWs found in the influence of the related plasma parameters for both cases of non-relativistic degenerate electrons and ultra-relativistic Fermi gas in Section 4. Finally, we present our conclusion in section 5.

II. Governing Fluid Equations

We consider the nonlinear propagation of a fully ionized un-magnetized collision less plasma system for weakly relativistic regime consisting of degenerate electrons, positrons and relativistic thermal ions. The densities of degenerate electrons and positrons are assumed to be obeying form the Fermi laws. At equilibrium, the charge neutrality condition is $n_{e0} = n_{p0} + n_{i0}$, where n_{i0}, n_{e0} and n_{p0} are unperturbed ion , unperturbed Fermi type electron and unperturbed Fermi type positron densities respectively. According to the conservation law of mass and momentum, the basic normalized ion continuity and momentum equations can be written as follows:

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial (\gamma u_i)}{\partial t} + u_i \frac{\partial (\gamma u_i)}{\partial x} + \frac{\partial \varphi}{\partial x} + \frac{\Omega}{n_i}\frac{\partial n_i}{\partial x} = 0 \tag{2}$$

where, n_i is the ion number density normalized by unperturbed ion density, u_i is ion fluid velocity normalized by ion acoustic speed $C_s = (T_e / m_i)^{1/2}$ (m_i is the mass of ions and T_e is the electron temperature), φ is electrostatic potential normalized by (T_e / e) (*e* is the electron charge), $\Omega = T_i / T_e$ is the ion to electron temperature ratio, *x* is the space variable normalized by the Debye length $\lambda_{De} = (T_e / 4\pi n_{eo} e^2)^{1/2}$ and *t* is the time variable normalized by plasma frequency $\omega_{pi}^{-1} = (m_i / 4\pi n_{eo} e^2)^{1/2}$. The Lorentz factor for relativistic thermal ions is defined as $\gamma = (1 - u_i^2 / c^2)^{-1/2}$, where c is the speed of light. In the case of weakly relativistic regime, it can be defined as $\gamma \approx 1 + u_i^2 / 2c^2$.

The system is closed via the following Poisson's equation:

$$\frac{\partial^2 \varphi}{\partial x^2} = n_e - n_p - n_i , \tag{3}$$

where, n_e and n_p indicates the densities of degenerate positrons and electrons respectively. The degenerate densities of electron and positrons are considered as Fermi types, which are obtained from the Thomas Fermi laws. Thus, the normalized densities of Fermi type electron and positron can be written in the following term:

$$n_e = (1 + \varphi)^{3/2} \text{ and } n_p = p(1 - \sigma\varphi)^{3/2} \tag{4}$$

where, p and σ denotes the unperturbed positron to electron density ratio and the Fermi type electron to positron temperature ratio respectively. It is notable that the ratio of Fermi type electron and positron temperature have defined as $\sigma = \alpha^{-2/3}$ for non-relativistic zero temperature with degenerate electrons and $\sigma = \alpha^{-1/3}$ for ultra-relativistic Fermi-gas. The normalized densities of electron and positron for a proper fraction of φ can be expanded in following form:

$$n_e = [1 + \frac{3}{2}\varphi + \frac{3}{8}\varphi^2 + \Lambda\,\Lambda\,\Lambda\,] \text{ , } n_p = p[1 - \frac{3}{2}\sigma\varphi + \frac{3}{8}\sigma^2\varphi^2 - \Lambda\,\Lambda\,] \tag{6}$$

It is notable that the basic fluid equations can be reduced in analyzing the nonlinear propagation of a three component un-magnetized plasma system consisting of Fermi type electrons, positrons and thermal ions as in the limit $\gamma = 1$.

III. Derivation and Analytical Solution to the Korteweg-de Vries (KdV) Equation

In this section, we have derived the KdV equation from the proposed governing fluid equations that are presented in section 2 via the well known reductive perturbation approach [36] and evaluated the analytical solution to this equation via the ansatz method [39, 40].

Firstly, we can be combined *x* and t by the following starched variables:

$$\eta = \varepsilon^{1/2}(x - \lambda_0 t),\ \ \tau = \varepsilon^{3/2} t, \tag{7}$$

where ε is a very small positive quantity and λ_0 is the phase velocity to be determined later. By setting eq. (7), the equations (1) to (3) can be converted in the following form:

$$\varepsilon \frac{\partial n_i}{\partial \tau} - \lambda_0 \frac{\partial n_i}{\partial \eta} + \frac{\partial}{\partial \eta}[n_i u_i] = 0 \tag{8}$$

$$\varepsilon \frac{\partial (\gamma u_i)}{\partial \tau} - \lambda_0 \frac{\partial (\gamma u_i)}{\partial \eta} + u_i \frac{\partial (\gamma u_i)}{\partial \eta} + \frac{\partial \varphi}{\partial \eta} + \frac{\Omega}{n_i}\frac{\partial n_i}{\partial \eta} = 0 \tag{9}$$

$$\varepsilon \frac{\partial^2 \varphi}{\partial \eta^2} = (1+\varphi)^{3/2} - p(1-\sigma\varphi)^{3/2} - n_i, \tag{10}$$

Secondly, we can be expanded the quantities n_i, u_i and φ as follows:

$$n_i = (1-p) + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \Lambda\ \Lambda\ \Lambda\ \Lambda$$
$$u_i = u_{i0} + \varepsilon u_i^{(1)} + \varepsilon^2 u_i^{(2)} + \Lambda\ \Lambda\ \Lambda \tag{11}$$
$$\varphi = \varepsilon \varphi^{(1)} + \varepsilon^2 \varphi^{(2)} + \Lambda\ \Lambda\ \Lambda\ \Lambda\ .$$

By substituting eq. (11) into the equations (8) to (10), we can be devolved equations in different order of ε. The lowest order of ε gives the following equations:

$$(\lambda_0 - u_{i0}) n_i^{(1)} = (1-p) u_i^{(1)}, \tag{12}$$

$$u_i^{(1)} = \frac{(\lambda_0 - u_{i0})}{[(\lambda_0 - u_{i0})^2 \gamma_{1i} - \Omega]} \varphi^{(1)}, \tag{13}$$

$$n_i^{(1)} = \frac{3}{2}(1+p\sigma)\varphi^{(1)} \tag{14}$$

where $\gamma_{1i} = 1 + 1.5\beta^2, \beta = u_{i0}/c$. By simplifying (12) equation to (14), the linear phase velocity can be obtained as follows:

$$\lambda_0 = u_{i0} + \sqrt{\frac{3\Omega(1+p\sigma) + 2(1-p)}{3\gamma_{1i}(1+p\sigma)}}. \tag{15}$$

The next order of ε gives the following nonlinear evolution equations (NLEEs):

$$\frac{\partial n_i^{(1)}}{\partial \tau} - (\lambda_0 - u_{i0}) \frac{\partial n_i^{(2)}}{\partial \eta} + (1-p)\frac{\partial u_i^{(2)}}{\partial \eta} + \frac{\partial}{\partial \eta}[n_i^{(1)} u_i^{(1)}] = 0, \tag{16}$$

$$\gamma_{1i} \frac{\partial u_i^{(1)}}{\partial \tau} - \gamma_{1i}(\lambda_0 - u_{i0}) \frac{\partial u_i^{(2)}}{\partial \eta} + (\gamma_{1i} - 2\gamma_{2i} \frac{\lambda_0 - u_{i0}}{u_{i0}}) u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \eta} + \frac{\partial \varphi^{(2)}}{\partial \eta} + \frac{\Omega}{1-p}\frac{\partial n_i^{(2)}}{\partial \eta} - \frac{\Omega}{(1-p)^2} n_i^{(1)} \frac{\partial n_i^{(1)}}{\partial \eta} = 0 \tag{17}$$

$$\frac{\partial^2 \varphi^{(1)}}{\partial \eta^2} = \frac{3(1+p\sigma)}{2}\varphi^{(2)} + \frac{3(1-p\sigma^2)}{8}(\varphi^{(1)})^2 - n_i^{(2)} \tag{18}$$

where $\gamma_{2i} = 1.5\beta^2$. Eliminating the second order perturbed quantities from the above NLEEs along with (12) to (15), we get

$$\frac{\partial \varphi^{(1)}}{\partial \tau} + L\varphi^{(1)} \frac{\partial \varphi^{(1)}}{\partial \eta} + M \frac{\partial^3 \varphi^{(1)}}{\partial \eta^3} = 0, \tag{19}$$

This is the well known K-dV equation and it is very useful for investigating the nonlinear propagation of IASWs for weakly relativistic regime in an un-magnetized three component plasma system. The nonlinear coefficient L and dispersive coefficients M are defined as follows:

$$L = [(1-p)(\frac{\lambda_0 - u_{i0}}{\kappa}) + (\frac{\lambda_0 - u_{i0}}{2\kappa\gamma_{1i}})(\gamma_{1i} - 2\gamma_{2i}\frac{\lambda_0 - u_{i0}}{u_{i0}}) - \frac{\kappa(1-p\sigma^2)}{4\gamma_{1i}(\lambda_0 - u_{i0})(1+p\sigma)} - \frac{\Omega}{2\kappa\gamma_{1i}(\lambda_0 - u_{i0})}\ ], \tag{20}$$

and

$$M = \frac{\kappa^2}{2\gamma_{1i}(1-p)^2(\lambda_0 - u_{i0})}\ , \tag{21}$$

where $\kappa = [\gamma_{1i}(\lambda_0 - u_{i0})^2 - \Omega]$.

To get the stationary solutions to the K-dV equation (19), we can be combined the starched variables by a compound variable ξ as $\varphi^{(1)}(\eta,\tau) = \varphi^{(1)}(\xi),\ \xi = \eta - V\tau,$ where V is the constant speed of the moving frame. Then the eq. (19) can be converted to a nonlinear ordinary differential equation as follows:

$$-V\frac{d\varphi^{(1)}}{d\xi}+\frac{L}{2}\frac{d(\varphi^{(1)})^2}{d\xi}+M\frac{d^3\varphi^{(1)}}{d\xi^3}=0, \qquad (22)$$

Integrating (22) once with regards to ξ and choosing the integrated constant to be zero for time homogeneity, we have

$$-V\varphi^{(1)}+\frac{L}{2}(\varphi^{(1)})^2+M\frac{d^2\varphi^{(1)}}{d\xi^2}=0, \qquad (23)$$

To investigate the bright Soliton, the solution of the eq. (23) can be considered according to the ansatz method as

$$\varphi^{(1)}(\xi)=\frac{A}{\cosh^q(B\xi)} \qquad (24)$$

where A and B denotes the amplitude and inverse width of the soliton respectively.

The first and second derivatives of $\varphi^{(1)}(\xi)$ with respect to ξ are given below:

$$\begin{aligned}\frac{d\varphi^{(1)}(\xi)}{d\xi}&=-ABq\cosh^{-1-q}(B\xi)\sinh(B\xi)\\ \frac{d^2\varphi^{(1)}(\xi)}{d\xi^2}&=AB^2q(1+q)\cosh^{-2-q}(B\xi)\sin^2h(B\xi)-AB^2q\cosh^{-q}(B\xi)\end{aligned} \qquad (25)$$

By substituting eq. (24) along with (25) into the equation (23), we obtain

$$\begin{aligned}&-VA\cosh^{-q}(B\xi)+\frac{L}{2}A^2\cosh^{-2q}(B\xi)-MAB^2q\cosh^{-q}(B\xi)\\ &\qquad+MAB^2q(1+q)\cosh^{-q}(B\xi)-MAB^2q(1+q)\cosh^{-2-q}(B\xi)=0,\end{aligned} \qquad (26)$$

By balancing the exponents of each pair of cosh that appeared in (26) yields $q=2$. Then we collect the coefficients of the same power in cosh and setting them to zeros, we get a system of algebraic equation, which are overlooked for simplicity. Solving the resulting system of equation, we get

$$A=\frac{3V}{L} \text{ and } B=\sqrt{\frac{V}{4M}} \qquad (27)$$

So, the stationary solitary wave solution to the equation (19) can be written in the following term:

$$\varphi_1^{(1)}(\eta,\tau)=\frac{3V}{L}\mathrm{sech}^2\left(\sqrt{\frac{V}{4M}}(\eta-V\tau)\right) \qquad (28)$$

As $V>0$, equation (28) evidently shows that the small IASWs, that is, positive bright soliton exists if $L>0$, negative bright soliton exists if $L<0$ and no soliton can exists around $L=0$. If we consider the stationary solution to the equation (19) according to the ansatz method as $\varphi^{(1)}(\xi)=A\tanh^q(B\xi)$, we observe that no dark solitons exists for the proposed model equations that are presented in section 2. One can be easily verified the stability of the KdV equation by impose the appropriate boundary conditions for localized perturbations, namely $\varphi^{(1)}\to 0, \frac{d\varphi^{(1)}}{d\xi}\to 0, \frac{d^2\varphi^{(1)}}{d\xi^2}\to 0$ at $\xi\to\pm\infty$ to investigate the asymptotic behavior of (23) and linearized it with regards to $\varphi^{(1)}$. As $V>0$, it should be notable that there will be a stable soliton if $M>0$, else there will be an oscillatory soliton.

IV. Results and Discussion

In this section, we have described the basic features of ion acoustic solitary waves for weakly relativistic regime in a three component unmagnetized plasma system containing Fermi type positrons, electrons and relativistic thermal ions.

The nonlinear oscillations of small amplitude IASWs in e-p-i plasmas have considered by analyzing the solution (28) to the KdV equation for weakly relativistic regime. We have investigated graphically the width, amplitude and phase velocity of solution, and electrostatic propagation of IASWs for both cases of ultra-relativistic and non-relativistic Fermi type electrons and positrons found by the variation of unperturbed Fermi type positron to electron density ratio(p), ion to electron temperature ratio (Ω), Fermi type electron to positron temperature ratio parameter(α) and relativistic streaming factor β . Fig 1 (a), 1(b) and 1(c) displays the effects of phase velocity $(v_0 - u_{i0})$ with relativistic streaming factor β at different values of α , Ω and p corresponds to the fixed values of the others parameters respectively for both cases of ultra-relativistic Fermi gas and non-relativistic degenerate electrons. From these Figures, we observe that the phase velocity increases in increasing of α and Ω, but decreases in increasing of p . Fig. 2 and 3 displays the variation of amplitude and width of soliton with β at different values of α , Ω and p corresponds to the fixed values of the other related plasma parameters for both cases respectively.

Fig 4 displays the 3D plot of increasing IASWs for both cases of ultra-relativistic and non-relativistic Fermi type electrons/positrons with fixed values of $\alpha = 0.3$, $u_{i0} = 0.9$, $\beta = 0.1$, $\Omega = 0.1$ and $p = 0.3$, whereas Fig 5 presents the variations of electrostatic IASWs for both cases at different values α with same values of the other parameters. It has been shown that the electrostatic potential have a remarkable effect on solitary waves and represents the solitary wave of bell type or positive bright soliton. The Fig. 5 shows that the peak amplitude as well as width of the soliton increases with increases of α . Fig. 6 exhibits the electrostatic effects of IASWs at different values of Ω for both cases with fixed values of $\alpha = 0.3$, $u_{i0} = 0.9$, $\beta = 0.1$ and $p = 0.5$. The Fig. 6 shows that the peak amplitude of the soliton decreases with increases of Ω and represent the narrow type solitary wave. Fig. 7 represents the electrostatic effects of IASWs at different values of p for both cases with fixed values of $\alpha = 0.3$, $u_{i0} = 0.9$, $\beta = 0.1$ and $\Omega = 0.1$. The Fig. 7 shows that the peak amplitude as well as width of the solitary wave decreases with increases of p and represent the bell type solitary wave for both cases of Fermi type electrons and positrons. Finally, Fig. 8 exhibits the variation of electrostatic IASWs potential in an increasing streaming velocity u_{i0} , that is $\beta = 0.04$ (red curve), $\beta = 0.1$ (green curve), $\beta = 0.4$ (golden curve), and $\beta = 0.7$ (blue curve) with fixed values of $\alpha = 0.5$, $p = 0.5$ and $\Omega = 0.3$ for both cases of ultra-relativistic Fermi gas and non-relativistic degenerate electrons. Form the Fig. 8, we observe that the peak amplitude of solitary wave increases but the width of the solitary wave decreases with increases of streaming velocity u_{i0} for both cases of Fermi type electrons and positrons. Thus we should point out the effects of Fermi type positrons and electrons on thermal ions that have been considered in this investigation would be very useful in perception to study the propagation of plasmas, where the particle velocity is comparable with the speed of light in interstellar, astrophysical and space plasmas, particularly in plasma sheet boundary layer of earth's magnetosphere, in laser-plasma interaction, quark-gluon plasma, relatively massive white dwarfs, dark-matter halos etc.

V. Concluding Remarks

In this research work, we have considered a three component un-magnetized plasma systems by composing of Fermi type electrons, positrons and the ions speed is comparable with the speed of lights. The reductive perturbation technique has implemented to derive the KdV equation from the proposed model equations in investigating the small but finite amplitude propagation of ion-acoustic solitary waves in electron-positron-ion plasma with ultra-relativistic or non-relativistic degenerate electrons and positrons. The solutions of this equation have evaluated by using the anstaz method. The basic properties (phase velocity, amplitude and width, electrostatic potential) of ion acoustic soliton for weakly relativistic regime have demonstrated graphically by analyzing the solution to this equation for both cases. We have found the only positive bright soliton for both cases of degenerate electrons and positrons due to the influences of the related plasma parameters. The results that are predicted graphically reveals that both supersonic and subsonic IASWs can propagate in ultra-relativistic case, whereas, only subsonic propagations can occur for non-relativistic case. Although both of supersonic and subsonic IASWs can propagate in ultra-relativistic degeneracy case, however, the former exist only for very small range of α. It is noted that the characteristics of nonlinear IASW oscillation differ in the mentioned cases. Thus we conclude that it may be perform in a laboratory experiment which will be able to identify the special new features of IASWs propagating in a e-p-i plasma for the ions speed is comparable with the speed of light with ultra-relativistic or non-relativistic degenerate electrons and positrons. In future, we can be investigated the nonlinear propagation of ion acoustic waves by composing of relativistic thermal ions, and kappa distributed electrons and positions.

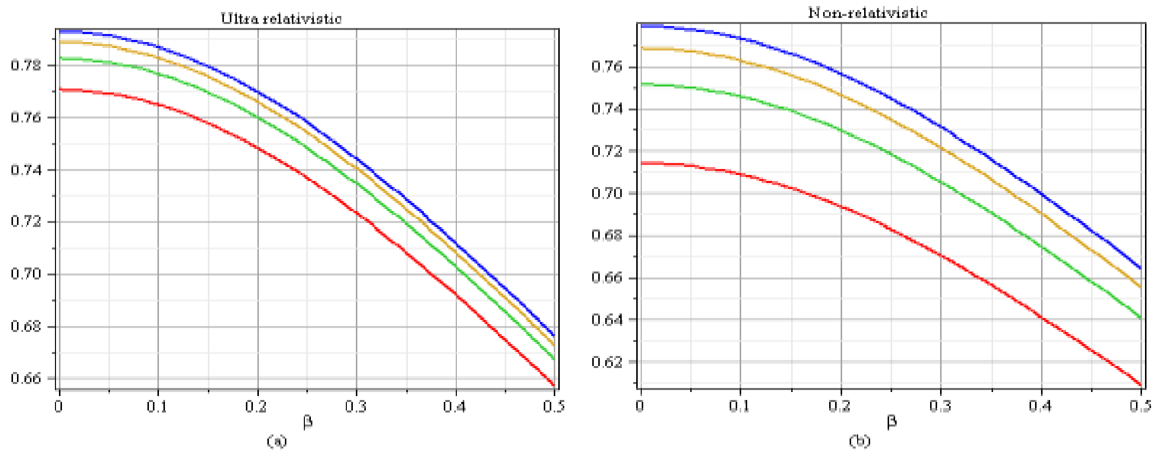

Fig.1 (a) Effects of phase velocity $(v_0 - u_{i0})$ corresponds to relativistic factor β at different values of $\alpha = 0.1$ (red curve), $\alpha = 0.2$ (green curve), $\alpha = 0.3$ (golden curve), $\alpha = 0.4$ (blue curve) and constant values of $\Omega = 0.1$ and $p = 0.1$.

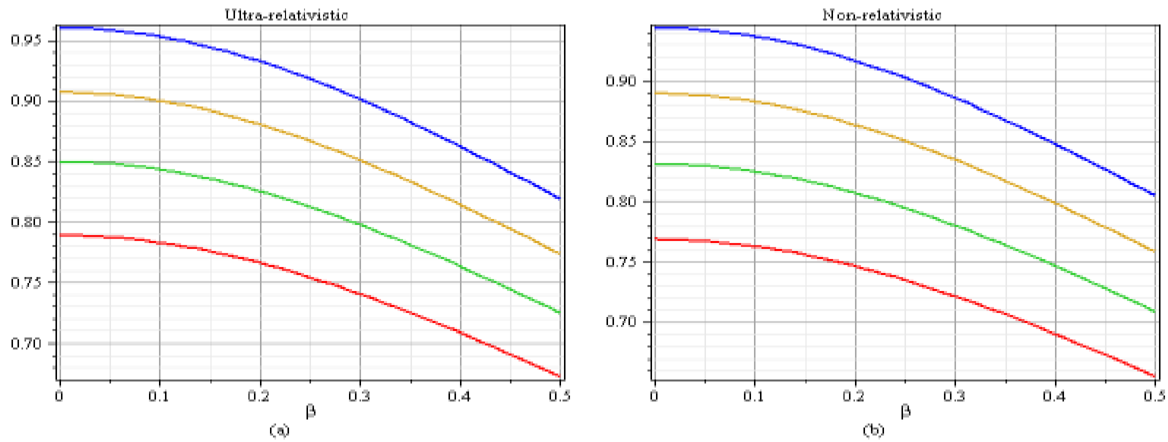

Fig. 1(b) Effects of phase velocity $(v_0 - u_{i0})$ corresponds to relativistic factor β at different values of $\Omega = 0.1$ (red curve), $\Omega = 0.2$ (green curve), $\Omega = 0.3$ (golden curve), $\Omega = 0.4$ (blue curve) and constant values of $\alpha = 0.3$ and $p = 0.1$.

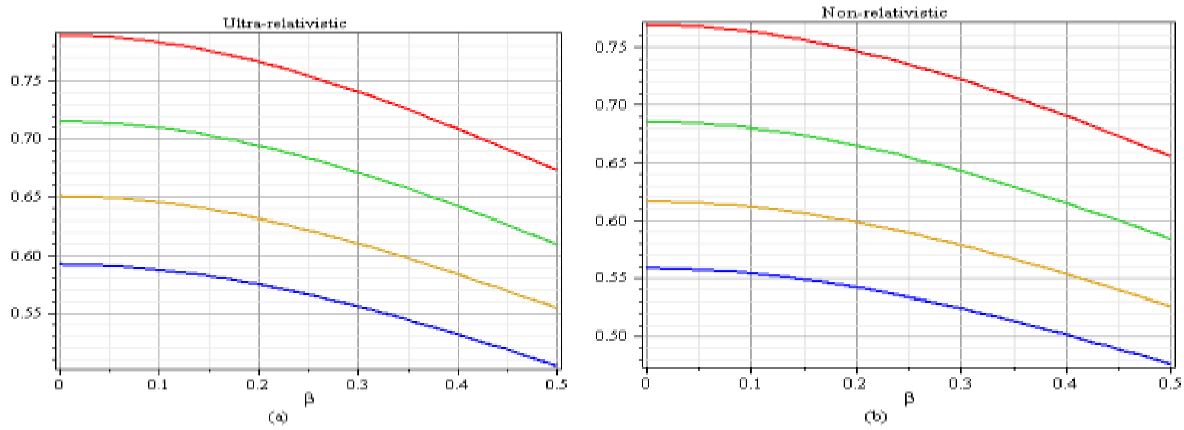

Fig. 1 (c) Effects of phase velocity $(v_0 - u_{i0})$ corresponds to relativistic factor β at different values of $p = 0.1$ (red curve), $p = 0.2$ (green curve), $p = 0.3$ (golden curve), $p = 0.4$ (blue curve) and constant values of $\alpha = 0.3$ and $\Omega = 0.1$.

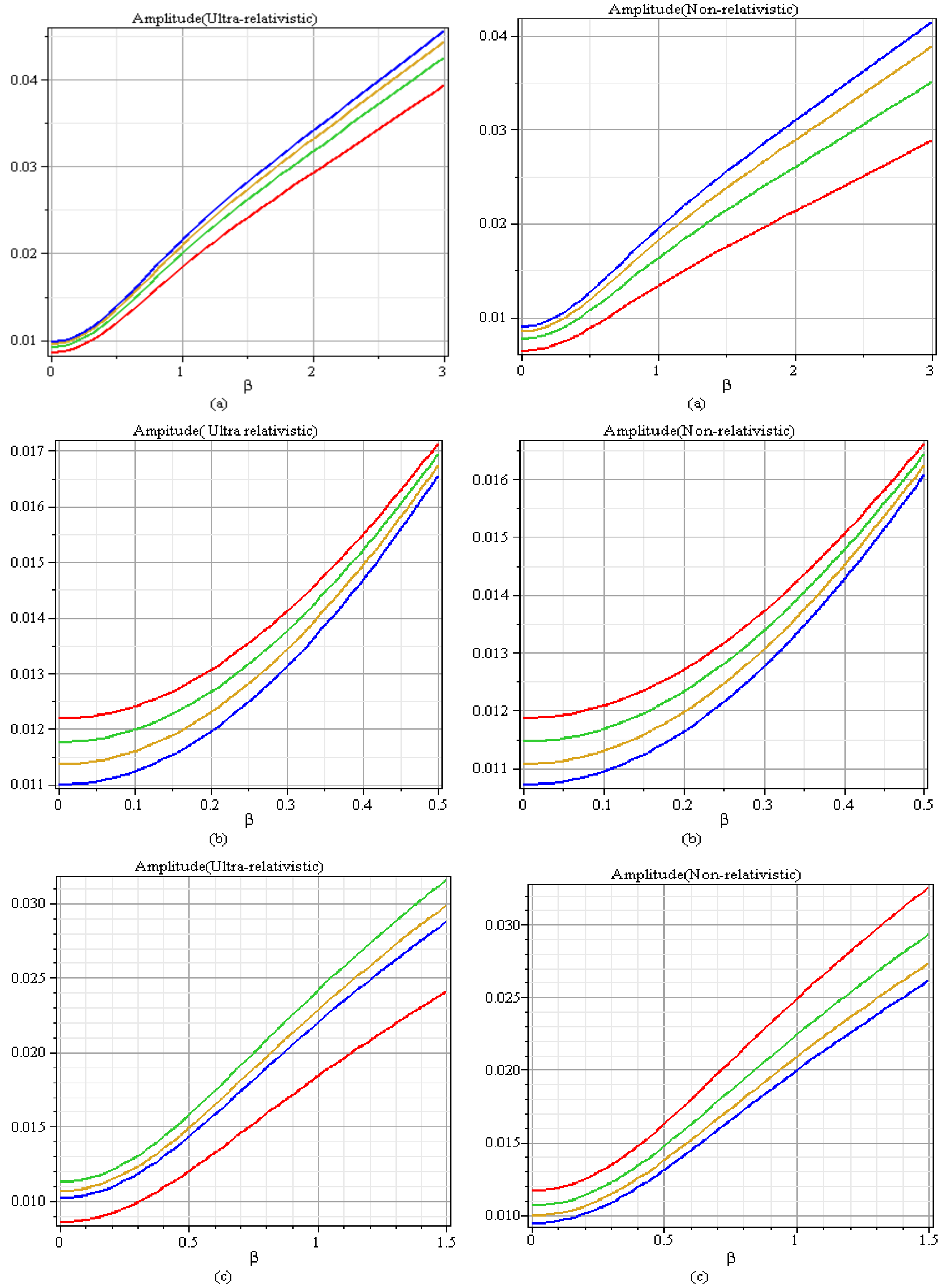

Fig. 2 Variation of amplitude of soliton for both cases of ultra relativistic and non relativistic with β at different values of **(a)** α for constant values of $\Omega = 0.1$ and $p = 0.5$; **(b)** Ω for constant values of $\alpha = 0.5$ and $p = 0.1$; **(c)** p and constant values of $\Omega = 0.1$ and $\alpha = 0.4$.

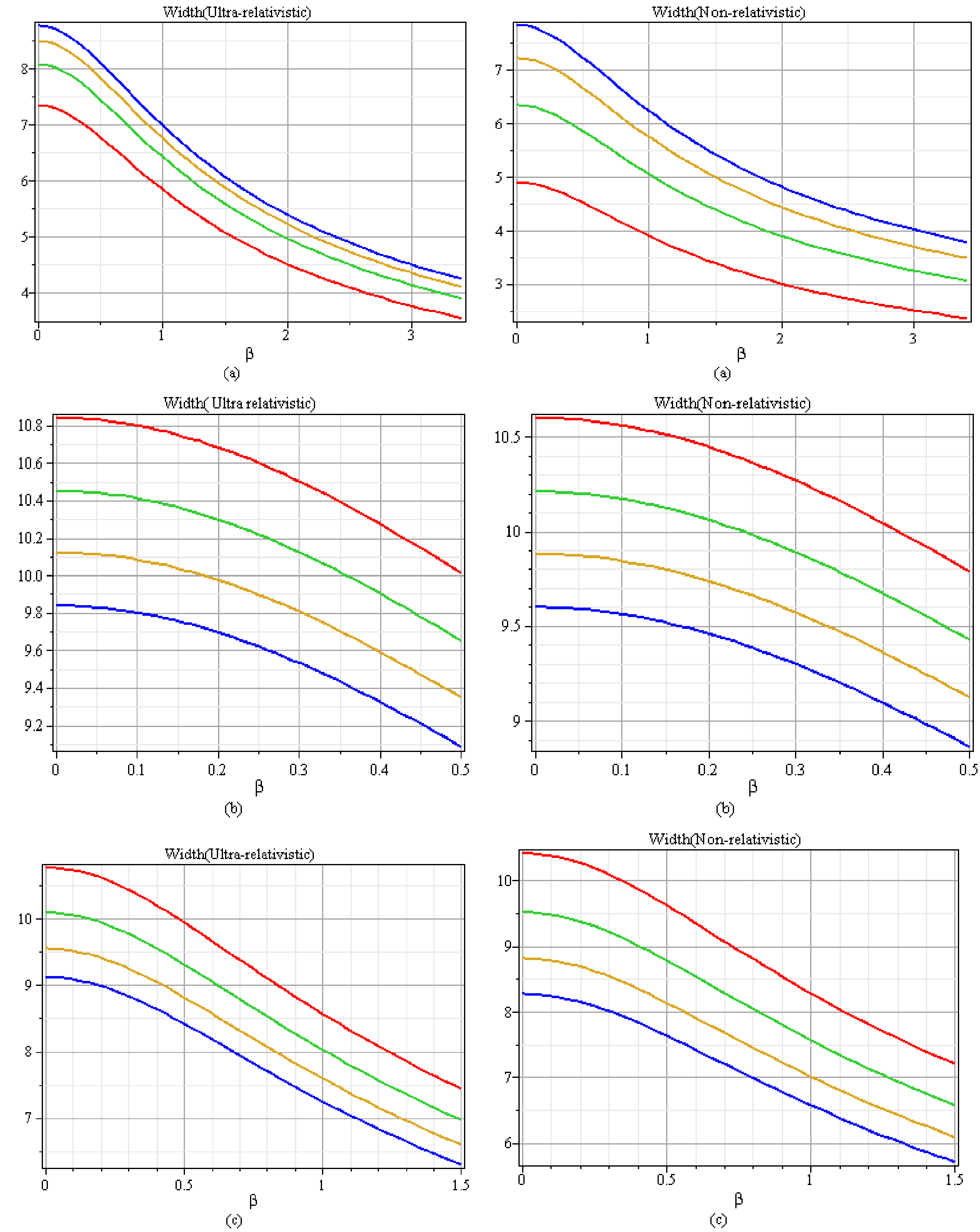

Fig. 3 Variation of width of soliton for both cases of ultra relativistic and non relativistic with β at different values of **(a)** α for constant values of $\Omega=0.1$ and $p=0.5$; **(b)** Ω for constant values of $\alpha=0.5$ and $p=0.1$; **(c)** p and constant values of $\Omega=0.1$ and $\alpha=0.4$.

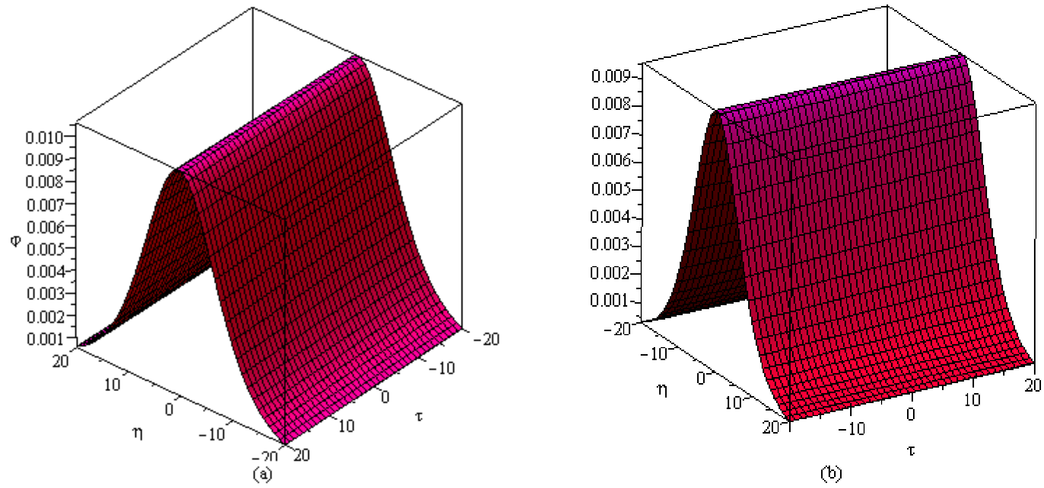

Fig. 4 3D plot of electrostatic IASWs for weakly relativistic regime in the case of **(a)** Ultra-relativistic and **(b)** non-relativistic with fixed values of $\alpha=0.3$, $u_{i0}=0.9$, $\beta=0.1$, $\Omega=0.1$ and $p=0.3$.

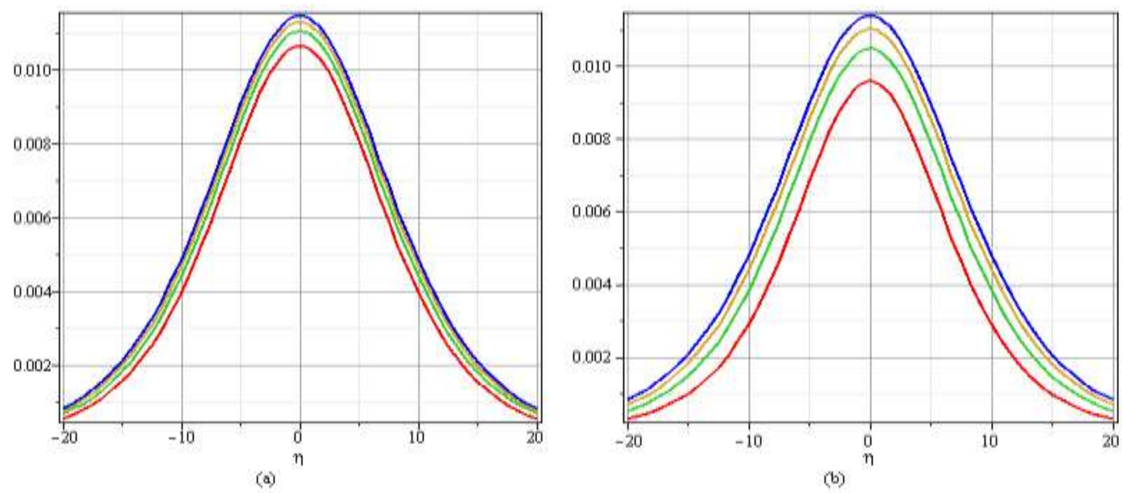

Fig. 5 Variation of electrostatic IASWs for weakly relativistic regime in the case of (a) Ultra-relativistic Fermi gas and **(b)** non-relativistic degenerate electrons at $\alpha=0.3$ (red curve), $\alpha=0.5$ (green curve), $\alpha=0.7$ (golden curve) and $\alpha=0.9$ (blue curve) with fixed values of $u_{i0}=0.9$, $\beta=0.1$, $\Omega=0.1$ and $p=0.3$.

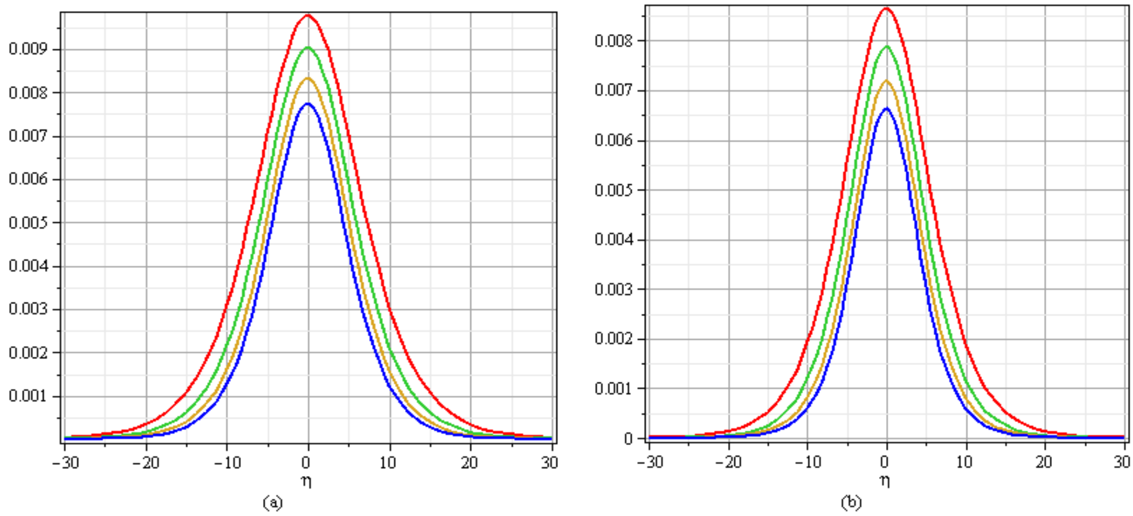

Fig. 6 Variation of electrostatic IASWs for weakly relativistic regime in the case of (a) Ultra-relativistic and **(b)** non-relativistic at $\Omega = 0.1$ (red curve), $\Omega = 0.3$ (green curve), $\Omega = 0.5$ (golden curve) and $\Omega = 0.7$ (blue curve) with fixed values of $\alpha = 0.3$, $u_{i0} = 0.9$, $\beta = 0.1$ and $p = 0.5$.

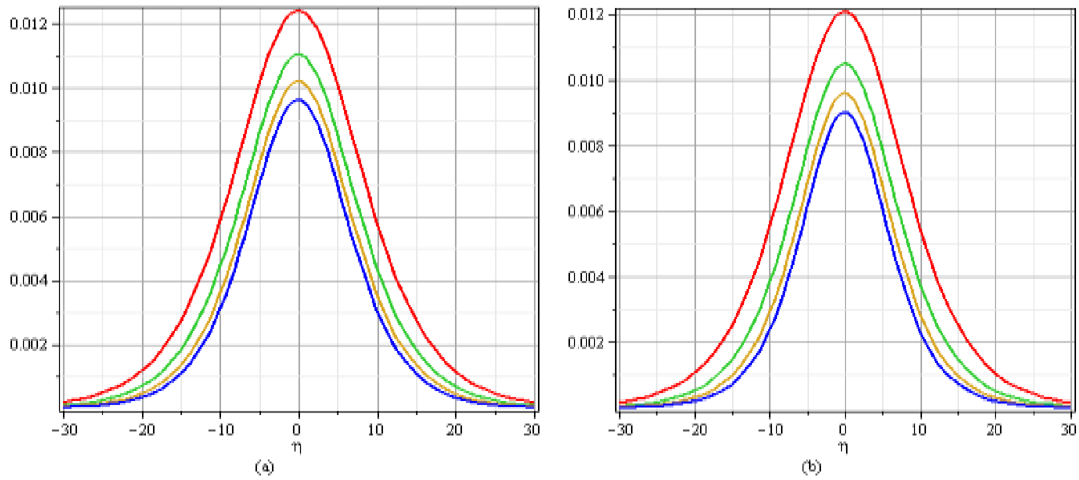

Fig. 7 Variation of electrostatic IASWs for weakly relativistic regime in the case of (a) Ultra-relativistic and **(b)** non-relativistic at $p = 0.1$ (red curve), $p = 0.3$ (green curve), $p = 0.5$ (golden curve) and $p = 0.7$ (blue curve) with fixed values of $\alpha = 0.3$, $u_{i0} = 0.9$, $\beta = 0.1$ and $\Omega = 0.1$.

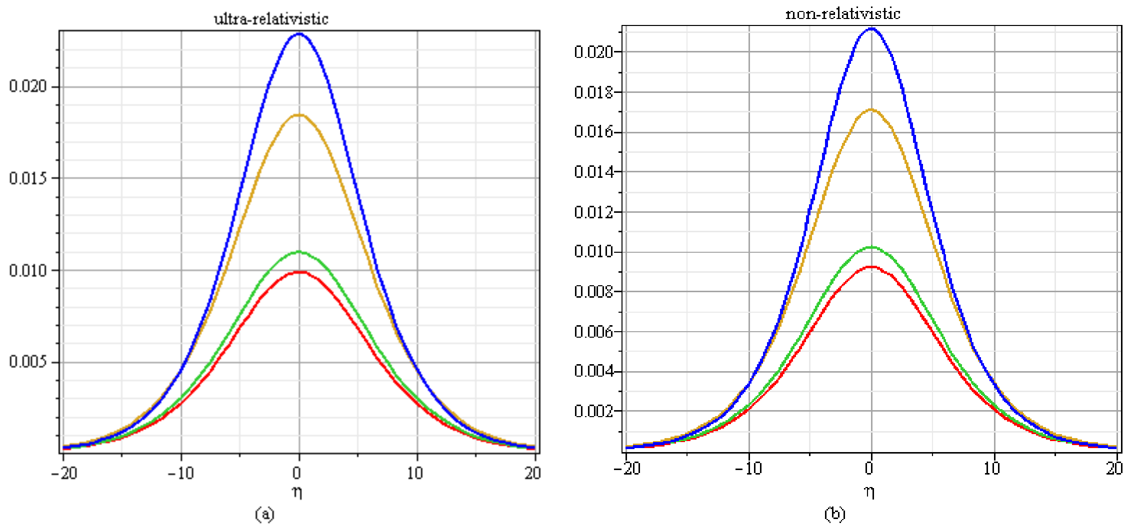

Fig. 8 Variation of electrostatic IASWs potential for weakly relativistic regime in an increasing streaming velocity u_{i0} , that is $\beta=0.04$ (red curve), $\beta=0.1$ (green curve), $\beta=0.4$ (golden curve), and $\beta=0.7$ (blue curve) with fixed values of $\alpha=0.5$, $p=0.5$ and $\Omega=0.3$ for both cases of Fermi type electrons and positrons.

References

[1] Grabbe C 1989 J. Geophys. Res. **94** 17299
[2] Shen B and Meyer-ter-Vehn J 2001 Phys. Rev. E 65 **016405**
[3] Liang E P, Wilks S C and Tabak M 1998 Phys. Rev. Lett. **81** 4887
[4] Michel F C 1991Theory of Neutron Star Magnetosphere (Chicago: Chicago University Press)
[5] Michel F C 1982 Rev. Mod. Phys. **54** 1
[6] Miller H R and Wiita P J 1987 Active Galactic Nuclei (Berlin: Springer)
[7] Burns M L 1983 Positron-Electron Pairs in Astrophysics (Melville: American Institute of Physics NY)
[8] Shukla P K, Rao N N , Yu M Y and Tsintsadze N L 1986 Phys. Reports **138**(1-2) 1
[9] Silva L O, Bingham R, Dawson J M, Mendona J T and Shukla P K 1999 Phys. Rev. Lett. **83** (14) 2703
[10] Mamun A A and Shukla P K 2010 Phy. Lett. A **374**(3) 472
[11] Mamun A A and Shukla P K 2010 Phy. Plasmas **17** 104504
[12] Mahmood S and Akhtar N 2008 Eur. Phys. J. D **49** 217
[13] Saleem H, Haque Q and Vranges J 2003 Phys. Rev. E **67** 057402
[14] Hasegawa H and Ohsawa Y 2004 J. Phys. Soc. Jpn. **73** 1764
[15] Salahuddin M, Saleem H and Saddiq M. 2002 Phys. Rev. E **66** 036407
[16] Zobaer M S, Roy N, Mamun A A 2014 Astrophys Space Sci. 350 231
[17] Saleem H and Mahmood S 2003 Phys. Plasmas **10** 2612
[18] Shukla P K, Mendonca J T and Bingham R 2004 Phys. Scr. T **113** 133
[19] Shah A, Haque Q and Mahmood S 2011 Astrophys. Space Sci. **335** 529
[20] Gill T S, Bains A S and Saini N S 2009 Can. J. Phys**. 87** 861
[21] Tribeche M and Djebarni L 2010 Phys. Plasmas 17 124502
[22] Masood W and Rizvi H 2010 Phys. Plasmas 17 052314
[23] Saeed R, Shah A and Noaman-ul-Haq M 2010 Phys. Plasmas 1**7** 102301
[24] Jung Y D 2001 Phys. Rev. Lett. 8 3842

[25] Marklund M and Shukla P K 2006 Rev. Mod. Phys. 78 591

[26] Killan T C 2006 Nature 441 297

[27] Becker K, Koutsospyros K, Yin S M, Christodoulatos C, Abramzon N, Joaquin J C and Brelles-Mariono G 2005 Nature 47 B 513

[28] Markowich P A, Ringhofer C A, and Schmeiser C 1990 Semiconductor Equations (New York: Springer) p.65.

[29] Chandrasekhar S 1939 An Introduction to the Study of Stellar Structure (Chicago: the University of Chicago press)

[30] Esfandyaari-Klajahi, Akbari-Moghanjoughi M and Saberian E 2010 Plasma Fusion Res. **5**, 045

[31] Zeba I, Moslem W M and Shukla P K 2012 Astrophys. J. 750 72

[32] Roy N, Tasnim S and Mamun A A 2012 Phys. Plasmas 19 033705

[33] Nejoh Y N 1996 Phys. Scr.49 967

[34] Zobaer M S, Roy N and Mamun A A 2013 J. Plasma Phys. 79 65

[35] Ali S, Moslem W M, Shukla P K and Schlickeiser R 2007 Phys. Plasmas 14 082307

[36] Mehdipoor M and Neirameh A 2012 Astrophys. Space Sci. **337** 269

[37] Hossen M R and Mamun A A 2014 Braz. J. Phys. 44 673

[38] Hossen M R, Nahar L and Mamun A A 2014 Braz. J. Phys 44 638

[39] H Triki and A M Wazwaz 2009 Phys. Lett. A. 373 2162

[40] A Biswas 2009 Appl. Math. Lett. 22 208