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# Effect of Rotation on Rayleigh Wave Propagation in an Initially Stressed Voigt-type Viscoelastic Layer in the Gravity Field

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***Abstract:***
*The present paper investigates the effect of angular rotation of the solid on Rayleigh surface waves in a homogeneous, isotropic initially stressed Voigt-type viscoelastic layer in its gravity field. After solving the basic equations with harmonic solution method, the dispersion relations for initially stressed viscoelastic layer and free layer are derived in rotating solid in its gravity field. Symmetric and anti-symmetric modes of Raleigh waves are studied in the special cases. Rayleigh waves in symmetric modes are independent of gravity and influenced by angular rotation, while Rayleigh waves in anti-symmetric modes are influenced by rotation in its gravity field. The numerical computation has been performed for the derived phase velocity of Rayleigh waves to discuss the effect of rotation, viscoelasticity and initial stress. From the graphical illustrations, it is observed that the angular rotation of the solid, viscoelastic parameters and initial stress have significant effect on the phase velocity.*

***Key Words**: Rayleigh Wave; Rotation; Gravity; Voigt-type; Initial stress; Viscoelastic layer.*

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## I. Introduction

The studies on Rayleigh wave propagation provide several applications on geophysics, seismology, acoustic, earthquake engineering, material sciences and telecommunication. The investigation of Rayleigh type surface waves in seismology context is very crucial for civil engineers in their constructions. It is necessary to know the wave propagation mechanism in layered media for interpretation of the geophysical data. There is no perfect elastic material in our realistic world, so by considering the earth as layered elastic media and some of its parts treated as "viscoelastic" in nature. From last few years, much attention has been given to the elastic wave propagation in layered media or different elastic solids. Many researchers[1,2,3,4,5] have been focused their attention on wave propagation in elastic layered media. The materials like sediments, salt, cool tar treated as viscoelastic materials and they are buried beneath the earth surface. When the elastic wave propagates through viscoelastic material, the viscoelasticity is responsible for attenuation.

Some researchers, Garg[6], Sharma et. al.[7] and Kielezynski et.al[8] investigated on viscoelastic materials. Our earth is model for initially stressed solid. Some physical causes like "pressure due to over burden layer, resulting from difference of temperature, cold working, process of quenching" may be arise of a quality of initial stress. Biot[9] discussed that the elastic wave propagation prominently influenced by initial stress. Ghorai et.al[10] studied the effect of gravity on Love waves in a fluid saturated porous media. The propagation of shear waves in a layered poroelastic media was discussed by Son and Kang[11]. In resent, Somaiah[12] discussed the effect of initial stress on Love wave propagation. Ravi kumar and Somaiah[13] studied the gravity and rotation effects on Rayleigh waves in micro polar elastic solid with stretch.

Some of the above authors have been studied the affect of initial stress on wave propagation in layered media. But in this article, the effect of the rotation is studied on Rayleigh surface wave propagation in an initially stressed Voigt type viscoelastic layer under the consideration of its gravity field. But the gravity field is not unique on different places of earth, so the study of gravity effect on surface wave propagation in Voigt-type viscoelastic layer will be produces many results for wave researchers.

## II. Basic Equations

With the usual notation of Biot[9], the equations of motion for rotating elastic solid under the effect of initial stress and in the absence of body forces are given by

$$\sigma_{ij,j} + S_{kj}\phi_{ik,j} + S_{ik}\phi_{jk,j} = \rho\left[\ddot{U}_i + \vec{\Omega}\times\left(\vec{\Omega}\times U_i\right) + 2\left(\vec{\Omega}\times U_i\right)\right] \;;\quad 1\le i,j,k\le 3 \tag{1}$$

where the displacement components $U_i$ are given by $U_i = (u,v,w)$, $S_{ij}$ are initial stress, $\phi_{ij}$ are micro rotational vector components, $\rho$ is mass density of the layer, $\sigma_{ij}$ are stress components, the angular rotation vector $\vec{\Omega}$ of the media is divided as two types of accelerations namely, Centripetal acceleration is given as $\vec{\Omega}\times\left(\vec{\Omega}\times U_i\right)$ and Coriolis acceleration is given by $2\left(\vec{\Omega}\times U_i\right)$. Subscript $j$ ; $(1\le j\le 3)$ followed by a comma indicates the partial derivative with respect to $j^{\text{th}}$ coordinate and superposed dot represents the partial derivative with respect to time $t$.

## III. Problem Formulation and Solution

Consider a homogeneous, isotropic elastic solid under the viscoelastic layer of thickness $T$. Assume that *Z*-axis is vertically downwards into the layer and the wave propagation along *X*-direction and medium is assumed to be rotating with angular speed $\vec{\Omega}$ along *Z*-axis. So displacement components $U_i$; $1\le j\le 3$ and angular rotation $\vec{\Omega}$ are given by

$$U_i = (u,0,w);\quad \vec{\Omega} = (0,0,\Omega)$$

where $u = u(x,z,t);\quad w = w(x,z,t);\quad \dfrac{\partial(-)}{\partial y} = 0$ (2)

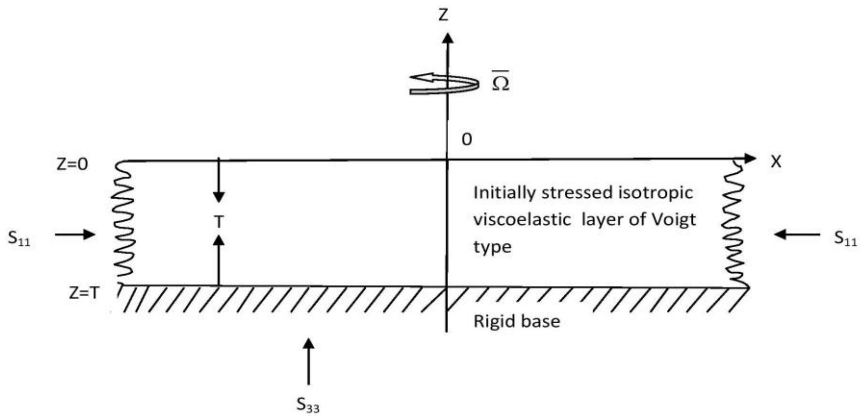


**Figure 1:** Geometry of the problem

In view of equation (2), equation (1) becomes

$$\frac{\partial\sigma_{11}}{\partial x} + \frac{\partial\sigma_{13}}{\partial z} + \left(S_{33} - S_{11}\right)\frac{\partial\phi_{13}}{\partial z} = \rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u\right] \tag{3}$$

$$\frac{\partial \sigma_{13}}{\partial x} + \frac{\partial \sigma_{33}}{\partial z} + \left(S_{33} - S_{11}\right)\frac{\partial \phi_{13}}{\partial x} = \rho \frac{\partial^2 w}{\partial t^2} \tag{4}$$

where

$$\sigma_{11} = \left(\overline{\lambda} + 2\overline{\mu}\right)\frac{\partial u}{\partial x} + \overline{\lambda}\frac{\partial w}{\partial z};\ \ \sigma_{33} = \left(\overline{\lambda} + 2\overline{\mu}\right)\frac{\partial w}{\partial z} + \overline{\lambda}\frac{\partial u}{\partial x};\ \ \sigma_{13} = \overline{\mu}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right);$$

Micro rotational components about *Y*-axis are

$$\phi_{13} = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right).\ \ \overline{\lambda} = \lambda + \lambda'\frac{\partial}{\partial t}; \tag{5}$$

$\overline{\mu} = \mu + \mu'\frac{\partial}{\partial t}$; $\lambda$, $\mu$ are Lame constants and $\lambda'$, $\mu'$ are parameters due to viscoelasticity.

With the help of equation (5), equations (3) and (4) becomes

$$\begin{gathered}\left[\lambda + 2\mu + \left(\lambda' + 2\mu'\right)\frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial x^2} + \left[\mu + \frac{1}{2}\left(S_{33} - S_{11}\right) + \mu'\frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial z^2} + \\ \left[\lambda + \mu + \frac{1}{2}\left(S_{11} - S_{33}\right) + \left(\lambda' + \mu'\right)\frac{\partial}{\partial t}\right]\frac{\partial^2 w}{\partial x \partial z} = \rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u\right]\end{gathered} \tag{6}$$

and

$$\begin{gathered}\left[\lambda + 2\mu + \frac{1}{2}\left(S_{33} - S_{11}\right) + \left(\lambda' + \mu'\right)\frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial x \partial z} + \left[\mu + \frac{1}{2}\left(S_{11} - S_{33}\right) + \mu'\frac{\partial}{\partial t}\right]\frac{\partial^2 w}{\partial x^2} + \\ \left[\lambda + 2\mu + \left(\lambda' + 2\mu'\right)\frac{\partial}{\partial t}\right]\frac{\partial^2 w}{\partial z^2} = \rho\frac{\partial^2 w}{\partial t^2}\end{gathered} \tag{7}$$

One can assume that the harmonic solution of equations (6) and (7) in the form:

$$\begin{gathered} u = U\left(z\right)e^{iq(x - vt)} \\ w = W\left(z\right)e^{iq(x - vt)} \end{gathered} \tag{8}$$

where $U$ and $W$ are amplitudes; the wave number $q$ and wave velocity $v$ are connected by $v = \frac{\omega}{q}$; where $\omega$ is angular frequency.

On using eq. (8) in eq. (6) and (7), it can obtain the following system of homogeneous equations

$$\begin{gathered} \left(\varepsilon_1 + \varepsilon_2 D^2\right)U\left(z\right) + \varepsilon_3 DW\left(z\right) = 0 \\ \delta_1 DU\left(z\right) + \left(\delta_2 + \delta_3 D^2\right)W\left(z\right) = 0 \end{gathered} \tag{9}$$

where

$$D \equiv \frac{\partial}{\partial z};\ \varepsilon_1 = \rho\left(\Omega^2 - q^2 v^2\right) + q^2\left[iqv\left(\lambda' + 2\mu'\right) - \left(\lambda + 2\mu\right)\right];$$

$$\varepsilon_2 = \mu + \frac{1}{2}\left(S_{33} - S_{11}\right) - iq\mu' v;\ \varepsilon_3 = \left(\lambda + \mu\right) + \frac{iq}{2}\left[(S_{11} - S_{33}) - 2i(\lambda' + \mu')qv\right];$$

$$\delta_1 = \left(\lambda' + \mu'\right) q^2 v + iq\left[\lambda + 2\mu + \frac{1}{2}\left(S_{33} - S_{11}\right)\right];$$

$$\delta_2 = q^2\{\rho v^2 + i\mu' qv - [\mu + \frac{1}{2}(S_{11} - S_{33})]\};\ \delta_3 = (\lambda + 2\mu) - (\lambda' + 2\mu')iqv \tag{10}$$

Solving the system (9), one can get the amplitudes $U(z)$ and $W(z)$ as

$$U(z) = A^* \cosh(m_1 z) + B^* \sinh(m_1 z) + C^* \cosh(m_2 z) + D^* \sinh(m_2 z) \tag{11}$$

and

$$W(z) = A^* \xi_1 \sinh(m_1 z) + B^* \xi_1 \cosh(m_1 z) + C^* \xi_2 \sinh(m_2 z) + D^* \xi_2 \cosh(m_2 z) \tag{12}$$

where $m_{1,} m_2 = \left[\frac{P \pm \sqrt{P^2 - 4Q}}{2}\right]^{\frac{1}{2}};\ P = \frac{\delta_3 \varepsilon_1 + \delta_2 \varepsilon_2 - \delta_1 \delta_3}{\varepsilon_2 \delta_3};\ Q = \frac{\varepsilon_1 \delta_2}{\varepsilon_2 \delta_3};$

$$\xi_j = \frac{\rho(q^2 v^2 - \Omega^2) + q^2\left[(\lambda + 2\mu) - iqv(\lambda' + 2\mu')\right] + \left[i\mu' qv + \frac{1}{2}(S_{11} - S_{33}) - \mu\right] m_j^2}{m_j\left[(\lambda + \mu) + \frac{iq}{2}\left\{(S_{11} - S_{33}) - 2iqv(\lambda' + \mu')\right\}\right]} \tag{13}$$

$j$=1,2 and $A^*, B^*, C^*, D^*$ are arbitrary constants.

On using equations (11) and (12) in eq. (8), one can obtain the displacement components $u$ and $w$ as

$$u(x,z,t) = \left[A^* \cosh(m_1 z) + B^* \sinh(m_1 z) + C^* \cosh(m_2 z) + D^* \sinh(m_2 z)\right] e^{iq(x - vt)} \tag{14}$$

and

$$w(x,z,t) = [A^* \xi_1 \sinh(m_1 z) + B^* \xi_1 \cosh(m_1 z) + C^* \xi_2 \sinh(m_2 z) + D^* \xi_2 \cosh(m_2 z)] e^{iq(x - vt)} \tag{15}$$

## IV. **Boundary Conditions and Secular Equations**

In the field of gravity, the suitable boundary conditions of the problem are stated by the following.
Traction free boundary conditions of upper boundary surface layer are

(i) $\sigma_{33} + \rho g w = 0$ at $z = 0$ (16)

(ii) $\sigma_{13} = 0$ at $z = 0$ (17)

Rigid boundary conditions of lower boundary surface are

(iii) $u = 0$ at $z = T$ (18)

(iv) $w = 0$ at $z = T$ (19)

On using equations (14) and (15) in the boundary conditions (16) to (19), it can obtain the following system of four homogeneous equations in $A^*$, $B^*$, $C^*$ and $D^*$

$$\left[\left\{\lambda+2\mu-\left(\lambda'+2\mu'\right)iqv\right\}\xi_1 m_1+(i\lambda+\lambda' qv)q\right]A^*+\rho g\xi_1 B^*+$$
$$\left[\left\{\lambda+2\mu-(\lambda'+2\mu')iqv\right\}\xi_2 m_2+(i\lambda+\lambda' qv)q)\right]C^*+\rho g\xi_2 D^*=0$$
$$\left[\left(\mu-i\mu' qv\right)m_1+\xi_1 q(i\mu+\mu' qv)\right]B^*+\left[\left(\mu-i\mu' qv\right)m_2+\xi_2 q(i\mu+\mu' qv)\right]D^*=0 \qquad (20)$$
$$\cosh(m_1T)A^*+\sinh(m_1T)B^*+\cosh(m_2T)C^*+\sinh(m_2T)D^*=0$$
$$\xi_1\sinh(m_1T)A^*+\xi_1\cosh(m_1T)B^*+\xi_2\sinh(m_2T)C^*+\xi_2\cosh(m_2T)D^*=0$$

On using Armend Salihu[14], new method to calculate the determinants of $n\times n\ (n\geq 3)$ matrix to the system (20), one can obtain the following dispersion relation which depends on an initial stress, gravity and rotation of the layer.

$$\begin{aligned}&\left[X_2\cosh(m_1T)-X_1\cosh(m_2T)\right]\\&\left[\rho g\xi_2+X_2\cosh(m_1T)\sinh(m_2T)\left(1-\frac{\xi_2}{\xi_1}\tanh(m_1T)\coth(m_2T)\right)\right]\\&=\left[\rho g\xi_2\cosh(m_2T)(X_2\tanh(m_2T)-1)+\left(\frac{m_2+iq\xi_2}{m_1+iq\xi_1}\right)\left(\rho g\xi_1\cosh(m_2T)-X_2\sinh(m_1T)\right)\right] \qquad (21)\\&\quad\left[X_2-X_1\cosh(m_1T)\cosh(m_2T)\left(1-\frac{\xi_2}{\xi_1}\tanh(m_1T)\tanh(m_2T)\right)\right]+\\&\quad\rho g\xi_1\sinh(m_1T)\cosh(m_2T)\left[1-\frac{\xi_2}{\xi_1}\coth(m_1T)\tanh(m_2T)\right]\end{aligned}$$

where $\quad X_j=(\bar\lambda+2\bar\mu)m_j\xi_j+i\bar\lambda q;\quad j=1,2 \qquad (22)$

When layer is free of viscoelasticity and initial stress i.e., $\lambda'=\mu'=S_{11}=S_{33}=0$, equation (21) reduces to

$$\begin{aligned}&\left[X_2^*\cosh(m_1^*T)-X_1^*\cosh(m_2^*T)\right]\\&\left[\rho g\xi_2^*+X_2^*\cosh(m_1^*T)\sinh(m_2^*T)\left(1-\frac{\xi_2^*}{\xi_1^*}\tanh(m_1^*T)\coth(m_2^*T)\right)\right]\\&=\left[\rho g\xi_2^*\cosh(m_2^*T)(X_2^*\tanh(m_2^*T)-1)+\left(\frac{m_2^*+iq\xi_2^*}{m_1^*+iq\xi_1^*}\right)\left(\rho g\xi_1^*\cosh(m_2^*T)-X_2^*\sinh(m_1^*T)\right)\right] \qquad (23)\\&\quad\left[X_2^*-X_1^*\cosh(m_1^*T)\cosh(m_2^*T)\left(1-\frac{\xi_2^*}{\xi_1^*}\tanh(m_1^*T)\tanh(m_2^*T)\right)\right]+\\&\quad\rho g\xi_1^*\sinh(m_1^*T)\cosh(m_2^*T)\left[1-\frac{\xi_2^*}{\xi_1^*}\coth(m_1^*T)\tanh(m_2^*T)\right]\end{aligned}$$

where $\quad X_j^*=(\bar\lambda+2\bar\mu)m_j^*\xi_j^*+i\bar\lambda q;\quad j=1,2$;

$$m_1^{*2},m_2^{*2}=\frac{1}{2}\left[P^*\pm\sqrt{P^{*2}-4Q^*}\right];$$

$$\xi_j^* = \frac{\left(\rho v^2 + \lambda + 2\mu\right)q^2 - \rho\Omega^2 - \mu m_j^{*^2}}{m_j^*(\lambda+\mu)} \quad ; \quad j = 1,2\,; \tag{24}$$

$$P^* = \frac{(\lambda+2\mu)\left[\rho\left(\Omega^2 - q^2v^2\right) - q^2\left(\lambda+2\mu\right)\right] + \mu\left(\rho q^2 v^2 - \mu q^2\right) - iq(\lambda+\mu)(\lambda+2\mu)}{(\lambda+2\mu)\mu}$$

and $$Q^* = \frac{\left(\rho q^2 v^2 - \mu q^2\right)\left[\rho\left(\Omega^2 - q^2v^2\right) - q^2\left(\lambda+2\mu\right)\right]}{(\lambda+2\mu)\mu}$$

Dispersion relations of Rayleigh waves given by equations (21) and (23) are influenced by angular rotation of the layer in its gravity field.

## V. Special cases

**Case (5.1): Symmetric waves**

The displacement components are given by equations (14) and (15) in symmetric modes can be expressed as

$$u(x,z,t) = \left[A^*\cosh(m_1 z) + C^*\cosh(m_2 z)\right]e^{iq(x-vt)} \tag{25}$$

and

$$w(x,z,t) = [A^*\xi_1\sinh(m_1 z) + C^*\xi_2\sinh(m_2 z)]e^{iq(x-vt)} \tag{26}$$

On using equations (25) and (26) in boundary conditions (16) to (19) and after eliminating arbitrary constants $A^*, C^*$, one can obtain the following dispersion equation

$$\frac{\left[\lambda+2\mu-(\lambda'+2\mu')iq\right]\xi_2 m_2 + (i\lambda+\lambda' qv)q}{\left[\lambda+2\mu-(\lambda'+2\mu')iq\right]\xi_1 m_1 + (i\lambda+\lambda' qv)q} = \frac{\xi_2\sinh(m_2T) + \cosh(m_2T)}{\xi_1\sinh(m_1T) + \cosh(m_1T)} \tag{27}$$

When layer is free of viscoelasticity and initial stress i.e., $\lambda' = \mu' = S_{11} = S_{33} = 0$, the equation (27) reduces to

$$\frac{\left[\lambda+2\mu-(\lambda'+2\mu')iq\right]\xi_2^* m_2^* + (i\lambda+\lambda' qv)q}{\left[\lambda+2\mu-(\lambda'+2\mu')iq\right]\xi_1^* m_1^* + (i\lambda+\lambda' qv)q} = \frac{\xi_2^*\sinh(m_2^*T) + \cosh(m_2^*T)}{\xi_1^*\sinh(m_1^*T) + \cosh(m_1^*T)} \tag{28}$$

Dispersion equations (27) and (28) are independent of gravity and influenced by angular rotation of the layer.

**Case(5.2) Anti-symmetric waves**

The displacement components given in equations (14) and (15) can be expressed in anti-symmetric modes as

$$u(x,z,t) = \left[B^*\sinh(m_1 z) + D^*\sinh(m_2 z)\right]e^{iq(x-vt)} \tag{29}$$

and

$$w(x,z,t) = [B^*\xi_1\cosh(m_1 z) + D^*\xi_2\cosh(m_2 z)]e^{iq(x-vt)} \tag{30}$$

Substituting equations (29) and (30) in the boundary conditions (16) to (19) and after eliminating $B^*$ and $D^*$, one can obtain the following dispersion equation

$$\frac{\mu\left(m_2 + iq\xi_2\right) + iq\mu' v\left(\xi_2 q - im_2\right)}{\mu\left(m_1 + iq\xi_1\right) + iq\mu' v\left(\xi_1 q - im_1\right)} = \frac{\left[\rho g + \cosh(m_2T)\right]\xi_2 + \sinh(m_2T)}{\left[\rho g + \cosh(m_1T)\right]\xi_1 + \sinh(m_1T)} \tag{31}$$

When layer is free of viscoelasticity and initial stress i.e., $\lambda' = \mu' = S_{11} = S_{33} = 0$, the equation (31)

becomes $$\frac{\mu\left(m_2^* + iq\xi_2^*\right) + iq\mu' v\left(\xi_2^* q - im_2^*\right)}{\mu\left(m_1^* + iq\xi_1^*\right) + iq\mu' v\left(\xi_1^* q - im_1^*\right)} = \frac{\left[\rho g + \cosh(m_2^*T)\right]\xi_2^* + \sinh(m_2^*T)}{\left[\rho g + \cosh(m_1^*T)\right]\xi_1^* + \sinh(m_1^*T)} \tag{32}$$

The dispersion equation (31) and (32) in anti-symmetric modes are influenced by angular rotation of the layer in the gravity field.

## VI. Numerical Computations

To discuss the effect of rotation of the solid on the speed of Rayleigh waves in symmetric and anti- symmetric modes in its gravity field, consider a particular numerical example with the data taken from Yu et.al[15] and Chattaraj et.al[16] as Elastic parameters: $\lambda = 6.12\times10^9\ N/m^2$; $\mu = 3.32\times10^9\ N/m^2$; $\rho = 1.5\times10^3\ kg/m^3$. Viscoelastic parameters: $\lambda^{'} = 2.774\times10^9\ N/m^2$; $\mu^{'} = 1.387\ N/m^2$. Natural frequency of the material is $\omega = 10Hz.$ $P = S_{11} - S_{33} = 0.5$ ; thickness $T = 0.2m$. We shall discuss the effect of rotation with angular speed $\Omega = 0$; $0.5\times10^5$ ; $1.5\times10^6$ rps on phase velocity of Rayleigh waves in the given range of non-dimensional wave number $q$ with $10 \le q \le 100$. The variation of wave number and phase velocity of Rayleigh waves in an initially stressed Voigt type non-rotating and rotating viscoelastic layer with rotational speeds $0\times10^5$ rps, $0.5\times10^5$ rps, $1.5\times10^6$ rps for symmetric and anti-symmetric modes are shown in figures (2) and (5) . From the comparative figures it is observed that the speed of Rayleigh waves are constant in a non-rotating initially stressed Voigt type viscoelastic layer. The phase velocity curves in symmetric and anti-symmetric modes on free surface (i.e., non-stressed and free of viscosity) solid are shown in figures (3) and (7). From figures (3) and (7), one can say that the Rayleigh waves are propagating with constant speed on free surface with low rotating solids and also from figures (4) and (8), one can say that they are vanishes in rotating and non-rotating free surface of solids. From figure (6), one can say that anti-symmetric waves are faster than symmetric waves in non- rotating initially stressed viscoelastic layer.

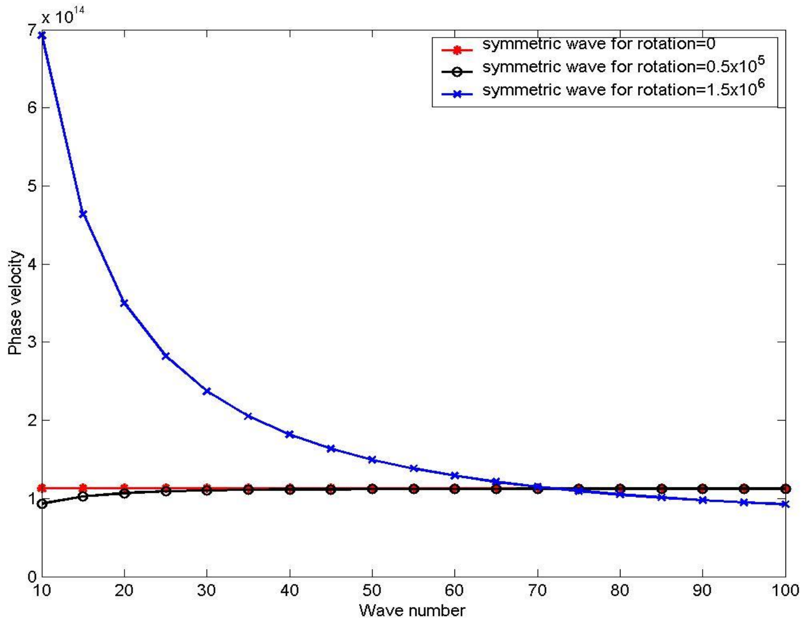


**Figure 2:** Wave number versus phase velocity of Symmetric wave on viscoelastic layer

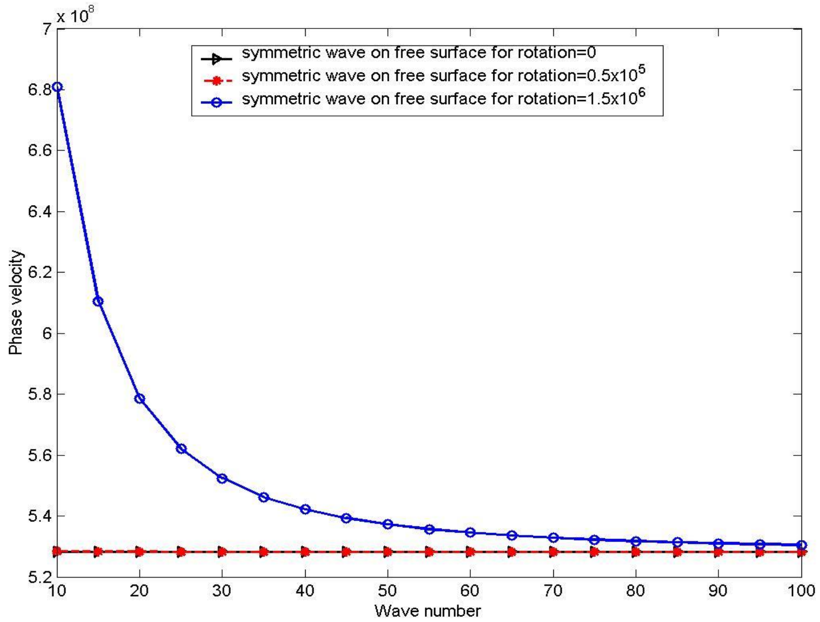


**Figure 3:** Wave number versus phase velocity of Symmetric wave on free surface

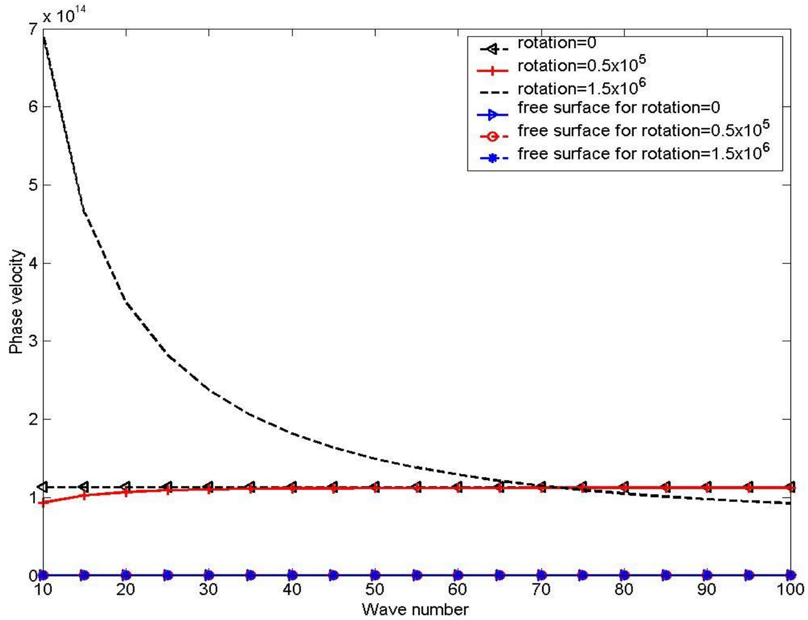


**Figure 4:** Wave number versus phase velocity of Symmetric wave on viscoelastic layer and free surface

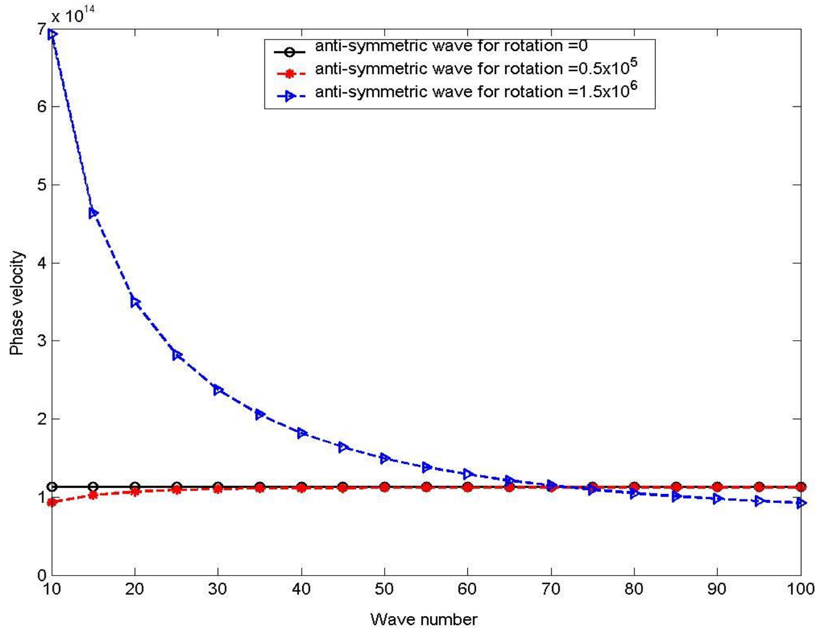


**Figure 5:** Wave number versus phase velocity of Anti-Symmetric wave on viscoelastic layer

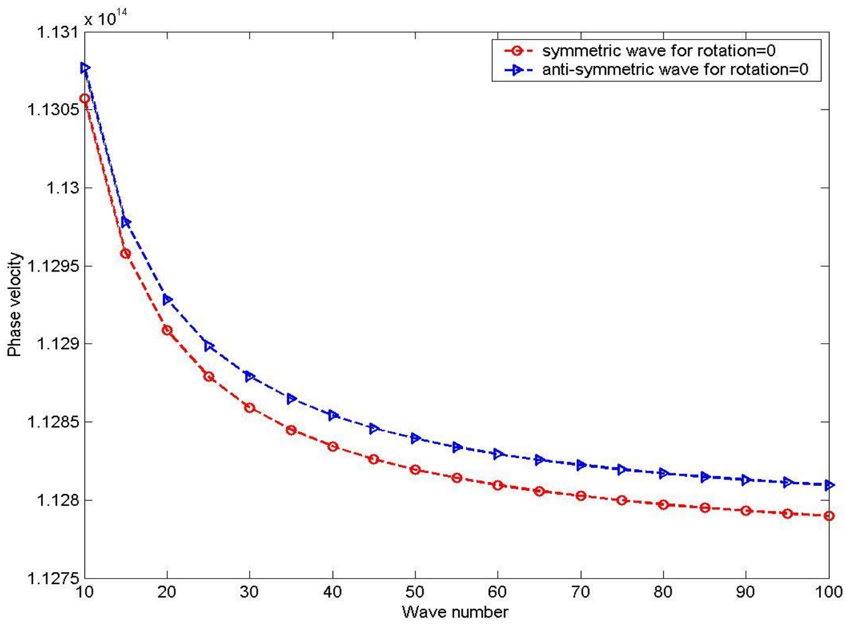


**Figure 6:** Wave number versus phase velocity of Symmetric & Anti-Symmetricwave for non-rotating viscoelastic layer

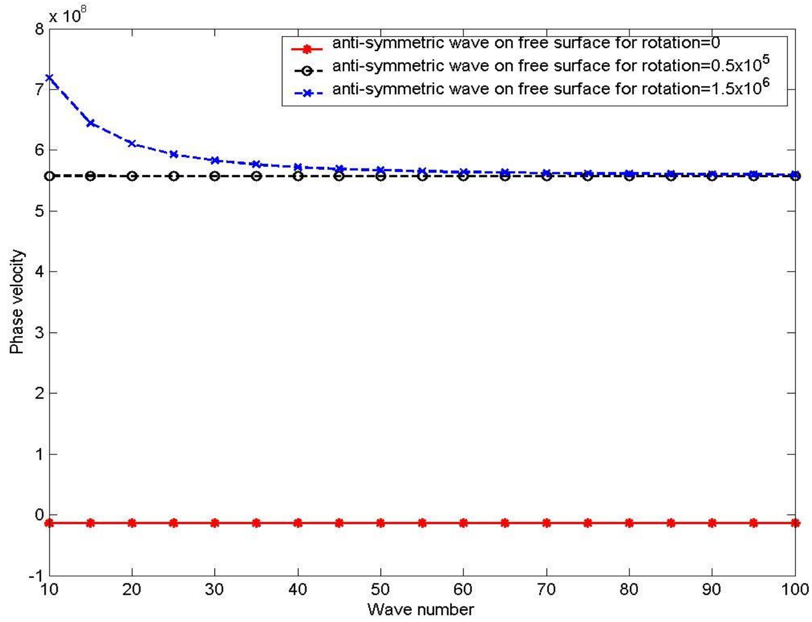


**Figure 7:** Wave number versus phase velocity of Anti-Symmetric wave on free surface

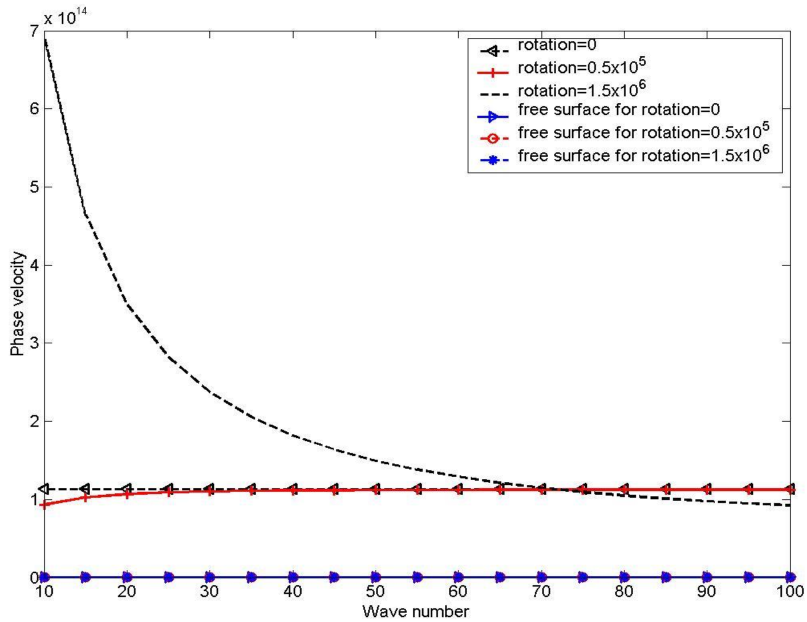


**Figure 8:** Wave number versus phase velocity of Anti-Symmetric wave on viscoelastic layer and free surface

## VII. Conclusion

From theoretical illustrations and numerical computations of Rayleigh waves one can conclude that :

i. Rayleigh waves are influenced by angular rotation of the viscoelastic layer in its gravity field.
ii. Two types of Rayleigh waves in symmetric and anti-symmetric modes arederived in an initially stressed viscoelastic and free layer.
iii. Symmetric waves are influenced by angular rotation of the viscoelastic layer with independent of gravity.
iv. Anti-symmetric waves are influenced by angular rotation in the gravity field.
v. The speed of Raleigh waves is constant in a non-rotating initially stressed viscoelastic layer.
vi. The speed of Raleigh wave also constant in low rotating free layer.
vii. The Rayleigh waves are vanishes in free layer.
viii. Anti-symmetric mode of wave is faster than the symmetric mode of wave in non-rotating initially stressed viscoelastic layer.

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