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A Model for the Dengue Virus Transmission Incorporating Educational Campaigning and Quarantining in Mombasa County, Kenya

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Abstract:
An infectious viral disease called dengue fever (DF) is prevalent in urban and peri-urban areas of the tropics and subtropics. This disease continues to be a threat to global public health. In this study, a dengue virus transmission based on an equation system for ordinary differentials was developed to study the dynamics of DF as a measure to prevent epidemics in Kenya, quarantines during treatment and health education are used to prevent transmission. The next generation matrix approach determines the effective basic reproduction number (R_o). The model's equilibrium points are identified, and their stability analysed. The effectiveness of DF health education and patient quarantining also examined through numerical simulation utilizing the MATLAB program. The results of the stability analysis demonstrate that the disease-free equilibrium is asymptotically stable both locally and globally when $R_0<1$ and the endemic equilibrium (EE) point was found to be locally asymptotically stable when $R_0 > 1$. Numerical simulation performed with MATLAB software demonstrates that when health education campaigns are effective, the number of DF-infected people falls o more quickly, suggesting that health education campaigns are essential for halting the spread of DF.

Keywords: *dengue fever, basic reproduction number, stability analysis, numerical stimulation*

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I. INTRODUCTION

When not treated in a timely manner, dengue fever, a viral infection disease, can be fatal. Dengue is caused by viruses of the genus *Togaviride*, subgenus Flavivirus. It can be brought on by any of the four serotypes, which are DEN 1, DEN 2, DEN 3, and DEN 4, and is spread by the genus Aedes, which has two varieties; dengue fever (DF) and dengue haemorrhagic fever (DHF), which can progress to a more serious form called dengue shock syndrome (DSS). The two species of Aedes transmitting dengue are Aedes aegypti and Aedes albopictus. The first is highly anthropophilic, living in busy places and biting throughout the day, while the second is less anthropophilic and lives in rural areas, according to WHO (2016). Dengue symptoms appear 3 to 14 days after infection. High temperature, headache, nausea, pain in the muscles and joints, and a characteristic skin rash are some of these symptoms. After contracting one of the four serotypes, a person will never contract that serotype again and will become more vulnerable to developing DHF in roughly 12 weeks.

Because the lifetime movement of Aedes aegypti is less than a kilometre, the spread of dengue virus is likely to be driven by human movement. Infected mosquitoes transmit infection by biting susceptible people, and when a susceptible mosquito bites an infected person, it becomes infected. As a result, humans serve as the primary vectors between localized mosquito populations. Because population growth, urbanization, and poverty increase the presence and transmission of infectious diseases, the primary method for controlling and preventing the spread of dengue virus is to combat vector population through various measures such as reducing mosquito habitat and exposure to bites. Temperature and precipitation have a significant impact on dengue virus transmission, according to Rueda et al (1990).

In developing countries, infectious diseases are still the main cause of mortality and morbidity. We must f i rst comprehend the dynamics of disease transmission and take into consideration all pertinent factors, such as vector dynamics, in order to limit dengue infection.

According to Lutomiah J et al. (2016), the earliest dengue outbreaks in East Africa were recorded in the late 1970s and early 1980s, including the one that took place in 1982 near the Kenyan coast. WHO (2020), received 500,000 reports of dengue cases and estimates that the disease poses a risk to nearly 2.5 billion people. With 553 instances of dengue fever epidemic in coastal Kenya in 2021 reported over the preceding four months of January, February, March, and April, more than 100 tropical and subtropical countries were affected. The transmission of infectious virus and the efficacy of prospective control strategies can both be studied effectively using mathematical models. In their 1992 proposal, Anderson and May suggested using mathematics to re- search infectious diseases. Sensitivity tests and a comparison of conjectures are made possible by the model's formulation and the availability of a simulation with parameter estimation Hethcote (2000). Consequently, DF remains a major cause of morbidity and mortality in Kenya and further study is required to comprehend its dynamics.

According to Whitehead et al. (2007), despite numerous attempts, no vaccine exists to protect against any of the virus's four serotypes. The DF can be eradicated if the mosquito population declines and evaluation of the impact of vector management on dengue virus trans- mission according to Yang et al (2008). Fischer et al. (2019) investigated optimal dengue vaccination and control strategies and discovered a positive effect on the number of infected people. According to Burattin et al. (2007), DF can be controlled by quarantining infected people and developing other control strategies.

II. THE MATHEMATICAL MODEL FORMULATION AND DESCRIPTION

To investigate the dynamics of DF transmission by implementing public health initiatives and isolating sick individuals. An equation system for mathematical model based on ordinary differential equations is created. In the study, both qualitative and quantitative analyses are conducted on the model. The next-generation matrix technique is used to calculate the (R_0). The equilibrium points of the model are, and its stability is assessed.

A mathematical model based on ordinary differential equations will be developed and used to study the dynamics of DF. There are two categories of population namely, human population N_h and vector population N_m. For convenience, human population shall be separate into four classes: Susceptible, Infectious, Quarantined, *Recovered* R_h there are two classes within the vector population. Susceptible, *Infectious* There will be natural death rate of human μ_h in all compartments of human and mosquito natural death rate will be μ_m in all mosquito compartments. Rates of DF infection-related deaths in infected and conned compartments is d_h. The model assumes that people and mosquitoes are mixed uniformly, giving each bite an equal chance of coming from any individual person. At a rate of β_h, the human population will be recruited to the susceptible compartment and at a rate of, Mosquito population will be recruited to susceptible compartment. The human will become infected at rate and mosquito will become infected at rate (t). Infected people enter quarantine at rate, and those quarantined individuals recover at . The mosquito-human inter- action rate is and the human-to-mosquito interaction is. Therefore (0<q<1) will be the decrease in mosquito-human contact because of the education campaign and (0<p<1) will be decreased human to mosquito interaction ratio as a result of education campaigns where p is a measure of education campaign efficiency human mosquito interaction and q is a measure of education campaign efficiency mosquito human interaction.

Table 1: Variables of the model

Description of variables	Symbol
Susceptible Individuals	
Infected Individuals	
Quarantined Individuals	
Recovered Individuals	
Susceptible Mosquitoes	
Infected Mosquitoes	

Table 2: Parameters of the model

Description of parameters	Symbol
Human population recruitment rate	
Death rate of human population	
Measure of education efficiency human mosquito inter- action	
Measure of education efficiency human mosquito inter- action	
Rate of transmission probabilities from human to mosquito	
Rate of quarantining of infected individuals	
Rate of recovery of quarantine individuals	
Mosquito population recruitment rate	
Death rate of mosquito's population	
Rate of transmission probabilities from mosquito to human	
Biting rate of susceptible mosquito	
Biting rate of infected mosquito	

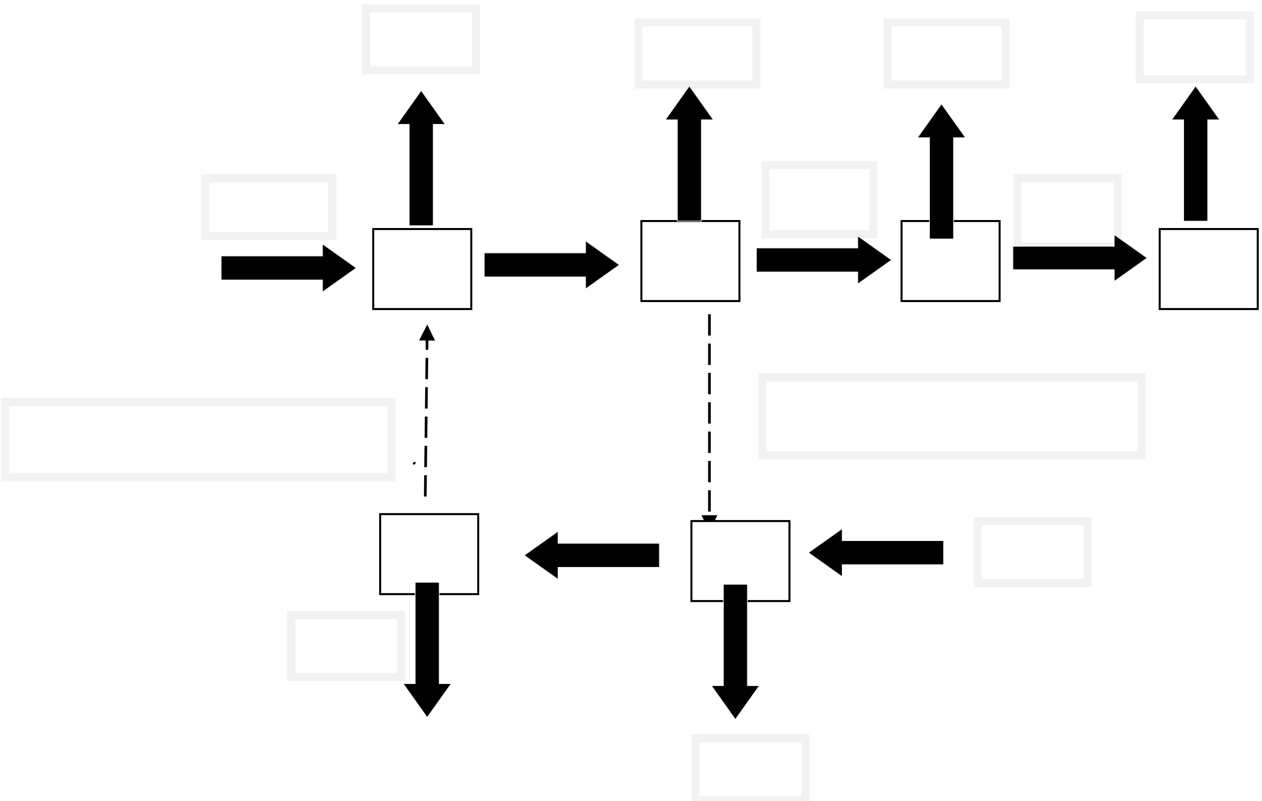
- ***Model assumption***

The model's underlying presumptions are as follows:

- Rates of birth and death are equal.
- All identified DF-infected people will be placed in quarantine.
- Those who have been quarantined will be treated.
- After recovery, there is no re-infection with another serotype.

Model flow chart and equations

Figure 1: Flow chart of the model



The equations of the model are;
Human population

=

=-

=-)

=

Mosquito population

=

= - -
=
=[
=

Model analysis

In this section, we talk about the model's equilibrium points, basic reproduction number, and positivity and boundedness of solutions.

Positivity and Boundedness of Solution

Theorem 2.1. *Let the solution of the equations 2.1 to 2..6 on the compact set be $\Gamma = \in$, t*

Proof; in an appropriate subset Γ, to demonstrate that the solutions are uniformly bounded, the model equations 2.1 to 2.6 are separated into the mosquito compartment N_m and the human compartment

Let be the model equation system's solution from equations 2.1 to 2.4, that determines the derivative of along the solution path of a model equation

Simplifying equation above

The integrating factor of the above is

Equation 2.8's two sides are multiplied by an integrating factor to get.

Integrating both sides of equation 2.10

C being a constant of integration. Thus, dividing equation above by

Applying initial conditions t=0,

Applying equation to the values of C that were found above

Applying differential inequality theorem

This demonstrates that is bounded and that all possible answers for the human component in the equations 2.1 through 2.4 system of the dengue fever model, beginning in the approach, enter or stay in the area where;

Similar to that, the feasible solution set for the mosquito population.

As a result of the aforementioned, both and are positively bounded, and all of the model's potential solutions that start in Γ will remain in the same region. = * for all t>0.The dengue disease equation system, which ranges from 2.1 to 2.6 is thus both biologically meaningful and theoretically properly formulated in the domain since is positively invariant.

Stability analysis
Local Stability of the Disease-Free Equilibrium Point (DFE)

The point at which no disease is present in a given population is known as the disease-free equilibrium. In the model, it is the moment at which the infected population equals zero, =0 and =0 is found at this point, therefore = and N_m=. Equations 2.1 to 2.6 are nonlinear ordinary differential equations, so the system is linearized to produce a Jacobian matrix in order to ascertain the local stability of a disease-free equilibrium. To determine whether this mathematical model is stable, we set the RHS of systems of differential equations 2.1 to 2.6 equal to zero, that is:

In absence of Dengue fever, this model has DFE. This means that = =0 as mention above. Therefore, = and $N_m = S_m$. Using a Jacobian matrix, we investigate the linear stability of the DFE. To derive the Jacobian matrix, each equation is partially differentiated with regard to S, I, Q and R in human population and S and I in mosquito population. ***Theorem 2.2***. *The system's disease-free equilibrium is locally asymptotically stable when $R_0 < 1$ and unstable when$R_0 > 1$.*

In human population, the linearized Jacobian matrix is

Equating equations 3.1 to 3.4 to zero, the equations 2.2, 2.3 and 2.4 gives
= = R_h = 0. Equations 2.1 becomes - =0
⇒ =
Due to our assumption that human population remain constant. This

Implies that
The Jacobian at is given by

The characteristic equation for the disease-free equilibrium is as follows:

On solving equation 2.18, the eigen values are;

Since all eigen values are negative, thus DFE is asymptotically stable and R_0<1

Equating the RHS of system 2.5 and 2.6 equal to 0 for the Aedes mosquito. Since = 0 from 2.6, we get −= 0.

Letting the subset, we obtain =

But the death rate is equal to the intake rate,

Implying that.

In mosquito population, the linearized Jacobian matrix is

Indicating that.

$\Rightarrow(-\mu_m - \lambda)\,(-\mu_m - \lambda) = 0$. It implies that $\lambda = -\mu_m$

Since the eigenvalues are negative, it means that the DFE is asymptotically stable if $R_o < 1$. Thus, the disease can be eradicated. Local Stability of the Endemic Equilibrium Point

When a disease reaches an endemic equilibrium, it remains in the population but cannot be totally eliminated. When endemic conditions are met, the classes but the population is still infected with the disease when. At the endemic equilibrium point in the human population, the linearized Jacobian matrix is given by

and the characteristics equation is given as

h

The above equation is of the form $\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$ which implies that

This clearly shows that a_1>0, a_3>0, a_4>0 and $a_1a_2a_3 > a_3^2 + a_1^2a_4$. According to Routh- Hurwitz criterion, all have negative real roots. Therefore, the endemic equilibrium point of the human population is locally asymptotically stable when R_0> 1 and disease can persist in human population.

The Jacobian matrix (J) for mosquito population is given by

The characteristic equation is equals to

Which is in the form of + + a = 0. Using

Thus a=1

Solving the equation 2.21 using quadratic formula

This clearly shows that real part of eigen values is negative. The Routh-Hurwitz criterion states that if R_0 >1, the endemic equilibrium is locally asymptotically stable. Therefore, the disease may continue to spread among mosquitoes.

Global Stability of the Endemic Equilibrium Point

Theorem 2.3 *If R_0 > 1, the system's Endemic equilibrium point is globally asymptotically stable.*
To proof global stability, application of Laselle (1976) is used through construction of Lyapunov function. For human population, the Lyapunov function will be;
H (S, I, R, Q) =+ + +

On differentiating the above
+ +

On substituting the values of

Rearranging equation 2.1 to 2.4

+

Substituting in the above

+

It implies that Thus if S=S_e, I= Q= and R=R_e, then =0. Based on Lasalle's invariant principle, the endemic equilibrium points of the system 2.1 to 2.4 is therefore globally asymptotically stable. For Mosquito population, the Lyapunov function will be;
+
Differentiating the above
=
On substituting the values of and from equation 2.5 and 2.6

Rearranging equation 2.5 and 2.6

Replacing the values of and μ_m

It implies that Thus if S=S_e and then.Therefore, the endemic equilibrium points of systems 2.5 and 2.6 is globally asymptotically stable according to Lasalle's invariant principle.

The Basic Reproduction Number (Ro)

In a perfectly sensitive population, the Basic Reproduction Number (R_0) is the total number of secondary illnesses caused by a single ill person.

Using the next-generation matrix approach developed by Van den Driessche and Watmough (2002) to ascertain the R_0. This approach uses to calculate the basic reproduction number. The susceptible population in the model are human and mosquito population. Just the infected compartments of the systems of differential equations 2.2 and 2.6 of the two populations mentioned above are utilized to calculate R_0 (Gaff et al., 2007).

In the human and vector model, the rate at which a new infection emerges
is known as the vector valued function, or f

Linearizing the matrix about DFE, it forms Jacobian of F

But

Individuals are moved from an infectious class by the following

The Jacobian matrix for removing people from infectious classes is equals to

Obtaining V inverse results in

The basic reproduction number is equal to the spectral radius of

Therefore;

R_o will be given by the greatest positive Eigen value of the above

Thus:

III. NUMERICAL SOLUTION

This chapter will address the numerical simulations to look at the state Variable dynamics. The parameter values are partly estimates, derived and obtained from literature. The KNBS-2011 population projection was used to get the human population. In 2011, there was an outbreak of Dengue fever in North Eastern Kenya, Mombasa and Mandera. The state variables have the initial values as estimates. MATLAB R2023a was used to generate numerical simulations, using the appropriate parameters and initial values for the variables as listed in Table 3 and the output is obtained in relation to in regard to the human population and vector population compartments. By performing a sensitivity analysis in the basic control reproduction number R_0 using the parameter values shown in the table, we are able to determine the contribution of each parameter in the model.

Description of parameters	Initial values	Source
	1208333	KNBS - 2011 population projection
	1208332	Assumed
	1	Assumed
	1	Assumed
	0	Assumed
	600	Chepkorir et al. (2014)
	599	Assumed
	1	Assumed

	0.75	Derouich et al.2006
	0.75	Derouich et al.2006
	48.33	Computed
	150	Computed
	0.00004	Iurii Bakach (2015)
	0.25	Iurii Bakach (2015)
p	0<p<1	Assumed
q	0<q<1	Assumed
	0.3	Assumed
	0.2857	WHO-2021
b_1	0.5	Derouich et al.2006
b_2	1.0	Derouich et al.2006

Table 3: Parameter and initial variable values of the model and their sources

The basic reproduction number R_0 and sensitivity analysis

This will be applied as the total number of secondary infections caused by a single infected person. Systemically, we obtain R_0 by the next generation matrix phenomenon. By applying the spectral radius theory, the R_0 will represent the spectral radius of the next generation matrix which will be evaluated as the greatest Eigen value of FV^{-1} and R_0=0.27931<1

Therefore, according to theorem 2.2, the systems DFE is locally asymptotically stable

From figure 2, it's observed that for different initial conditions, solutions' trajectories converge to (635, 0, 40, 0). This result agrees with our proposition that the disease-free equilibrium is globally asymptotically stable when $R_0 < 1$. It's important to observe in this case that, R_0 is a decreasing function related to self-protection awareness. Which result means identifying ways to reduce the community's dengue virus outbreak. Reduction in the basic re- production number will be instrumental in controlling such spread. It's also instructional to note that the self-protection awareness includes.

By performing a sensitivity analysis in the control reproduction number R_0 using the values given in Table 3, we are able to determine the contribution of each parameter in the model.

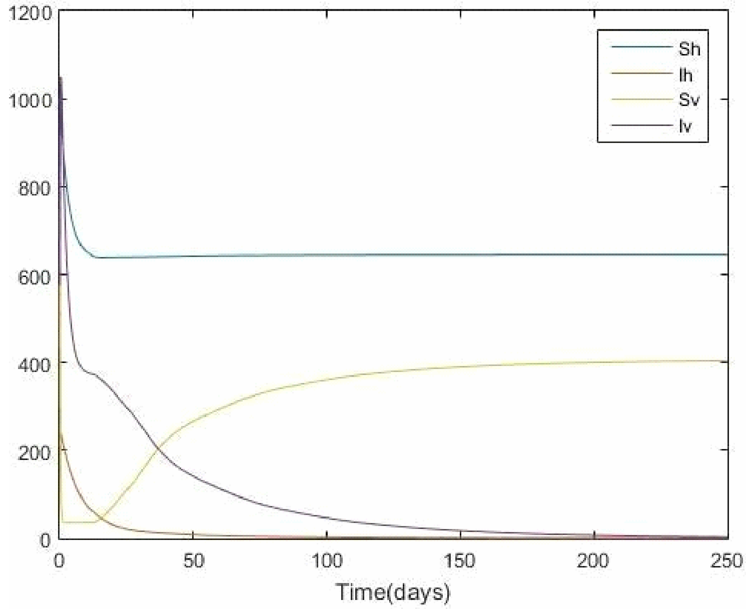


Figure 2: Simulation 1(The disease-free equilibrium is globally asymptotically) stable if $R_0 < 1$.

Elasticity indices

The formula for a parameter α elasticity index is

ε_α=

Therefore, it is a measurement of the proportional change in R_0 to the proportional change in α. The spread of the disease in the populations is caused by the parameter with the largest elasticity magnitude, which has the greatest effect on R_0. The factor the scaling factor normally referred to as the normalization of ε_α. We compute the elasticity of

=

Which is an indication of a linear relationship between β_h, , b_1, b_2, and the basic reproduction number. Therefore, an increase of the rate of one unit to these parameters will increase the rate of transmission at the same rate.

Therefore, an increase in p and q of 3 units will lead to a decrease of the same rate in the R_0 thus necessitating a decrease in the disease transmission rate. = −2−= $\varepsilon_\mu m$ meaning that decrease in and by two units will increase the basic reproduction number.

Finally, , hence a rise in will cause a nominal decrease in R_o

Elasticity is a concept used to measure how the transmission of the virus will vary with change in different parameters. It's clear from our study that the parameters with the most elastic properties are the education/campaign

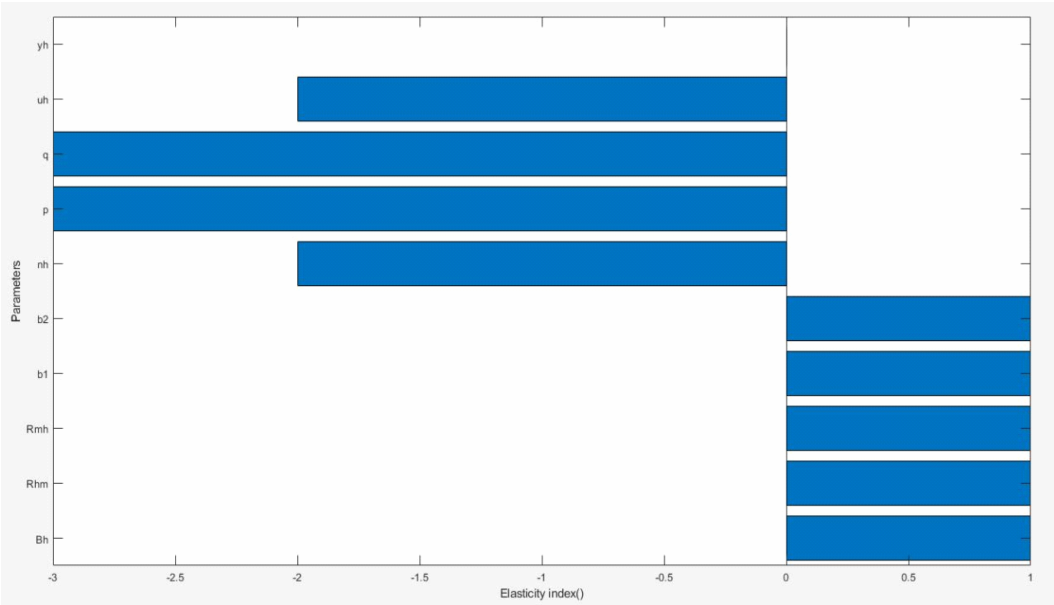


Figure 3: Elasticity of different parameters α.

Parameter is in relation to disease transmission dynamics. A parameterise considered to be elastic if the parameter changes more than proportionally then the system increases or decreases.

IV. Discussion

We consider the dynamics of transmission of the dengue virus by analysing variation of model state variables over a period of time. The variables under consideration will be

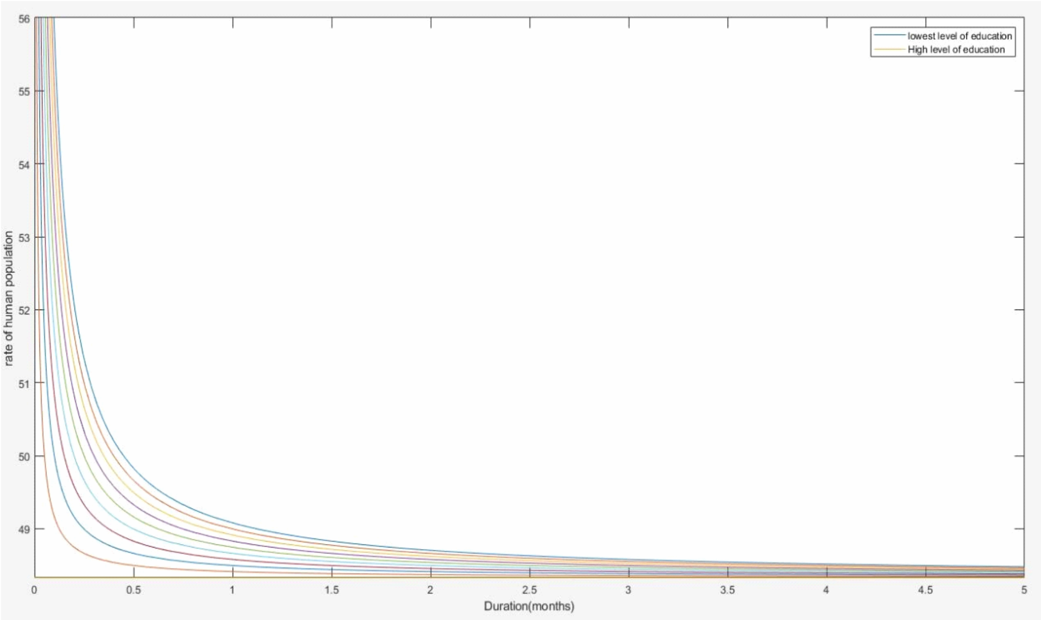


Figure 4: Population rate of susceptible humans with time.

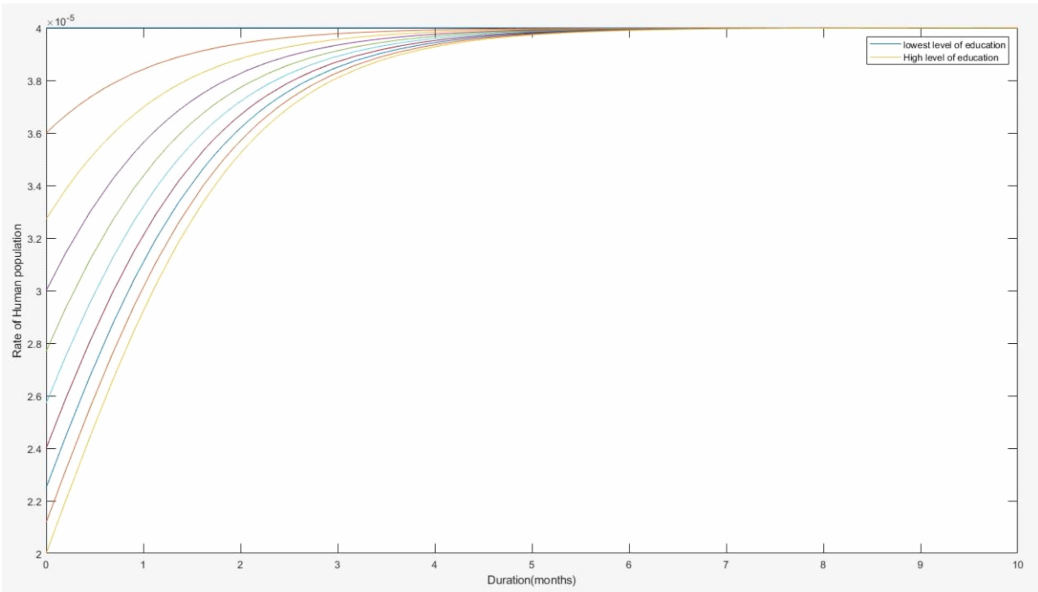


Figure 5: Variation infected human population with time.

From the figure above, it is noted that as time passes, the number of susceptible humans decreases up to some equilibrium point which is attained in five months. This could be partly because of the rise of infected persons from one individual and also due to the education campaign. From the graph, we note that there is a variation in the rate of change susceptible humans with time based on the education campaign index. It can be noted that a higher rate of education campaign will have a higher effect on reducing the susceptible population.

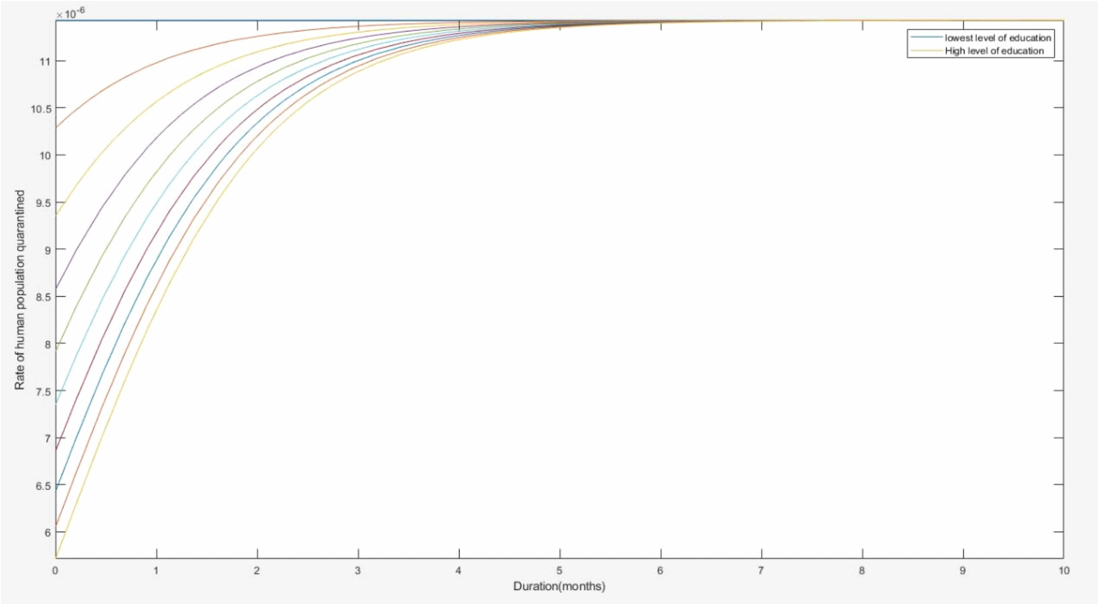


Figure 6: Variation of quarantined population of humans over time.

The population of infected humans is expected to grow with time, but at a decreasing rate, as is evident. As the infected transmission vector comes with human populations, the human population get infected with the virus at the rates as displayed in the figure above. The rate of vector to human infection of observed to decrease with the increase in the education efficiency for human vector interaction. Note that the rate of quarantine starts from zero and increases gradually over time. Still, the variation of p is seen to have an effect on the quarantine rate. Just as observed in figure 3, figure 4 implies that there is the effect of education on quarantined human. The higher the p the lower the quarantined population.

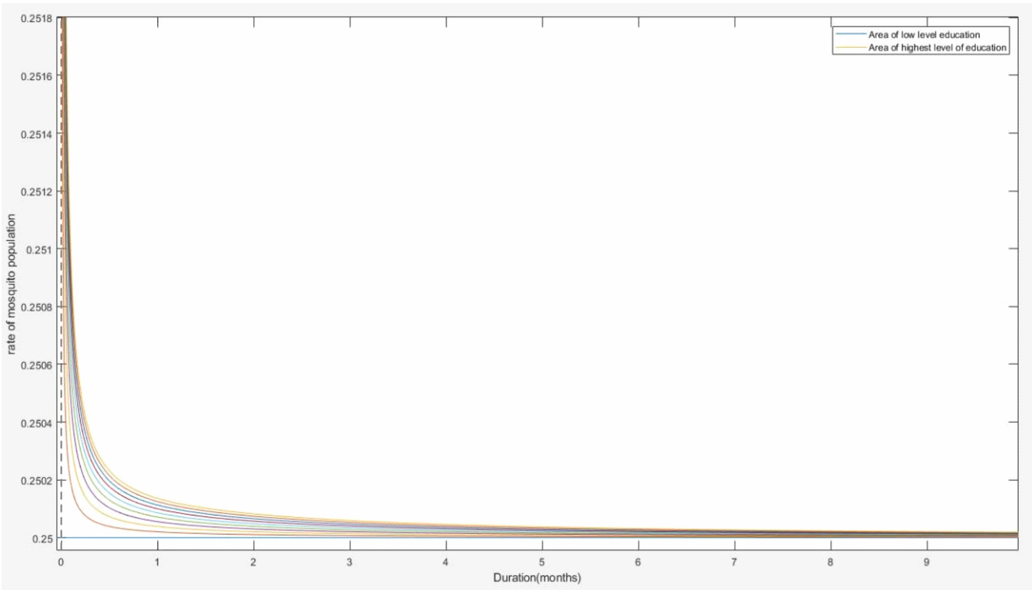


Figure 7: Variation of susceptible mosquito population with time.

Figure 7, the number of susceptible mosquitoes come in contact with infected persons, they do get infected and thus reducing their population. It is also clear that q has an effect on this since the lower the q, the higher the decrease in the rate of susceptible mosquito population. Where as in figure 6, as the susceptible population decrease, the infected population increase over the same period. The two vary mutually but in an opposite manner.

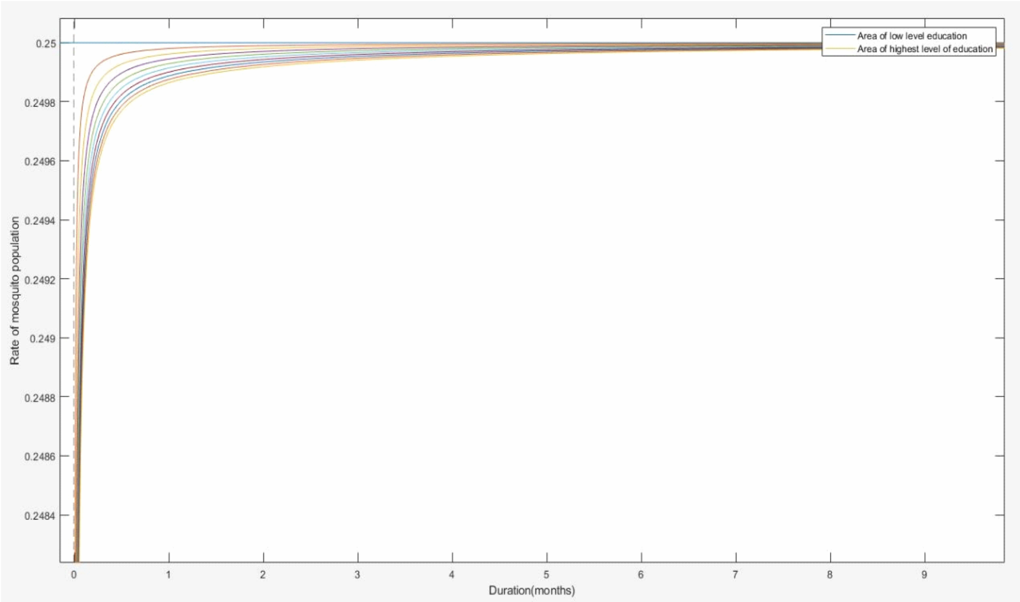


Figure 8: Variation of infected mosquito population with time.

V. CONCLUSION

The mathematical model to study the dynamics of the dengue virus was formulated and analysed for equilibrium points. Using the spectral radius theory, the R_0 is acquired as the subsequent generation matrix's spectral radius, which will be assessed as the largest Eigen value of FV^{-1} and R_0<1. The system DFE is thus locally asymptotically stable, as stated by Theorem 2.2. The education efficiency in this study is observed as the most elastic variable and is seen to have an impact on the rates of decline in number of susceptible populations of both humans and mosquito. It also affected the increase of the infected humans, mosquitoes and quarantined humans substantially. Recovery rate as observed is assumed to be higher than the mortality rate. Evidence of the importance of DF health education programs in enhancing knowledge and emphasizing application of that knowledge is shown by this study. Regular and more intensive health education campaigns about human-mosquito and mosquito-human interaction as well as the broader control of the mosquito population using currently available eco- logically friendly approaches could lead to even greater improvement. Social and community mobilization is also useful for in raising awareness and transforming knowledge into practice on control of Dengue Fever transmission. It has been shown that quarantining significantly lowers the infection rates in populations of mosquito's and humans. This is an aspect of control that its effect to the population is noted based on the rate high rate of decline in susceptible populations.

VI. Recommendations

Observation of outbreak trends and indicator surveillance will be useful in preparation of the health infrastructure and prevention mitigating measure. Accurate surveillance should inform the programs to be implemented and at what time. This surveillance however ought to be routine. The availability of a safe and effective vaccine would improve on measures dengue prevention. Vaccine development May be costly and time consuming but with long term benefits

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