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Semantic-Deductive Characterization Of The Original Gamma ExistentialGraphs

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Abstract.
In this paper, the paraconsistent propositional logic LG is presented, along with its semantic characterization. It is shown that the set of theorems of LG corresponds to the set of valid existential graphs of Charles Sanders Peirce's Gamma system. All evidence is presented in a complete, rigorous, and detailed manner. This result is generalized by constructing the paraconsistent systems of existential graphs GEG[FX]^{I}, and their semantic-deductive characterization. Finally, Zeman's Gamma-4, Gamma-4.2, and Gamma-5 existential graph systems are proven to be paraconsistent.

Keywords.*Gamma existential graphs, paraconsistent logic, semantics.*

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I. PRESENTATION

The existential graphs, alpha, beta, and gamma, were created by Peirce in the late nineteenth century, see Roberts (1992) and Peirce (1965). Alpha graphs correspond to classical propositional calculus, beta graphs correspond to classical logic of first-order relations. Gamma graphs were introduced by Peirce, and later extended by Zeman (1963), constructing existential graphs for modal logics S4, S4.2 and S5. On the other hand, Brade and Trymble (2000) have proposed categorical models for alpha existential graphs. Recently, Oostra (2010, 2011, 2012, and 2021) presented existential graphs for the intuitionist propositional calculus, for the intuitionist relationship calculus, and for the modal logics S4, S4.2, and S5, in their intuitionist versions. Sierra-Aristizabal (2022) presented existential graphs for the KT4P paraconsistent propositional logic system.Sierra (2023) presented the paraconsistent systems GT and GT4, which characterizes the GET and GET4 system of existential graphs, it is proven that GET4 matches Zeman's Gamma-4.

In this paper, the paraconsistent propositional logic LG is presented, along with its semantic characterization. LG's set of theorems is shown to correspond to the set of valid existential graphs of Charles Sanders Peirce's Original Gamma system. All evidence is presented in a complete, rigorous, and detailed manner.

This result is generalized by constructing the paraconsistent systems of existential graphs GEG,[FX]^{I}, and their semantic-deductive characterization. Finally, Zeman's Gamma-4, Gamma-4.2, and Gamma-5 existential graph systems are proven to be paraconsistent.

II. DEDUCTIVE SYSTEM LG

In this section, the deductive system of propositional logic, LG (Gamma Logic), is presented, its connections with classical propositional calculus, and some of its theorems.

Definition 1. The set of formulas, FL, of the deductive system, LG, is constructed from a set FA of atomic formulas, from the constant $\perp$, the unary connective weak negation,{$-$}, and the binary connective conditional,{$\supset$}, as follows.
1) $P\in FA$ implies $P\in FL$. 2) $\perp\in FL$. 3) $X\in FL$ implies $-X\in FL$. 4) $X,Y\in FL$ implies $X\supset Y, X\cap Y\in FL$.

Classical negation, strong affirmation, weak affirmation, disjunction, lambda and biconditional are defined as:
1) $\sim X = X\supset\perp$. 2) $+X = \sim-X$. 3) $\otimes X = -\sim X$. 4) $X\cup Y = \sim X\supset Y$. 5)$\lambda = \sim\perp$. 6) $X\equiv Y = (X\supset Y)\cap(Y\supset X)$.

Doi:

Definition 2. The LG deductive system consists of the axioms (where X,Y,Z∈FL):
Ax1)⊥⊃X.Ax2)X⊃(Y⊃X).Ax3) [X⊃(Y⊃Z)]⊃[(X⊃Y)⊃(X⊃Z)].Ax4) [(X⊃Y)⊃X]⊃X.Ax5)(X⊃⊥)⊃–X.
Ax6)–X⊃–(X∩Y).Ax7) (X∩Y)⊃X.Ax8) (X∩Y)⊃Y.Ax9)(X⊃Y)⊃[(X⊃Z)⊃(X⊃{Y∩Z})].
Ax10)–(Y∩–X)⊃–(Y∩(⊥⊃X)).Ax11. –(Y∩–(Z∩(⊥⊃X)))⊃–(Y∩–(Z∩–X)).
The only rule of inference is the *modus ponens*Mp: Z is inferred from X and X⊃Z.

Definition 3. LetX, X_1, …, X_n∈FL. X is a *theorem* of LG, denoted X∈TL, if there is a *proof* of X from the axioms using the rule Mp, i.e., X is the last row of a finite sequence of lines, in which, each of the lines is an axiom, or is inferred from two preceding rows, using the inference rule Mp. The number of lines in the sequence is referenced as the *length* of the X proof. Y is a *theorem (or consequence)* of {X_1, …, X_n},denoted{X_1, …, X_n}>>Y, if there is a proof of Y, from the axioms and assumptions {X_1, …, X_n}.

Proposition 1. Let them be X,Y,X_1, …, X_n∈FL. If {X_1, …, X_n, X}>>Y then LG, then {X_1, …, X_n}>>X⊃Y.
Proof. Axioms 2, 3 and 4, with the single inference rule Mp, determine the calculus for the classical implication CIC Rasiowa (1974), in which the deduction theorem applies.

Proposition 2. Sean X,Y∈FL. The following formulas are LG theorems:
1) (X⊃~Y)⊃(Y⊃~X). 2) ~(X⊃X)⊃Y. 3) X∪~X. 4) X⊃~~X. 5) ~~X⊃X. 6) (X⊃Y)⊃(~Y⊃~X). 7) (~Y⊃~X)⊃(X⊃Y).
Proof. 1) Suppose X⊃(Y⊃⊥), Y, X. By Mp is derived Y⊃⊥, again by Mp is inferred ⊥. Applying proposition 1, 3 times and using the definition of ~, we conclude (X⊃~Y)⊃(Y⊃~X).
2) Suppose ~(X⊃X), i.e. (X⊃X)⊃⊥, but X⊃X is a CIC theorem, resulting in ⊥, using Ax5 follows Y. Applying proposition 1 concludes ~(X⊃X)⊃Y.
3) By the principle of identity of the CIC we have ~X⊃~X, by the definition we conclude X∪~X.
4) Suppose X, X⊃⊥. By Mp it follows⊥ , applying proposition 1, 2 times and definition concludes X⊃~~X.
5) Suppose ~~X, i.e., ~X⊃⊥, by Ax1 we have ⊥⊃X, as (~X⊃⊥)⊃[(⊥⊃X)⊃(~X⊃X)] is a theorem of the CIC deduces ~X⊃X, i.e. (X⊃⊥)⊃X, using Ax4 implies X. Proposition 1 concludes ~~X⊃X.
6) and 7). Direct consequences for 1), 4) and 5).

Proposition 3. Let them be X,Y,Z∈FL. The following formulas are theorems of L:
1) X⊃(X∪Y). 2) X⊃(Y∪X). 3) (X⊃Y)⊃[(Z⊃Y)⊃({X∪Z}⊃Y)].
Proof. 1) Suppose X, ~X, i.e., X⊃⊥, by Mp we obtain ⊥, according to Ax1 we derive Y. Applying proposition 1, 2 times we conclude X⊃(~X⊃Y), i.e., X⊃(X∪Y).
2) By first part we conclude X⊃(~X⊃Y), using proposition 2, it can be said that X⊃(~Y⊃X), i.e., X⊃(Y∪X).
3) Suppose X⊃Y, Z⊃Y, X∪Z, i.e., ~X⊃Z, by CIC we infer ~X⊃Y, by proposition 2 we derive ~Y⊃X, by CIC we infer ~Y⊃X, i.e. (Y⊃–λ)⊃Y, by Ax4 we get Y. Applying proposition 1, 3 times we get (X⊃Y)⊃[(Z⊃Y)⊃({X∪Z}⊃Y)].

Proposition 4. For X,Y∈FL. X⊃[Y⊃(X∩Y)]∈TL
Proof. Suppose X, Y. Ax2 results in Ax1⊃X, Ax1⊃Y, Ax8 results in Ax1⊃(X∩Y), Mp results in X∩Y. Applying proposition 1, 2 times concludes X⊃[Y⊃(X∩Y)].

Proposition 5. The classical propositional calculus CPC with the language {⊃, ∩, ∪, ≡, ~} is included in the propositional calculus LG.
Proof. Axioms 2, 3, 4, 7, 8 and 9 along with propositions 2, 3, 4 and the inference rule Mp determine CPC Rasiowa (1974).

Proposition 6. Sean X,Y∈FL. The following formulas are theorems of L:
1) – –λ⊃λ. 2) X∪–X. 3) ~X⊃–X. 4) –X≡~+X. 5) +X⊃X.
Proof. 1) By Ax2 we have λ⊃(– –λ⊃λ), in addition to Ax1 of has ⊥⊃⊥, i.e., ~⊥, which means λ, applying Mp we conclude that – –λ⊃λ.

2) By definition in Ax5,result$X\cup-X$.

3) Ax5.

4) By definition we have $\sim-X\equiv+X$, applying proposition 2 we conclude $-X\equiv\sim+X$.

5) By Ax5 we have $\sim X\supset-X$, applying CPC we deduce $\sim-X\supset X$, i.e., $+X\supset X$.

Proposition 7. Sean $X,Y\in FL$. The following formulas are theorems of L:

1) $\sim+\sim X\equiv\otimes X$. 2) $+\sim X\equiv\sim\otimes X$. 3) $\sim+X\equiv-X$. 4) $X\supset\otimes X$. 5) $-X\supset\otimes\sim X$.

Proof. 1) By proposition 6 we have $-\sim X\equiv\sim+\sim X$, by definition it results $\otimes X\equiv\sim+\sim X$.

2) By CPC we conclude $\sim\otimes X\equiv+\sim X$.

3) By definition you have $\sim-X\equiv+X$, by CPC you get $-X\equiv\sim+X$.

4) By proposition 6 we have $+\sim X\supset\sim X$, using CPC we deduce$X\supset\sim+\sim X$, according to the previous paragraph we conclude $X\supset\otimes X$.

5) By definition we have $-\sim X\supset\otimes\sim\sim X$, by CPC we conclude $-X\supset\otimes\sim X$.

III. SEMANTICS

In this section, the semantics for the LG system are presented, in proposition 10 it is proved that the theorems of the LG system are valid formulas in the proposed semantics.This semantics follows the ideas presented by Batens & De Clercq (2004).

Definition 4.$M=(V_M, v)$ is a *model* for LG, it means that, V_M is a function of FL in {0,1}, v is a *function* of $-FL$ in {0, 1}, where $-FL = \{-X : X\in FL\}$.

Definition 5. In the model $M=(V_M, v)$, with $X,Y\in FL$.

$V_M(X)=1$ is abbreviated as $M(X)=1$, and means that in the M model, the formula X is *true*.

$V_M(X)=0$ is abbreviated as $M(X)=0$, and means that in the M model, the formula X is *false*.

The V_M function satisfies the following rules:

1) $V\bot$. $M(\bot)=0$. 2) $V\supset$. $M(X\supset Y)=1$ equals $M(X)=1$ implies $M(Y)=1$.3) $V-$. $M(-X)=1$ equals $M(X)=0$ or $v(-X)=1$. 4) $V-\cap$. $M(-X)=1$ implies $M(-(X\cap Y))=1$. 5) $V\cap$. $M(X\cap Y)=1$ equals $M(X)=M(Y)=1$.6) $V-\sim$. $M(-(Y\cap-X))=1$ implies $M(-(Y\cap\sim X))=1$. 7) $V--\sim$. $M(-(Y\cap-(Z\cap\sim X)))=1$ implies $M(-(Y\cap-(Z\cap-X)))=1$.

Proposition 8. For $X,Y\in FL$.1) $V\sim$. $M(\sim X)=1$ equals $M(X)=0$. 2) $V\cup$. $M(X\cup Y)=1$ equals $M(X)=1$ o $M(Y)=1$.

3) $V\equiv$. $M(X\equiv Y)=1$ equals $M(X)=M(Y)$.4) $V+$. $M(+X)=1$ equals $M(-X)=0$. 5) $V\otimes$. $M(\otimes X)=1$ equals $M(-\sim X)=1$.

6) $M(+X)=1$ implies $M(X)=1$. 7) $M(-X)=0$ implies $M(X)=1$. 8)$V\lambda$. $M(\lambda)=1$

Proof.1), 2) y 3), resulting from CPC semantics.

4) $M(+X)=1$, equals $M(\sim-X)=1$), for part a means $M(-X)=0$).

5)$M(\otimes X)=1$ by definition means $M(-\sim X)=1$.

6) If $M(+X)=1$, for the part 4), we have $M(-X)=0$, applying $V-$ we infer $M(X)=1$.

7) Direct consequence of $V-$.

8) If $M(\lambda)=0$, by $V\sim$ we say $M(\sim\lambda)=1$, this means $M(\bot)=1$, which is not the case.

Definition 6. For $X,X_1, \ldots, X_n\in FL$.A formula X is said to be *valid*, denoted $X\in VL$, if and only if X is true in all models for LG. It is said that$\{X_1, \ldots, X_n\}$*validates* Y if and only if $(X_1\cap X_2\cap\ldots\cap X_n)\supset Y\in VL$.

Proposition 9. Let $X\in FL$. If X is an axiom of LG, then $X\in VL$.

Proof. Ax1)$\bot\supset X$. Suppose $\bot\supset X\notin VL$, so there exists a model M, such that $M(\bot\supset X)=0$, by $V\supset$ we have $M(\bot)=1$, which contradicts $V\bot$. Hence, $Ax1\in VL$.

Ax2, Ax3, Ax4. If X is one of the axioms Ax2, Ax3, Ax4, using the rule $V\supset$ and proceeding as usual for the validity of CPC in van Dalen (2004), it is concluded that $X\in VL$, i.e., Ax2, Ax3, $Ax4\in VL$.

Ax5)$(X\supset\bot)\supset-X$.Suppose that $(X\supset\bot)\supset-X\notin VL$, so there is a model, M such that $M((X\supset\bot)\supset-X)=0$, by $V\supset$ results $M(X\supset\bot)=1$ and $M(-X)=0$, according to $V-$,$M(X)=1$ and $v(-X)=0$, are derived, applying $V\supset$ it follows that $M(\bot)=1$, which contradicts $\bot V$. Hence, $Ax5\in VL$.

Ax6)$-X\supset-(X\cap Y)$. Suppose that $-X\supset-(X\cap Y)\notin VL$, so there is a model M, such thatM$((-X\supset-(X\cap Y))=0$, by $V\supset$ results $M(-X)=1$ and $M(-(X\cap Y))=0$, according to $V-\cap$ it is derived, $M(-X)=0$, which is not the case. Hence, $Ax6\in VL$.

Ax7)$(X\cap Y)\supset X$ and Ax8)$(X\cap Y)\supset X$. From $V\cap$ it follows that $M(X\cap Y)=1$ implies $M(X)=M(Y)=1$.Hence, $Ax7\in VL$ and $Ax8\in VL$.

Ax9)(X⊃Y)⊃[(X⊃Z)⊃(X⊃{Y∩Z})]. Suppose that Ax9∉VL, so there is a model M, such that [(X⊃Y)⊃M(X⊃Z)⊃(X⊃{Y∩Z})])=0, by V⊃ results M(X⊃Y)=1, M(X⊃Z)=1, M(X)=1, M(Y∩Z)=0, applying V⊃ we derive M(Y)=1, M(Z)=1, which by V∩ means M(Y∩Z)=1, which is not the case. Hence, Ax9∈VL.

Ax10)–(Y∩–X)⊃–(Y∩(⊥⊃X)). It is satisfied by the rule V–~: M(–(Y∩–X))=1 implies M(–(Y∩~X))=1.Hence, Ax10∈VL.

Ax11)–(Y∩–(Z∩(⊥⊃X)))⊃–(Y∩–(Z∩–X)). It is satisfied by rule V––~: M(–(Y∩–(Z∩(⊥⊃X))))=1 implies M–(Y∩–(Z∩–X)))=1.Hence, Ax11∈VL.

Proposition 10. Sean X,Y∈FL. 1) If X∈TL then X∈VL.2) If {X_1, …, X_n}>>Y then {X_1, ...,X_n} *validates* Y.

Proof. 1) Suppose X∈TL. X∈VL is proved by induction over the length, L, of the proof of X.

Base step L=1. It means that X is an axiom, which from proposition 9 it follows that X∈VL.

Induction step. As an inductive hypothesis, we have that for every formula Y, if Y∈TL and the length of the proof of Y is less than L (where L>1) then Y∈VL. If X∈TL and the length of the proof of X is L, then X is an axiom or X is a consequence of applying Mp in earlier steps of the proof. In the first case, we proceed as in the base step. In the second case, we have for some formula Y, proofs of Y and Y⊃X, where the length of both proofs is less than L, using the inductive hypothesis it is inferred that Y∈VL and Y⊃X∈VL, so that, in any model M, we have M(Y)=1 and M(Y⊃X)=1, by V⊃ it turns out that M(X)=1, consequently, X∈VL. Using the principle of mathematical induction, it has been proven that, for every X∈FL, X∈TL implies X∈VL.

2) Suppose that{X_1, …, X_n}>>Y, applying CPC, we have (X_1∩X_2∩…∩X_n)⊃Y∈TL, from the part 1 is inferred, (X_1∩X_2∩…∩X_n)⊃Y∈VL, which by definition means that {X_1, …, X_n}*validates* Y.

IV. SEMANTIC-DEDUCTIVE CHARACTERIZATION

In this section, the characterization of LG with the semantics of the previous section is presented. In proposition 14 we have completeness and in proposition 15 we have semantic-deductive characterization.

Definition 7. An extension of a set of formulas C of LG, denotedC∈EXT(LG),is obtained by altering the set of formulas of C in such a way that the theorems of C are preserved, and that the language of the extension matches the language of LG. An extension is *locally consistent* if there is no X∈FL such that both X and ~X are extension theorems. A set of formulas is *locally inconsistent* if a contradiction Z∩~Z for some Z∈FL is derived from them. An extension is *locally complete* if for all X∈FL, either X is an extension theorem or ~X is an extension theorem.

Proposition 11. For X∈FL. 1)LT is *locally* consistent.2) If E∈EXT(LG), X∉TL-E, and E_x∈EXT(LG) are obtained by adding ~X as a new formula to E, then E_x is locally consistent.

Proof. 1) Suppose that LG is not *locally* consistent, so that there must be Z∈FL such that Z∩~Z∈TL, i.e. Z∩(Z⊃⊥)∈TL, by CPC results ⊥∈TL, by the validity theorem it is concluded that ⊥∈VL, i.e., for every model M, M(⊥)=1, which contradicts rule V⊥. Therefore, LG is *locally* consistent.

2) Let X∉TL-E, and let $E_x$the extension obtained by adding X as a new formula to E. Suppose that $E_x$is locally inconsistent, so that, for some Z∈FL, we have Z,~Z∈TL-E_x, by CPC we get ⊥∈TL-E_x, by Ax1 we derive X∈TL-E_x. But E_x differs from E only in that it has ~X as an additional axiom, so 'X is a theorem of E_x' is equivalent to 'X is a theorem of E from the set {~X}'. By proposition 1 it follows that ~X⊃X∈TL-E, and by CPC it is inferred that X∈TT-E, which is not the case, therefore E_x is locally consistent.

Proposition 12. If E∈EXT(LG) is locally consistent, then there is E'∈EXT(LG) that is *locally* consistent and complete.

Proof. Let be X_0, X $_1$, X_2, . . . an enumeration of all LG formulas. A sequence E'$_0$, E'$_1$, E'$_2$, . . . of extensions of E as follows: Let E'$_0$ = E. If X_0∈TL-E'$_0$, is E'$_1$ = E'$_0$, otherwise add ~X_0as a new formula to get E'$_1$from E'$_0$. In general, given t ≥ 1, to construct E'$_t$ from E'$_{t-1}$, we proceed as follows: if X_{t-1}∈TL-E'$_{t-1}$, then E'$_t$ = E'$_{t-1}$, otherwise let E'$_t$ be the extension of E'$_{t-1}$obtained by adding ~X_{t-1}as a new formula. The proof is widely known, details in Sierra (2023).

Proposition 13. If E'EXT∈(LG) is *locally* consistent, then there is a model in which all X∈TL-E' is true.

Proof. The model MF=(V_{MF}, v) is defined as follows: each extension F is associated with an MF model. For each MF and for each X∈FL, V_{MF}(X)=1 if X∈F; and V_{MF}(X)=0 if ~X∈F; v(–X)=1 if and only if

$-X \in F$, where F is the *locally* consistent and complete extension associated with MF. Note that V_{MF} is functional because F is *locally* consistent and complete. To claim that MF is a model, rules 1 through 7 of the model definitions must be guaranteed.

1. By CPC we have $\perp \supset \perp \in TL$, so $\sim\perp \in F$, i.e. $V_{MF}(\perp)=0$. Therefore, $V\perp$ is satisfied.

2. Using CPC we have the following chain of equivalences: $V_{MF}(X \supset Y)=0$, i.e. $\sim(X \supset Y) \in F$, by CPC we follow $X \cap \sim Y \in F$, resulting by CPC that $X \in F$ and $\sim Y \in F$, which means that $V_{MF}(X)=1$ y $V_{MF}(Y)=0$, so $V\supset$ is satisfied.

3. Suppose that $V_{MF}(-Z)=1$, so $-Z \in F$, from which $v(-Z)=1$, and then $V_{MF}(Z)=0$ o $v(-Z)=1$.

To prove the reciprocal, suppose $V_{MF}(Z)=0$ or $v(-Z)=1$. For the case $V_{MF}(Z)=0$, this means that $Z \notin F$, since F is complete, it is inferred that $\sim Z \in F$, using Ax5 can be assured that $-Z \in F$, i.e. $V_{MF}(-Z)=1$. For the case $v(-Z)=1$, this means $V_{MF}(-Z)=1$. So, if $V_{MF}(Z)=0$ o $v(-Z)=1$ then $V_{MF}(-Z)=1$. Since the reciprocal was initially proved, it is concluded that $V-$ is satisfied.

4. Suppose $V_{MF}(-X)=1$, so $-X \in F$, using Ax6, is derived $-(X \cap Y) \in F$, i.e. $V_{MF}(-(X \cap Y))=1$. Therefore, $V\cap$ is satisfied.

5. Suppose that $V_{MF}(X \cap Y)=1$, so $X \cap Y \in F$, applying Ax6 and Ax7 derive $X \in F$ and $Y \in F$, i.e. $V_{MF}(X)=1$ and $V_{MF(}Y)=1$. To prove the reciprocal, suppose $V_{MF}(X)=1$ and $V_{MF}(Y)=1$, which means that $X \in F$ and $Y \in F$, using Ax8 results in $X \cap Y \in F$, consequently, $V_{MF}(X \cap Y)=1$, Since the reciprocal was initially proved, it is concluded that $V\cap$ is satisfied.

6.Suppose that $V_{MF}(-(Y \cap -X))=1$, i.e., $-(Y \cap -X) \in F$, using Ax10 infers $-(Y \cap \sim X) \in F$, which means $V_{MF}(-(Y \cap \sim X))=1$. Therefore, $V-\sim$ is satisfied.

7. Suppose that $V_{MF}(-(Y \cap -(Z \cap \sim X)))=1$, i.e$-(Y \cap -(Z \cap \sim X))) \in F$, using Ax11 infers $-(Y \cap -(Z \cap -X)) \in F$, which means $V_{MF}(-(Y \cap -(Z \cap -X)))=1$. Therefore, $V--\sim$ is satisfied.

Based on the above analysis, it is inferred that M is an LG model.

To conclude the proof, let X be a theorem of E', so $X \in E'$. Therefore, using the definition of V_{ME}, it turns out that $V_{ME}(X)=1$, i.e., X is true in the model $ME=(V_{ME},v)$.

Proposition 14. For $X,X_1,\ldots,X_n \in FL$. 1) If $X \in VL$ then $X \in TL$.2) If $\{X_1, \ldots, X_n\}$ *validates* Y then $\{X_1, \ldots,X_n\}$>>Y.

Proof. 1) If $X \notin TL$, then, by proposition 11, the extension E', obtained by adding $\sim X$ as a new formula, is locally consistent. Thus, according to proposition 13, there is a model ME such that every theorem of E' is true in ME, and since $\sim X \in TL\text{-}E'$, then $\sim X$ is true in ME, i.e., X is false in ME, hence $X \notin VL$. It has been proven that$X \notin TL$ implies $X \notin VL$, i.e., $X \in VL$ implies $X \in TL$.

2)Suppose$\{X_1, \ldots, X_n\}$*validates* Y, i.e., $(X_1 \cap X_2 \cap \ldots \cap X_n) \supset Y \in VL$, by part 1, it follows that, $(X_1 \cap X_2 \cap \ldots \cap X_n) \supset Y \in TL$. If $\{X_1, \ldots, X_n\}$are assumed, by CPC we infer Y, hence $\{X_1, \ldots,X_n\}$>>Y.

Proposition 15. For $X,Y,X_1,\ldots,X_n \in FL$. 1) $X \in VL$ if and only if $X \in TL$.2)$\{X_1, \ldots, X_n\}$*validates* Y if and only if $\{X_1, \ldots, X_n\}$>>Y.

Proof. Direct consequence of propositions 10 and 14.

V. EXISTENTIAL GRAPHS GEG

This section presents the original gamma existential graphs, GEG, proposed in 4.516 of Peirce's *Collected Papers* (1965). For the construction of existential graphs, a variant of notation is used, proposed by Peirce in 4.378 of *Peirce's Collected Papers* (1965).

Definition 8. The setof graphs, GG, of *original existential gamma* graphs, GEG, is constructed from a set of *atomic graphs*, GA, and the constant λ (*empty graph*, $\boldsymbol{\lambda}$='_'), as follows.

1) $P \in GA$ implies $P \in GG$. 2) $\lambda \in GG$. 3) $X \in GG$ implies $\{X\},(X) \in GG$. 4) $X,Y \in GG$ implies $(X(Y)),XY \in GG$.

Definition 9. On the graph (X(Y)) it is called a *conditional graph.* The outer parentheses determine the external cut of the conditional, and the internal parentheses determine the *internal cut of the conditional*. X is called *antecedent* and Y *consequent*. Conditional cuts are called *continuous cuts.*

In the {Z} graph, the keys determine the *broken cut*. The part where Z is located is called the inner region of the broken cut or simply the *region of the broken cut*.

Definition 10. Let them be X,Y,Z∈GG. A graph X is said to be in an even *region*, denoted X_p, if X is surrounded by an even number of *cuts (continuous* and/or *broken).* X is in an odd *region*, denoted X_i, if X is surrounded by an odd number of *cuts (continuous* and/or *broken).*X_{nc} means that the graph X is in a region surrounded by *n* continuous and/or broken cuts (n = 0, 1, 2, 3, ...), where *n* can be *odd or even*. X_{pc} indicates an even number of continuous cuts. X_{1c} indicates an odd number of continuous cuts. X_{ncc} means that X is in a *region of continuous cuts only*, i.e., no *broken cuts*appear. X_{1cq} means that X is in a region with at least one broken cut.

Definition 11. Let be X∈GG. *Lambda* is defined as the assertion sheet **λ**= '_'. Strong *graph* is defined as *X = ({X}). *Total falsehood* is defined as ⊥={_}.

Definition 12. The system consists of the following RTRA primitive transformation rules:

A1) Alpha Rules. The primitive transformation rules of Pierce's Alpha existential graph system are primitive transformation rules of the GEG system. These rules are: Erase and Write, Iteration and Deiteration in regions of continuous cuts only or no cut, Write and Erase the empty double cut. The assertion sheet, λ, is the only axiom of the Alpha system.

A2) Writing graphs in broken cut region. On a broken cut that is written on the assertion sheet, any graph can be written. EG{}. {X}|⇒{XY}.

A3) Writing and erasing in the cuts. A continuous cut can be *partially erased* (generating a broken cut) when it is in an *even* region. 3a. Bc. $(X)_p$|⇒ {X}.

A broken cut can be *completed* (generating a continuous cut) when it is in an *even* region. 3b. Eccl. $\{X\}_i$|⇒(X).

In addition to the primitive rules, you have the following implicit rules:

RI1) Concatenation. Two graphs that are in the same region can be concatenated. Conversely, two graphs that are concatenated can be separated in the same region. Conc. X, Y⇔ YX, in any region.

RI2) Commutativity. Two concatenated graphs can be rewritten by changing the order. Com. XY⇔YX, in any region.

RI3) Associativity. In three graphs that are concatenated, *the order* in which they were concatenated is irrelevant. Initially, the first is concatenated with the second and this result is concatenated with the third, or the first is concatenated with the result of concatenating the second with the third. Aso. XY, Z ⇔ X,YZ ⇔ XYZ, in any region.

Remark. Rules RI1, RI2 and RI3 are called implicit rules, since, given their obviousness and graphic naturalness, they may not be referenced, but they are applied.

Definition 13. For X∈GG. X is a *graphical theorem* ofGEG, denoted X∈TG, if there is a *proof* of X from the graph λ, using the graph transformation rules, i.e., X is the last row of a finite sequence of lines, in which each of the lines is λ, or is inferred from previous rows, using the transformation rules. Or to put it briefly, X∈TG yes and only if λ>>X. The number of lines, of the finite sequence, is referenced as the *length* of the proof of X. Y>>X, means that X is obtained from Y using a finite number of transformation rules.

Proposition 16. For X,Y∈GG. Let be R∈RTRA and R≠R2. If $X_p \overset{R}{\Longrightarrow} Y$ then there exists R'∈RTRA such that $Y_i \overset{R'}{\Longrightarrow} X$.

Proof: by simple inspection of the primitive rules.

Proposition 17. For X,Y,Z∈GG. When you have an inference, in every *even* region of the antecedent you infer the consequent, provided you don't use rule R2. X>>Z impliesX_p>>Z.

When an inference is made, in every *odd* region of the consequent the antecedent is inferred, if rule R2 is not used. X>>Z implies Z_i>>X.

Proof: For X,Z∈GK, suppose X>>Z. it must be proved that [X_p>>Z and Z_i>>X].

If X>>Z then there are $R_1, \ldots, R_n$∈RTRA, and there are $X_1, \ldots, X_{n-1}$∈GG, such that $XR_1X_1R_2X_2 \ldots X_{n-1}R_nZ$, and the *length* of the transformation of X>>Z is said to be n and denoted by $X>>_nZ$.

The proof is performed by induction on the length of the transformation.

Base step. n=1. It means that only one of the primitive rules was applied, and since X is in an even region, then R must be of the form $X_p \overset{R}{\Longrightarrow} Z$ with R∈RTRA. From proposition 16 it is inferred that there is R', $Z_1 \overset{R'}{\Longrightarrow} X$ with R'∈RTRA.

Inductive step. Inductive hypothesis (∀n>1)[W>>$_n$K⇒{W_p>>K y K_i>>W}]. If X>>$_{n+1}$Z, then $XR_1X_1R_2X_2 \ldots X_{n-1}R_nX_nR_{n+1}Z$, i.e., $XR_1X_1R_2X_2 \ldots X_{n-1}R_nX_n$ and $X_nR_{n+1}Z$, so X>>$_n$$X_n$ and $X_nR_{n+1}Z$. Applying the inductive hypothesis and proposition 16 we get X_p>>X_n y $X_nR_{n+1}Z$, X_{ni}>>X and $Z_iR'_{n+1}X_n$. So, X_p>>Z_p and Z_i>>X.

By the principle of mathematical induction, the truth of the proposition is concluded.

Proposition 18. For X,Y,Z∈GG. A conditional graph can be written when the consequent is inferred from the antecedent, if the R2 rule is not used. Egc. X>>Z |⇒ (X(Z)).

A conditional graph can be erased when you have the antecedent. Bgc. X(X(Z))|⇒ X Z.

Proof. $\lambda \overset{R1}{\Rightarrow} ((\_)) \overset{R1}{\Rightarrow} (X(\_)) \overset{R1}{\Rightarrow} (X(X)) \xrightarrow{X\gg Y \text{ y proposición } 17} (X(Y))$. Hence, X>>Y ⇒ (X(Y)).

VI. EQUIVALENCE

In this section, the equivalence between LG and GEG is presented, initially, in proposition 20, it is proved that LG's theorems are graphical theorems of, in proposition 23, it is proved that the graphical theorems of are valid in the semantics of possible worlds, in proposition 26, it is proved that LEG's theorems are exactly the graph theorems.

Definition 14. FA=GA (atomic formulas are the same atomic graphs). Translation function [_]' of FL in GG. Let be X,Y∈FL and P∈FA.

1) P'=P. 2) [X⊃Y]'=(X'(Y)). 3) [X∨Y]'=((X')(Y')). 4) (−X)'={X'}. 5) λ'=λ. 6) (X∩Y)'=X'Y'. 7) (~X)'=(X').

Proposition 19. Let X∈FL be. If X is LG's axiom, then X'∈TG.

Proof. Using primitive rules, you have:

Ax1)⊥⊃X. By R1 we have ((_)), according to R1 we have ((X')(_)), i.e. (⊥⊃X)'. Therefore, (Ax1)' is a graphical theorem.

Ax2, Ax3, Ax4, Ax7, Ax8 and Ax9. Their translations are valid thanks to R1, since these are axioms of CPC, which is validated by the Alpha system.

Ax5)(X⊃−λ)⊃−X. (X')>>{X'} is satisfied by rule R3. It is concluded that (Ax5)' is a graphical theorem.

Ax6)−X⊃−(X∩Y). {X'}>>{X'Y'} is satisfied by rule R2. It is concluded that (Ax6)' is a graphical theorem.

Ax10)−(Y∩−X)⊃−(Y∩(⊥⊃X)). By R1 we have the sequence ((_)), so ({Y{X'}}(_)), we derive ({Y{X'}}({Y{X'}})), applying R4 we infer ({Y{X'}}({Y(X')})). It is concluded that (Ax10)' is a graphical theorem.

Ax11)−(Y∩−(Z∩(⊥⊃X)))⊃−(Y∩−(Z∩−X)). By R1 wehavethesequence((_)), so ({Y{Z(X')}}(_)), isderived ({Y{Z(X')}}({Y{Z(X')}})), applyingR4infers ({Y{Z(X')}} {Y{Z{X'}} })). It is concluded that (Ax11)' is a graphical theorem.

Proposition 20. For X∈FL. 1) If X∈TL then X'∈TG. 2) If X>>Y then X'>>Y'.

Proof. 1) Induction about the length of the X demonstration in LG.

Base step. If the length of the proof is 1, then X is an axiom, by the proposition 19 X'∈TG.

Induction step. The inductive hypothesis is: if Y∈TG and the length of the proof of Y is less than L, then Y'∈TG. Suppose X∈TG and that the length of the proof of X is L, so X is an axiom or obtained from previous steps using Mp. In the first case, proceed as in the base step. In the second case, Y and Y⊃X are taken in previous steps of the proof of X, i.e., the lengths of the proofs of Y and Y⊃X are less than L, by the inductive hypothesis it turns out that Y'∈TG and (Y'(X'))∈TG, applying R1 infers ((X'))∈TG, using R1 concludes X'∈TG.

By the principle of mathematical induction, LG's theorems are proved to be graphical theorems.

2) If X>>Y, then X⊃Y∈TL, by the part 1, (X'(Y'))∈TG, i.e., λ>>(X'(Y'))), if X' is assumed, by R1 follows ((Y')), applying R1 results in Y', so X'>>Y'.

Definition 15. Translation function, *(_)"* of GG in FG. For X,Y∈FL and P∈GA.

1) P''=P. 2) λ''= λ. 3) (X(Y))''=X''⊃Y''. 4) ((X)(Y))''=X''∪Y''. 5) {X}''=–X''.6) [XY]''= X''∩Y''.

7) (X)''=~X''.

Proposition 21. Rules R1 and R2 are valid rules in LG semantics

Proof. R1)Peirce's Alpha system rules are validated by CPC. Therefore, R1" is valid.

R2) {X}|⇒{XY}. Consider an arbitrary model M=(V_M,v).By V–∩M(–X")=1 implies M(–(X''∩Y''))=1. Therefore, R2",is a valid rule in LG.

Proposition 22. The R3. $(X)_p$|⇒{X} and $\{X\}_i$|⇒(X) are valid in LG's semantics.

Proof. Induction in the number, *n*, of negations surrounding X.

Base step. *n*=1. (X)|⇒{X}. Let M=(V_M,v) be any model. Suppose that V_M(~X)=1, by V it turns out that V_M(X)=0, applying V– we infer V_M(–X")=1. Therefore, V_M(~X")=1 implies V_M(–X")= 1, so R3 is satisfied for n=1.

n=2. There are 2 possibilities, {Y{X}}⇒{Y(X)} and (Y{X})⇒(Y(X)). For the first case, by the rule V–~ we have M(–(Y"∩–X"))=1 implies M(–(Y"∩~X"))=1, so the rule is satisfied. For the second case, let M=(V_M,v) be any model, suppose that V_M(~(Y"∩–X"))=1, i.e. V_M(Y"∩–X")=0, resulting in V_M(Y")=0 or V_M(–X")=0, using the result when n=1, deduces V_M(Y")=0 or V_M(~X")=0, which means that it is not the case that V_M(Y"∩~X")=1, and then V_M(~(Y"∩~X"))=1,has been tested, V_M(~(Y"∩–X"))=1 implies V_M(~(Y"∩~X"))=1, so the rule is satisfied. Therefore, R3 is satisfied for n=2.

n=3. There are 2 possibilities, {Y{Z(X)}}⇒{Y{Z{X}}} and (Y{Z(X)})⇒(Y{Z{X}}). For the first case, by rule V––~we have M(–(Y"∩–(Z"∩~X")))=1 implies M(–(Y"∩–(Z"∩–X")))=1, so the rule is satisfied. For the second case, let M=(VM,v) be any model, suppose that V_M((~(Y"∩–(Z"∩~X")))=1, i.e. V_M(Y"∩–(Z"∩~X"))=0, resulting in V_M(Y")=0 o V_M(–(Z"∩~X"))=0, using the result when n=2, we deduce V_M(Y")=0 o $V_{M(}$~(Z"∩~X"))=0, which means that it is not the case that V_M(Y"∩–(Z"∩–X"))=1,and then V_M(~(Y"∩–(Z"∩–X")))=1,has been tested, V_M(~(Y"∩–(Z"∩~X")))=1 implies V_M(~(Y"∩–(Z"∩–X")))=1, so the rule is satisfied. Therefore, R3 is satisfied for n=3.

Inductive step. Rule 3a. As an inductive hypothesis we have that, if (X") is surrounded by 2*n* negations, then {Y"{Z"(X")}}⇒{Y"{Z"{X"}}} and (Y"{Z"(X")})⇒(Y"{Z"{X"}}), are the only cases in which two other negations can be added to X, and they result in valid rules, as proved in the base step when n=3. Therefore, if X is surrounded by 2n+2 slices, i.e., by 2(*n*+1) slices, then R3 is satisfied.

Rule 3b. As an inductive hypothesis it is that, if (X) is surrounded by 2n+1 negations, then {Y"{Z"(X")}}⇒{Y"{Z"{X"}}} and (Y"{Z"(X")})⇒(Y"{Z"{X"}}), are the only cases in which, to X, two other negations can be added, and they result in valid rules, as proved in the base step when *n*=3. Therefore, if X is surrounded by 2n+3 cuts, i.e. by 2(*n+1)*+1 cuts, then R3 is satisfied.

By the principle of mathematical induction, the validity of R3 has been tested.

Proposition 23. For X∈GG. 1) The primitive rules of G are valid rules in the semantics of LG. 2) If X∈TG then X"∈VL.

Proof. 1)Direct consequence of propositions 21 and 22.

2) If X∈TG then λ>>X', so there are R_1, …, R_n∈RTRA, and there are X_1, …, X_{n-1}∈GE, such that $\lambda R_1X_1R_2X_2\ldots X_{n-1}R_nX$ (Proof length is n).

The proof is performed by induction over the length L of the demonstration.

Base step. L=1. It means that only one of the primitive rules was applied, then X"∈VG.

Inductive step. Inductive hypothesis: The proposition is valid if L<=n with n>0. Let L=n+1, so $\lambda R_1X_1R_2X_2\ldots X_{n-1}R_nX_nR_{n+1}X$, i.e., $\lambda R_1X_1R_2X_2\ldots X_{n-1}R_nX_n$ and $X_nR_{n+1}X$, both demonstrations with length less than n+1. Applying the inductive hypothesis is it turns out that X''_n∈VG and *from*X''_n *is validly inferred* X", hence X"∈VG.

By the principle of mathematical induction, the truth of the proposition is concluded.

Proposition 24. For X,Y∈GG. 1) If X∈TG then X"∈TL. 2) If X>>Y, then X">>Y".

Proof. 1) By Proposition 15 we have that, X"∈VL if and only if X"∈TL, and by Proposition 23 we have that, if X∈TG then X"∈VL. Therefore, if X∈TG then X"∈TL.

2) Direct consequence of part a and proposition 15.

Proposition 25. The translations presented in definitions 14 and 15 are inverse functions.
For$G\in GG$ and $X\in FL$.1) $[X']''=X$. 2) $[G'']'=G$.

Proof. The proof is presented in Sierra (2023).

Proposition 26. For $G,H\in GG$ and $X,Y\in FL$. 1) $G\in TG$ if and only if $G''\in TL$. 2) $X'\in TG$ if and only if $X\in TL$.

3) $G>>H$ if and only if $G''>>H''$.4) $X'>>Y'$ if and only if $X>>Y$.

Proof. 1) By proposition 24 we have that, if $G\in TG$ then $G''\in TL$, in addition, by proposition 20 we have that, if $G''\in TL$ then $(G'')'\in TG$, but by proposition 25 we know that, $(G'')'=G$, resulting that, if $G''\in TL$ then $G\in TG$, and since we have the reciprocal, we conclude that, $G\in TG$ if and only if $G''\in TL$.

2) By proposition 20 we have that, if $X\in TL$ then $X'\in TG$, in addition, by proposition 23 we have that, if $X'\in TG$ then $(X')''\in TL$, but by proposition 25 we know that, $(X')''=X$, resulting that, if $X'\in TG$ then $X\in TL$, and since we have the reciprocal, we conclude that, $X'\in TG$ if and only if $X\in TL$.

3) By proposition 20 we have, if $X''>>Y''$ then $[X'']'>>[Y'']'$, by proposition 25 we have $[X'']'=X$ and $[Y'']'=Y$, so if $X''>>Y''$ then $X>>Y$, in addition by proposition 24 we have the reciprocal. Therefore, $G>>H$ if and only if $G''>>H''$.

4) by proposition 24 we have, if $X'>>Y'$ then $[X']''>>[Y']''$, by proposition 25 we have $[X']''=X$ and $[Y']''=Y$, so if $X'>>Y'$ then $X>>Y$, by proposition 20 we have the reciprocal. Therefore, $X'>>Y'$ if and only if $X>>Y$.

VII. CONCLUSIONS

In this section, in conclusion 1, it is proved that the Original Gamma existential graphs are paraconsistent. This result is generalized by constructing the paraconsistent systems of existential graphs $GE[FX]^I$, and in conclusion 4, the semantic-deductive characterization of them is presented. Finally, in conclusion 6, it is proven that Gamma-4, Gamma-4.2 and Gamma-5 systems are paraconsistent.

Definition 16. Let SD be a deductive system with a negation operator N and let X be a formula for SD. SD is said *to be paraconsistent* when SD does not derive all SD formulas from X and NX.

Conclusion 1. For $G,H,K\in GG$. 1) $G''\supset(-G''\supset H'')\notin VG$.2) GEG is paraconsistent. 3) LG is paraconsistent.

Proof. 1) Consider a model $M=(V_M,v)$, such that $V_M(G'')=1$, $V_M(H'')=0$ and $v(-G'')=1$. As $v(-G'')=1$, then $V_M(-G'')=1$, and as $V_M(H'')=0$, then $V_M(-G''\supset H'')=0$, but also $V_M(-G'')=1$, consequently $V_M(G''\supset(-G''\supset H''))=0$. Therefore, $G''\supset(-G''\supset H'')\notin VG$.

2) Applying proposition 26 yields $(G((\{G\}(H))))\notin TG$, which implies that this is *not* the case: $G\{G\}>>H$. Therefore, GEG is paraconsistent.

3)Using proposition 26 it turns out that LG is paraconsistent.

In paraconsistent logics, for example, paraconsistent logics presented in Sierra (2005, 2007), the so-called *good behavior* operator is used, which simply allows the paraconsistent negation of a given formula to behave like the classical negation. In LG's case, the good behavior is defined below.

Definition 17. Let $X\in FL$ be. X is *incompatible with the negation paraconsistent*, denoted X^I, meaning that $-X\supset\sim X$. Now, consider a model $M=(V_M,v)$, if $V_M(-X\supset\sim X)=0$ then $V_M(X)=1$ and $V_M(-X)=1$, which implies that $V_M(X)=0$ o $v(-X)=1$, i.e. $v(-X)=1$. Consequently, for$V_M(-X\supset\sim X)=1$, it is necessary that $v(-X)=0$. Therefore, $V_M(X^I)=1$ can be defined as $v(X)=0$.

Conclusion 2. By CPC, $-X\supset\sim X$ (in GEG it would be $\{X'\}\Rightarrow(X')$) is equivalent to $X\supset\sim -X$ (in GEG it would be $X'\Rightarrow(\{X'\})$), i.e.,$X\supset +X$. From Ax5 the reciprocal ones are inferred. Therefore, X^I means that both negations coincide ($\sim X\equiv -X$) and both affirmations coincide ($X\equiv +X$). In GEG you would have $\{X'\}\Leftrightarrow(X')$ and $(\{X'\})\Leftrightarrow X'$ when X^I. Therefore, X'^I means that $X'\Rightarrow(\{X'\})$.

Conclusion 3. As a direct consequence of the definitions, if $X\in FL$ and $M=(V_M,v)$ is an LG model, then:

1)$V_M(X^I)=1$ equals $V_M(-X)=1$ implies $V_M(X)=0$, which also equals $v(-X)=0$.

2) $V_M M(X^I)=0$ equals $V_M(X)=1$ and $V_M(-X)=1$, which also equals $v(-X)=1$.

Now, if for all $X\in FL$ we have $V_M(X^I)=1$, then LG coincides with CPC, and GEG coincides with Alpha. But, if for all $X\in FL$ we have $V_M([+X]^I)=1$, i.e. $v(-+X)=0$, and then it turns out that $\sim+X\equiv-+X$, but it is not the case that $-X\supset\sim X$, since it has not been asked that $v(-X)=0$.

On the other hand, in LG, if $V_M(-+X\supset-X)=0$ then $V_M(-+X)=1$ y $V_M(-X)=0$, which implies that $V_M(+X)=0$ o $v(-+X)=1$, i.e. $V_M(-X)=1$ o $v(-+X)=1$, so $v(-+X)=1$. Therefore, $V_M(-+X\supset-X)=1$ if and only if $v(-+X)=0$.

As a consequence of the above, new deductive systems and the corresponding existential graphs can be constructed, as shown below.

Definition 18. For $X\in FL$,FX an LG formula, which contains X as a subformulaand GX'the graph associated with FX, which contains X' as a subgraph. The deductive system $LG[FX]^I$is constructed by adding to the LG system the axiom Ax12: $[FX]^I$. The $Sem[FX]^I$ semantics is constructed by adding to the LG semantics (denoted as Sem), the rule $V[FX]^I$: $V_M([FX]^I)=1$. The existential graph system $GEG[FX]^I$ is constructed by adding to the system GEG, the rule: $[GX']^I$.

Conclusion 4. The deductive system $LG[FX]^I$ is characterized by $Sem[FX]^I$ semantics, and is equivalent to the existential graph system $GEG[FX]^I$. In addition, $LG[FX]^I$ and $GEG[FX]^I$ are paraconsistent, when this is not the case that $FX\equiv X$.

Proof. All the scaffolding with which the equivalence between LG and GEG was tested is used.

To prove that the theorems of $LG[FX]^I$ are valid in $Sem[FX]^I$, it is sufficient to add to proposition 9 the validity of the axiom $[FX]^I$, which is satisfied by the rule $V[FX]^I$. To prove that the valid formulas in $Sem[FX]^I$ are theorems of $LG[FX]^I$, it is sufficient to guarantee in proposition 13 the validity of the rule $V[FX]^I$, which is achieved by the axiom $[FX]^I$. To prove that, if Z is a theorem of $LG[FX]^I$ then Z' is a graphical theorem of $GEG[GX']^I$, it is sufficient to guarantee in proposition 19, $([GX]^I)'$ is a graphical theorem of $GEG[GX']^I$, which is true thanks to the rule $[GX']^I$.

To prove the reciprocal, it is sufficient to guarantee in proposition 22 that the rule $[GX']^I$ is valid in $Sem[FX]^I$, which is a fact since the rule $[GX']^I$ is the translation of the axiom $[FX]^I$. Finally, these systems are paraconsistent when $FX\equiv X$ is not the case, since this means that there exists a model $M=(V_M,v)$, such that $V_M[X]^I=0$, i.e. $V_M(X)=1$ and $V_M(-X)=1$.

Definition 19. The $LG\lambda^I$ deductive system is constructed by adding to the LG system, the axiom Ax12: $\sim-\lambda$. The $Sem\lambda^I$semantics is constructed by adding to the LG semantics, the rule $V\sim-\lambda$: $V_M(\sim-\lambda)=1$. The system of existential graphs $GEG\lambda^I$is constructed by adding to the system GEG, the rule: $(\{\lambda\})$. Notice what $\sim-\lambda$means λ^I.

The LG4 deductive system builds by adding to the LG system, the axiom Ax12: $+X\supset++X$. The Sem4 semantics is constructed by adding to Sem, the V++ rule: $V_M(+X\supset++X)=1$. The GEG4 system of existential graphs is constructed by adding to the GEG system, the rule: $*X'|\Rightarrow**X'$.

Note. Ax12 means $[+X]^I$. $\sim-\lambda$ means λ^I. In addition, ({_}) is not valid in GEG, since to validate it is required that, $V_M(\sim-\lambda)=1$ (which means $[+\lambda]^I$), i.e. $V_M(-\lambda)=0$, deriving that $v(-\lambda)=0$, which is not in the semantics of LG.

Conclusion 5.1) The deductive system $LG\lambda^I$is characterized by $Sem\lambda^I$semantics and is equivalent to the existential graph system $GEG\lambda^I$;2) the deductive system LG4 is characterized by Sem4 semantics and is equivalent to the existential graph system GEG4.

Remark. GEG4 does not coincide with Zeman's (1964) Gamma-4, since in Gamma-4 there is the rule erased in even regions, which involve broken cuts or not, but in GEG4 this does not occur, for example, the transformation $\{\{X'Y'\}\}\Rightarrow\{\{X'\}\}$, corresponding to the formula $--(X\cap Y)\supset--X$, is not valid in GEG4, since it is refuted by the model $M1=(V_{M1},v1)$ with$v1(--(X\cap Y))=v1(-X)=1$ and $v1(--X)=0$; and by the model $M2=(V_{M2},v2)$ where $V_{M2}(X)=V_{M2}(Y)=v2(-X)=1$ and $v2(--X)=v2(-(X\cap Y))=0$.

Conclusion 6. The Gamma-4, Gamma-4.2 and Gamma-5 systems presented by Zeman are paraconsistent.

Proof. Gamma-4, Gamma-4.2 and Gamma-5 correspond to the modal logic systems S4, S4.2 and S5 which are characterized by semantics of possible worlds, in which the broken cut corresponds to the possibility of the classical negation, so the rule {X}⇒(X), corresponds to the modal formula ⊗~X⊃~X, which by CPC is equivalent to X⊃~⊗~X, i.e., X⊃□X (where □is the operator of necessity of such systems), and in such systems the reciprocal is valid, so we would have X≡□X, if ⊗~X⊃~Xwere valid, but the formula X≡□X, in fact, is not valid in such semantics (and should not be, since, in that case, the modalities would make no difference with the statement of classical logic, S4, S4.2 and S5 would collapse into CPC), for details see Hughes and Cresswell (1968), consequently, ⊗~X⊃~X is not valid, neither in Gamma-4, nor in Gamma-4.2 nor in Gamma-5. Therefore, the 3 systems of existential graphs are paraconsistent.

Remark. In Sierra (2023) is presented the paraconsistent system GT4,whichcharacterizes the GET4 system of existential graphs. It is proven that GET4 matches Zeman's Gamma-4.

Definition 20. LG_0=LG–{Ax6, Ax10, Ax11}. GEG_0=GG–{R2, R3}. Sem_0=Sem–{V–∩, V– ~, V––~}.

$LG_{0.1}$=LG–{Ax6}. $GEG_{0.1}$=GG–{R2}. $Sem_{0.1}$=Sem–{V–∩}.

Conclusion 7. 1)LG_0 equals GEG_0, and LG_0 is characterized by Sem_0.

2) $LG_{0.1}$ equals $GEG_{0.1}$, and $LG_{0.1}$ is characterized by $Sem_{0.1}$.

Existential Graphs	*ParaconsistentDeductive System*
GEG_0	*LG_0.LBPc. CLuN*
$GEG_{0.1}$	*$LG_{0.1}$*
GEG	*LG*
GEGλ[I]	*LGλ[I]*
GEG[+X][I]. GEG4	LG[+X][I]. LG4
GEG[FX][I]	LG[FX][I]
GET	*GT*
GET4. Gamma-4.	*GT4. S4*
Gamma-4.2	*S4.2*
Gamma-5	*S5*

Summary

Remark. From Sierra (2005), LG_0 equals LBPc (Basic Logic Paraconsistent without classical negation). From Batens & De Clercq (2004), LG_0 equals CluN (basic paraconsistent logic).

In Sierra (2023) is presented the paraconsistent system GT, which characterizes the GET system of existential graphs.It is proven that GET is an intermediate system, it is located between GEG and GET.

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