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An Overview on Polar Plots $r = 1 + 2^m cos\ (\theta)$

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Abstract *In this article we studied various polar plots* $r = 1 + 2^m cos\ (\theta)$ *. We consider only five cases* $m = 0$, $m = 1$; $m = 2, m = 3$; $m = 4$*; and* $m = 5$. *In the application part we established some designs with the help of these polar plots.*

Key Words *Cartesian coordinate System***,** *Polar Coordinate System, Polar Equation, Plotting.*

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I. Introduction

The polar coordinate system is the most beautiful two- dimensional coordinate system with wide applications. In this system each point on a plane is laid down by a distance from a reference point (analogous to the origin of a cartesian coordinate system) with angle from a reference direction. More precisely the reference point is called the pole, and the distance in the reference direction from the pole is known as polar axis. The distance from pole is nominated as radial coordinate (radial distance, or radius in the simple sense), and the angle is nominated as the angular coordinate (Polar angle, or azimuth). Angels in polar notation measured in either degrees or radians ($2\pi\ radians = 360^0$). We have a huge collection of literature in the form of books, research articles, etc. that provides the introduction of the polar coordinate system and its applications in the field of experimental sciences as per references [1], [2], [3], [4], [5], [6], [1], [7], [8], [9], etc. The current study provides some nice results on polar plots $r = 1 + 2^m cos\ (\theta)$. We consider only five cases $m = 0,\ m = 1;\ m = 2, m = 3;\ m = 4$; and $m = 5$. After that we settled some designs with the help of these polar plots.

1. Case Under the Consideration $m = 0$, the equation $r = 1 + 2^m cos\ (\theta)$ will be $r = 1 + cos\ (\theta)$. The plot of this equation will be evaluated by Wolfram Alpha for $0 \le \theta \le 2\pi$ as-

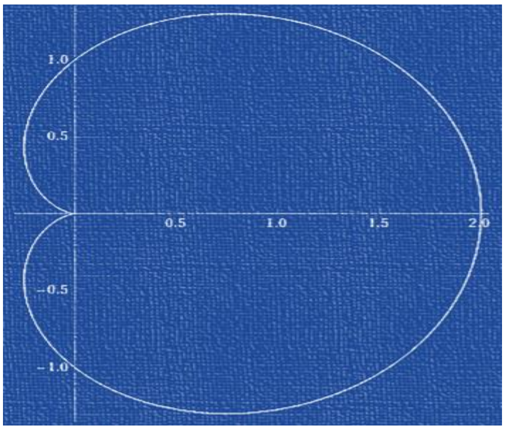
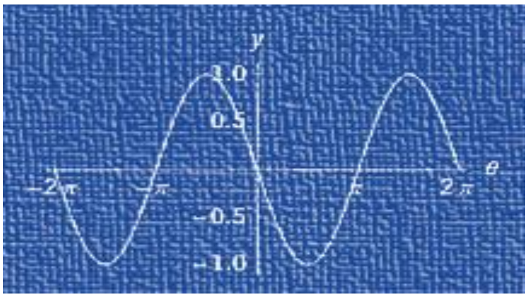


Figure (2.1)

1.1 Derivative The derivative of the equation $r = 1 + cos\ (\theta)$ evaluated by differentiate the sum term by term, i.e.,

$$\frac{dr}{d\theta} = \frac{d}{d\theta}\big(1 + cos\ (\theta)\big) = \frac{d}{d\theta}(1) + \frac{d}{d\theta}(cos\ (\theta) = -\sin\theta$$

A. Plot of $\frac{d}{d\theta}\big(1 + cos\ (\theta)\big)$- , $-2\pi \le \theta \le 2\pi$-, $-26 < \theta < 26$.



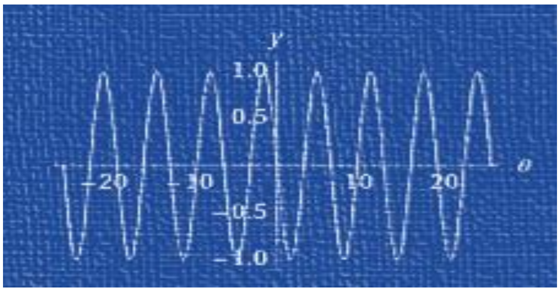


Figure (2.1-a, b)

B. Alternate Form of this equation is $\frac{1}{2}\ i(e^{i\theta} - e^{-i\theta})$.
C. Roots of this equation is $\theta = n\pi, n \in Z$.
D. Integer Root $\theta = 0$.
E. Properties as a function (Approximate forms)- It is 2π periodic in θ, with odd parity.
F. Taylor Series expansion at $\theta = 0$ evaluated as

$$-\theta + \frac{\theta^3}{6} - \frac{\theta^5}{120} + O(\theta^6)$$

G. Indefinite Integral evaluated as

$$\int - \sin\theta\, d\theta = \cos\theta + c, \text{ where } c \text{ is some constant}$$

H. Definite Integral over interval 0 to π evaluated as

$$\int_0^{\pi} (-\sin\theta)\, d\,\theta = -2$$

I. Definite Integral mean square over interval 0 to 2π evaluated as

$$\int_0^{2\pi} \left(\frac{\sin^2\theta}{2\pi}\right) d\theta = \frac{1}{2}$$

J. Global Maxima and Minima evaluated as

$$\max[-\sin\theta] = 1,\ \text{at } \theta = 2\pi n - \frac{\pi}{2}, n \in Z; \text{ and } \theta = 2\pi n + \frac{3\pi}{2}, n \in Z$$
$$\min[-\sin\theta] = -1,\ \text{at } \theta = 2\pi n + \frac{\pi}{2}, n \in Z$$

2. Case Under the Consideration $m = 1$, the equation $r = 1 + 2^m cos\ (\theta)$ will be $r = 1 + 2cos\ (\theta)$. The plot of this equation will be evaluated by Wolfram Alpha for $0 \le \theta \le 2\pi$ as-

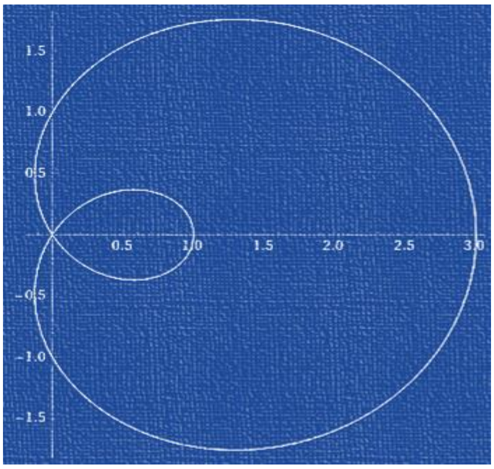
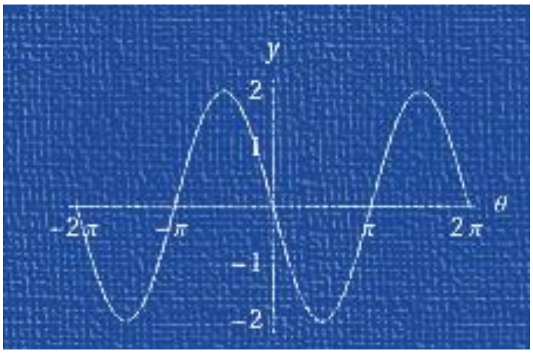


Figure (3.1)

2.1 Derivative The derivative of the equation $r = 1 + 2cos\ (\theta)$ evaluated by differentiate the sum term by term, i.e.,

$$\frac{dr}{d\theta} = \frac{d}{d\theta}\big(1 + 2cos\ (\theta)\big) = \frac{d}{d\theta}(1) + \frac{d}{d\theta}(2cos\ (\theta) = -2\sin\theta$$

A. Plot of $\frac{d}{d\theta}\big(1 + 2cos\ (\theta)\big)$- , $-2\pi \le \theta \le 2\pi$-, $-26 < \theta < 26$.



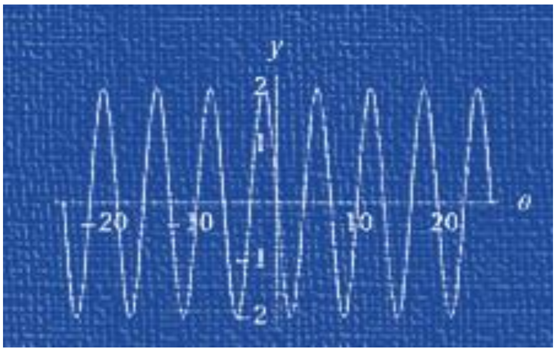


Figure (3.1-a, b)

B. Alternate Form of this equation is $i(e^{i\theta} - e^{-i\theta})$.
C. Roots of this equation is $\theta = n\pi, n \in Z$.
D. Integer Root $\theta = 0$.
E. Properties as a function (Approximate forms)- It is 2π periodic in θ, with odd parity.
F. Taylor Series expansion at $\theta = 0$ evaluated as

$$-2\theta + \frac{\theta^3}{3} - \frac{\theta^5}{60} + O(\theta^6)$$

G. Indefinite Integral evaluated as

$\int -2\sin\theta\, d\theta = 2\cos\theta + c$, where c is some constant

H. Definite Integral over interval 0 to π evaluated as

$$\int_0^{\pi} (-2\sin\theta)\, d\,\theta = -4$$

I. Definite Integral mean square over interval 0 to 2π evaluated as

$$\int_0^{2\pi} \left(\frac{2\sin^2\theta}{\pi}\right) d\theta = 2$$

J. Global Maxima and Minima evaluated as

$$\max[-\sin\theta] = 2,\ \text{at } \theta = 2\pi n - \tfrac{\pi}{2}, n \in Z; \text{and } \theta = 2\pi n + \tfrac{3\pi}{2}, n \in Z$$
$$\min[-\sin\theta] = -2,\ \text{at } \theta = 2\pi n + \tfrac{\pi}{2}, n \in Z$$

3. **Case** Under the Consideration $m = 2$, the equation $r = 1 + 2^m cos\ (\theta)$ will be $r = 1 + 4\ cos\ (\theta)$. The plot of this equation will be evaluated by Wolfram Alpha for $0 \le \theta \le 2\pi$ as-

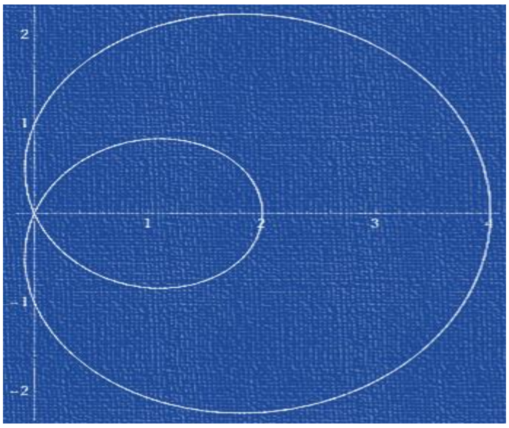


Figure (4.1)

3.1 **Derivative** The derivative of the equation $r = 1 + 4cos\ (\theta)$ evaluated by differentiate the sum term by term, i.e.,

$$\frac{dr}{d\theta} = \frac{d}{d\theta}\big(1 + 4cos\ (\theta)\big) = \frac{d}{d\theta}(1) + \frac{d}{d\theta}(4cos\ (\theta) = -4\sin\theta$$

A. Plot of $\frac{d}{d\theta}\big(1 + 4cos\ (\theta)\big)$- , $-2\pi \le \theta \le 2\pi$-, $-26 < \theta < 26$.

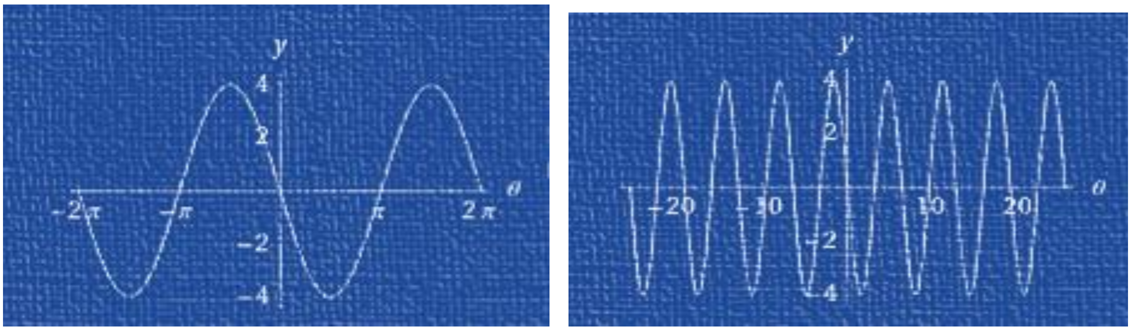


Figure (4.1-a, b)

B. Alternate Form of this equation is $2\, i(e^{i\theta} - e^{-i\theta})$.
C. Roots of this equation is $\theta = n\pi, n \in Z$.
D. Integer Root $\theta = 0$.
E. Properties as a function (Approximate forms)- It is 2π periodic in θ, with odd parity.
F. Taylor Series expansion at $\theta = 0$ evaluated as

$$-\theta + \frac{\theta^3}{6} - \frac{\theta^5}{120} + O(\theta^6)$$

G. Indefinite Integral evaluated as

$$\int -4\sin\theta\, d\theta = 4\cos\theta + c, \text{ where } c \text{ is some constant}$$

H. Definite Integral over interval 0 to π evaluated as

$$\int_0^{\pi} (-4\sin\theta)\, d\,\theta = -8$$

I. Definite Integral mean square over interval 0 to 2π evaluated as

$$\int_0^{2\pi} \left(\frac{\sin^2\theta}{2\pi}\right) d\theta = 8$$

J. Global Maxima and Minima evaluated as

$$\max[-\sin\theta] = 4, \text{ at } \theta = 2\pi n - \frac{\pi}{2}, n \in Z; \text{ and } \theta = 2\pi n + \frac{3\pi}{2}, n \in Z$$

$$\min[-\sin\theta] = -4, \text{ at } \theta = 2\pi n + \frac{\pi}{2}, n \in Z$$

4. Case Under the Consideration $m = 3$, the equation $r = 1 + 2^m cos\ (\theta)$ will be $r = 1 + 8\ cos\ (\theta)$. The plot of this equation will be evaluated by Wolfram Alpha for $0 \le \theta \le 2\pi$ as-

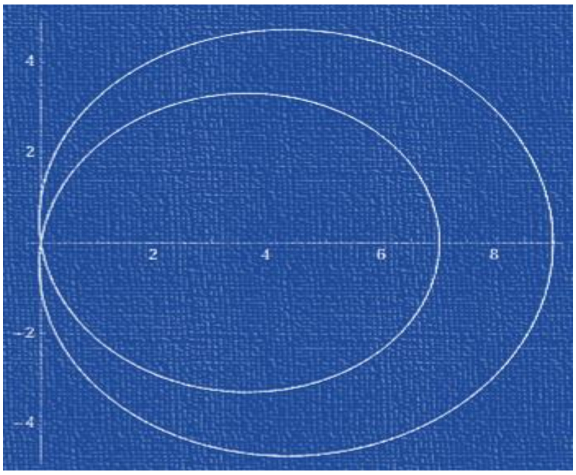
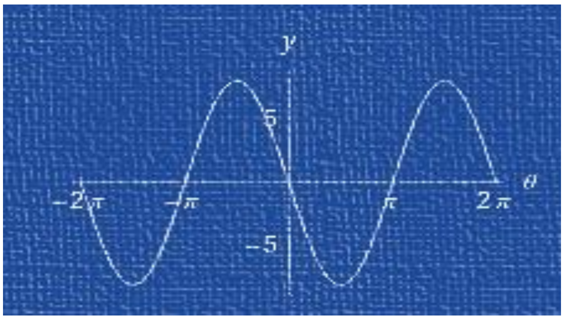


Figure (5.1)

4.1 Derivative The derivative of the equation $r = 1 + cos\ (\theta)$ evaluated by differentiate the sum term by term, i.e.,

$$\frac{dr}{d\theta} = \frac{d}{d\theta}\big(1 + 8cos\ (\theta)\big) = \frac{d}{d\theta}(1) + \frac{d}{d\theta}(8\ cos\ (\theta)) = -8\sin\theta$$

A. Plot of $\frac{d}{d\theta}\big(1 + 8\ cos\ (\theta)\big)$- , $-2\pi \le \theta \le 2\pi$-, $-26 < \theta < 26$.



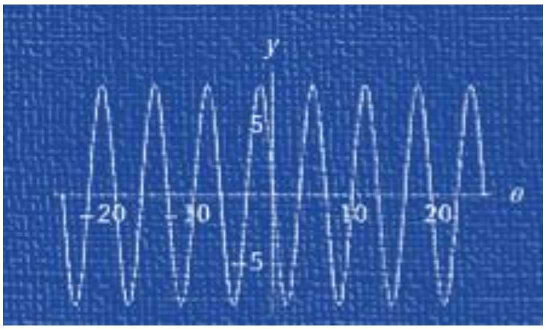


Figure (5.1-a, b)

B. Alternate Form of this equation is $4\, i(e^{i\theta} - e^{-i\theta})$.
C. Roots of this equation is $\theta = n\pi, n \in Z$.
D. Integer Root $\theta = 0$.
E. Properties as a function (Approximate forms)- It is 2π periodic in θ, with odd parity.
F. Taylor Series expansion at $\theta = 0$ evaluated as

$$-8\,\theta + \frac{4\,\theta^3}{3} - \frac{\theta^5}{15} + O(\theta^6)$$

G. Indefinite Integral evaluated as

$$\int -8\sin\theta\, d\theta = 8\cos\theta + c, \text{ where } c \text{ is some constant}$$

H. Definite Integral over interval 0 to π evaluated as

$$\int_0^{\pi} (-8\ \sin\theta)\, d\,\theta = -16$$

I. Definite Integral mean square over interval 0 to 2π evaluated as

$$\int_0^{2\pi} \left(32\frac{\sin^2\theta}{\pi}\right) d\theta = 32$$

J. Global Maxima and Minima evaluated as

$$\max[-\sin\theta] = 8,\ \text{at } \theta = 2\pi n - \frac{\pi}{2}, n \in Z; \text{ and } \theta = 2\pi n + \frac{3\pi}{2}, n \in Z$$

$$\min[-\sin\theta] = -8,\ \text{at } \theta = 2\pi n + \frac{\pi}{2}, n \in Z$$

5. **Case** Under the Consideration $m = 4$, the equation $r = 1 + 2^m cos\ (\theta)$ will be $r = 1 + 16\ cos\ (\theta)$. The plot of this equation will be evaluated by Wolfram Alpha for $0 \le \theta \le 2\pi$ as-

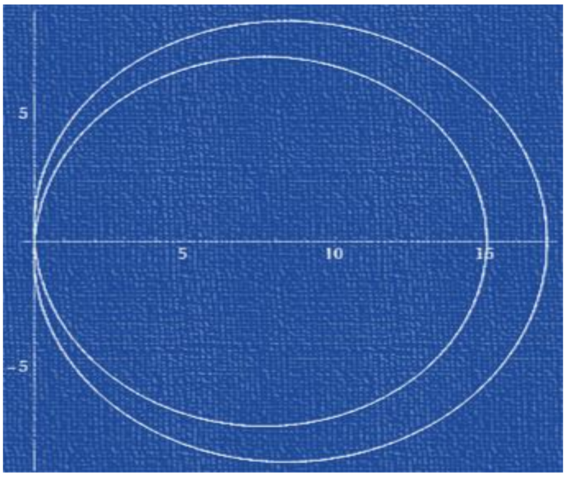
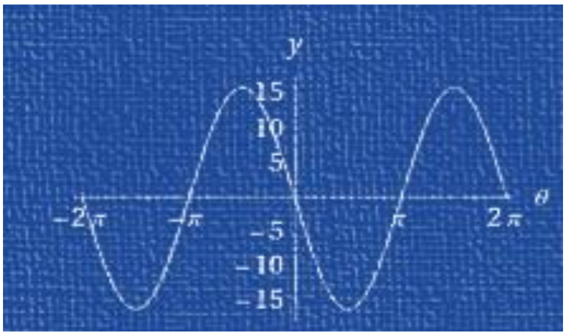


Figure (6.1)

5.1 **Derivative** The derivative of the equation $r = 1 + 16\ cos\ (\theta)$ evaluated by differentiate the sum term by term, i.e.,

$$\frac{dr}{d\theta} = \frac{d}{d\theta}\big(1 + 16\ cos\ (\theta)\big) = \frac{d}{d\theta}(1) + \frac{d}{d\theta}(16\ cos\ (\theta) = -16\sin\theta$$

A. Plot of $\frac{d}{d\theta}\big(1 + 16\ cos\ (\theta)\big)$- , $-2\pi \le \theta \le 2\pi$-, $-26 < \theta < 26$.



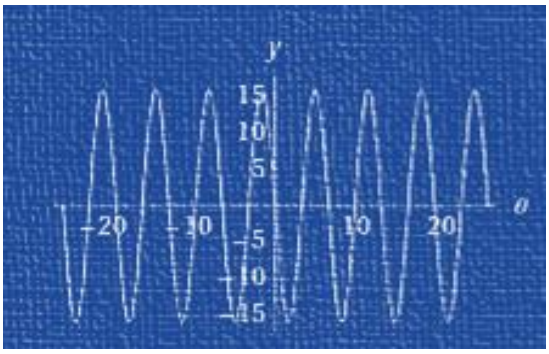


Figure (6.1-a, b)

B. Alternate Form of this equation is $8i(e^{i\theta} - e^{-i\theta})$.
C. Roots of this equation is $\theta = n\pi, n \in Z$.
D. Integer Root $\theta = 0$.
E. Properties as a function (Approximate forms)- It is 2π periodic in θ, with odd parity.
F. Taylor Series expansion at $\theta = 0$ evaluated as

$$-16\,\theta + \frac{8\theta^3}{3} - \frac{2\theta^5}{15} + O(\theta^6)$$

G. Indefinite Integral evaluated as

$\int -16 \sin\theta\, d\theta = 16\cos\theta + c$, where c is some constant

H. Definite Integral over interval 0 to π evaluated as

$$\int_0^{\pi} (-16\sin\theta)\, d\,\theta = -32$$

I. Definite Integral mean square over interval 0 to 2π evaluated as

$$\int_0^{2\pi} \left(\frac{128\sin^2\theta}{\pi}\right) d\theta = 128$$

J. Global Maxima and Minima evaluated as

$$\max[-16\sin\theta] = 16,\ \text{at } \theta = 2\pi n - \frac{\pi}{2}, n \in Z;\ \text{and } \theta = 2\pi n + \frac{3\pi}{2}, n \in Z$$
$$\min[-16\sin\theta] = -16,\ \text{at } \theta = 2\pi n + \frac{\pi}{2}, n \in Z$$

6. **Case** Under the Consideration $m = 4$, the equation $r = 1 + 2^m cos\,(\theta)$ will be $r = 1 + 32\,cos\,(\theta)$. The plot of this equation will be evaluated by Wolfram Alpha for $0 \le \theta \le 2\pi$ as-

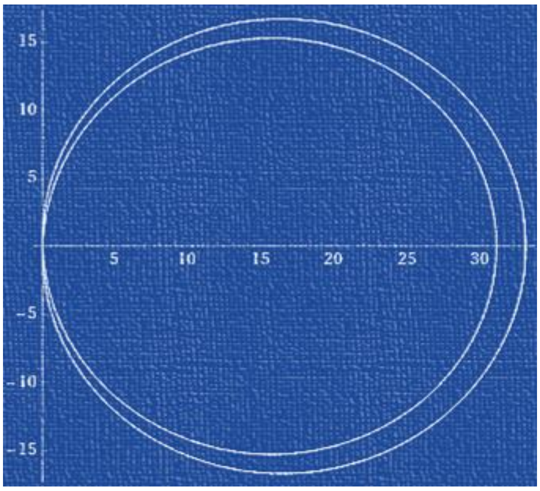
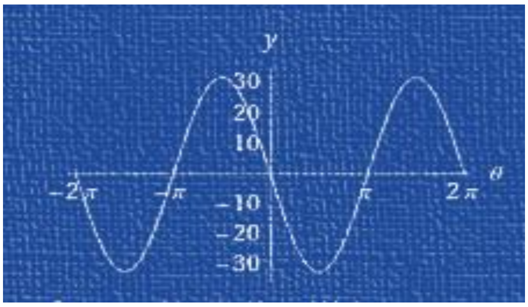


Figure (7.1)

6.1 **Derivative** The derivative of the equation $r = 1 + 32\,cos\,(\theta)$ evaluated by differentiate the sum term by term, i.e.,

$$\frac{dr}{d\theta} = \frac{d}{d\theta}\big(1 + 32\ cos\ (\theta)\big) = \frac{d}{d\theta}(1) + \frac{d}{d\theta}(32\ cos\ (\theta) = -32\sin\theta$$

A. Plot of $\frac{d}{d\theta}\big(1 + 32\ cos\ (\theta)\big)$- , $-2\pi \le \theta \le 2\pi$-, $-26 < \theta < 26$.



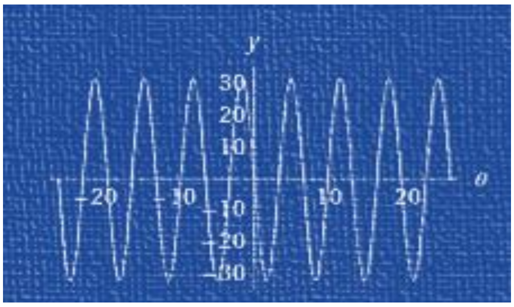


Figure (2.1-a, b)

B. Alternate Form of this equation is $16\ i(e^{i\theta} - e^{-i\theta})$.
C. Roots of this equation is $\theta = n\pi, n \in Z$.
D. Integer Root $\theta = 0$.
E. Properties as a function (Approximate forms)- It is 2π periodic in θ, with odd parity.
F. Taylor Series expansion at $\theta = 0$ evaluated as

$$-32\ \theta + \frac{16\ \theta^3}{3} - \frac{4\ \theta^5}{15} + O(\theta^6)$$

G. Indefinite Integral evaluated as

$$\int -32\ \sin\theta\ d\theta = 32\ \cos\theta + c, \text{ where } c \text{ is some constant}$$

H. Definite Integral over interval 0 to π evaluated as

$$\int_0^{\pi} (-32\ \sin\theta)\ d\ \theta = -64$$

I. Definite Integral mean square over interval 0 to 2π evaluated as

$$\int_0^{2\pi} \left(\frac{512\ \sin^2\theta}{\pi}\right) d\theta = 512$$

J. Global Maxima and Minima evaluated as

$$\max[-\sin\theta] = 32,\ \text{ at } \theta = 2\pi n - \frac{\pi}{2}, n \in Z; \text{ and } \theta = 2\pi n + \frac{3\pi}{2}, n \in Z$$

$$\min[-\sin\theta] = -32,\ \text{ at } \theta = 2\pi n + \frac{\pi}{2}, n \in Z$$

7. Designs Here, we develop the following designs with the help of above discussed polar plots.
7.1 (i) Desigen from $r = 1 + cos\ (\theta)$- Choose polar plot of this function and develop such as

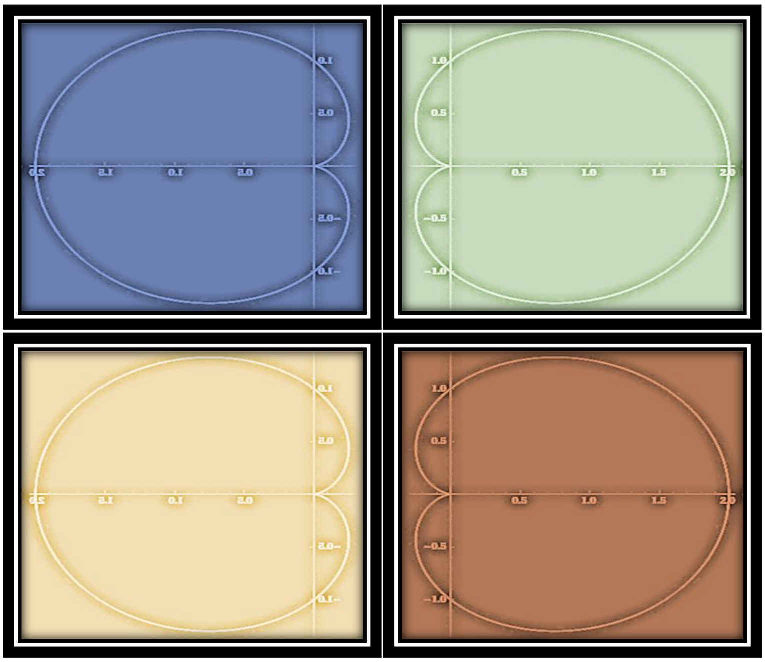


Figure 8.1 (a)

8.1 (b) Design from Plot of $\frac{d}{d\theta}\left(1 + cos\,(\theta)\right)$- , $-2\pi \leq \theta \leq 2\pi$-, $-26 < \theta < 26$.

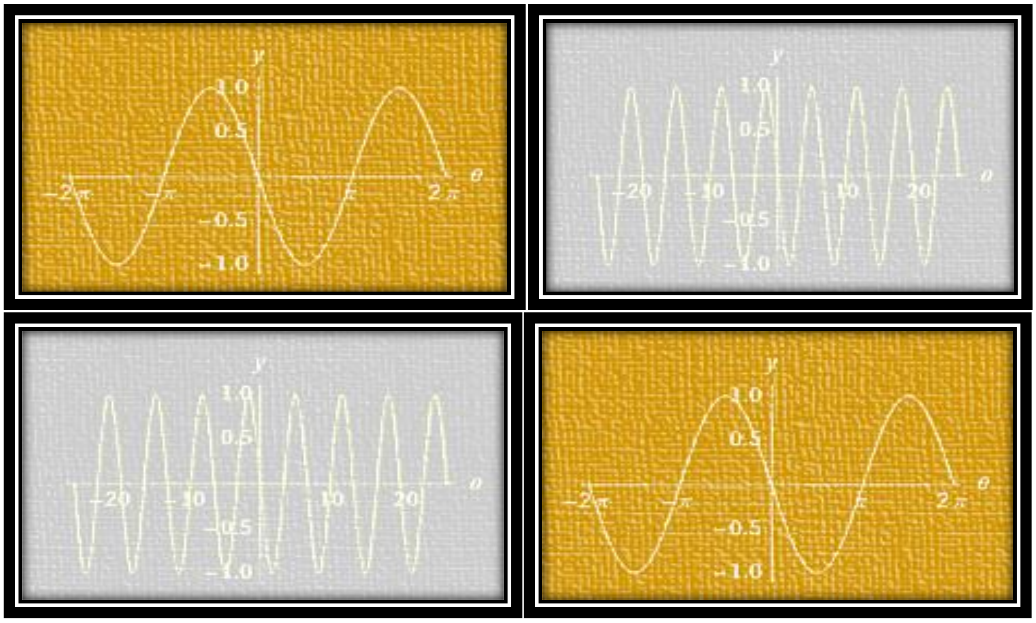


Figure 8.1 (a)

7.2 (a) Desigen from $r = 1 + 2\ cos\ (\theta)$- Choose polar plot of this function and develop such as

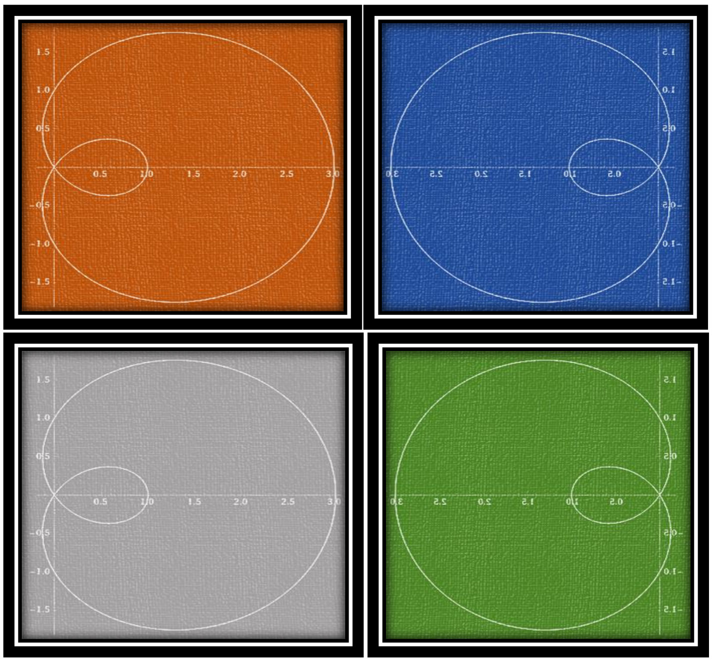


Figure 8.2 (a)

8.1 (b) Design from Plot of $\frac{d}{d\theta}\left(1 + 2\ cos\ (\theta)\right)$- , $-2\pi \leq \theta \leq 2\pi$-, $-26 < \theta < 26$.

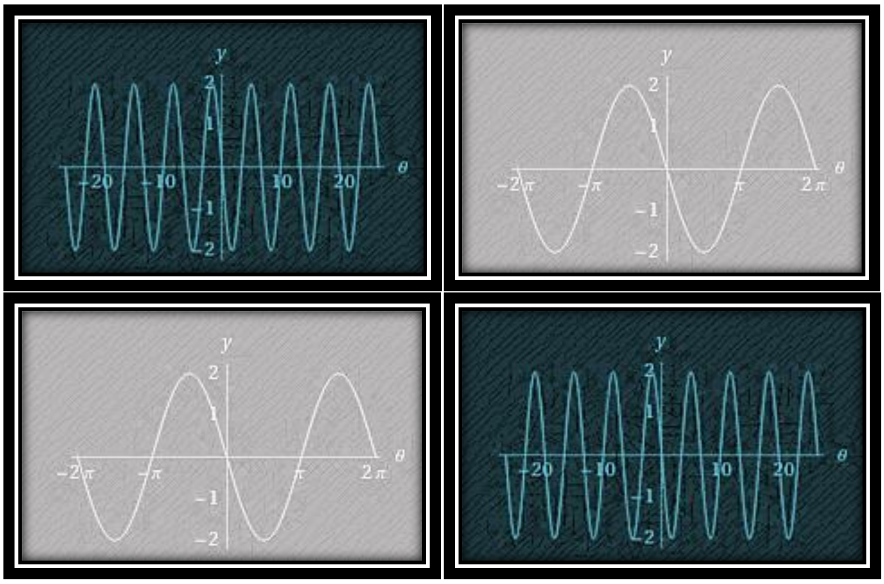


Figure 8.2 (b)

7.3 (a) Desigen from $r = 1 + 4\ cos\ (\theta)$- Choose polar plot of this function and develop such as

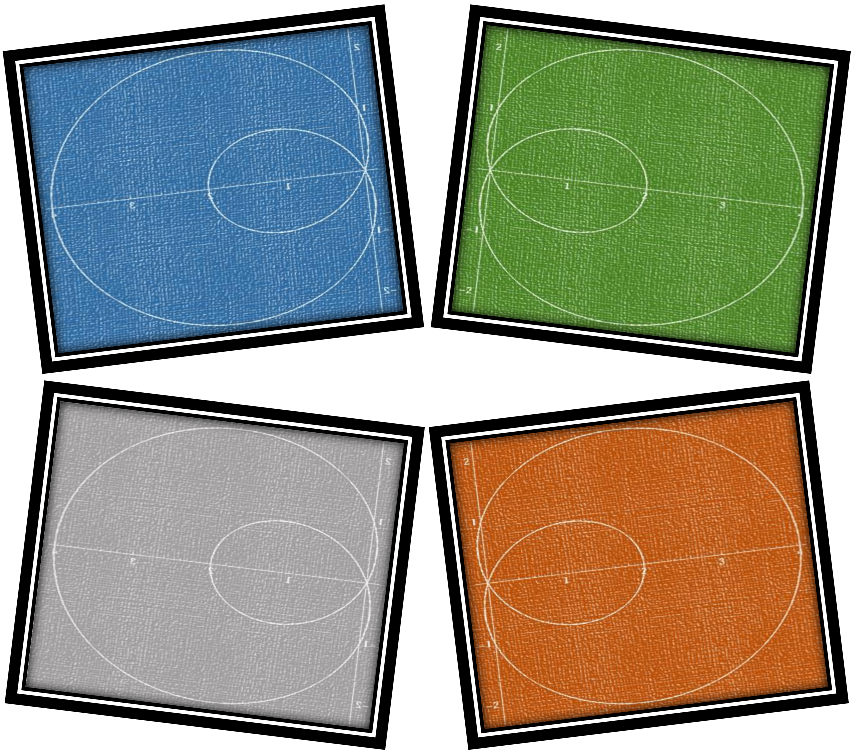


Figure 8.3 (a)

8.3 (b) Design from Plot of $\frac{d}{d\theta}\left(1 + 4\ cos\ (\theta)\right)$- , $-2\pi \leq \theta \leq 2\pi$-, $-26 < \theta < 26$. In this case we find the same designs as figure 8.1(b), 8.2(b).

8.4 (a) Desigen from $r = 1 + 8\ cos\ (\theta)$- Choose polar plot of this function and develop such as

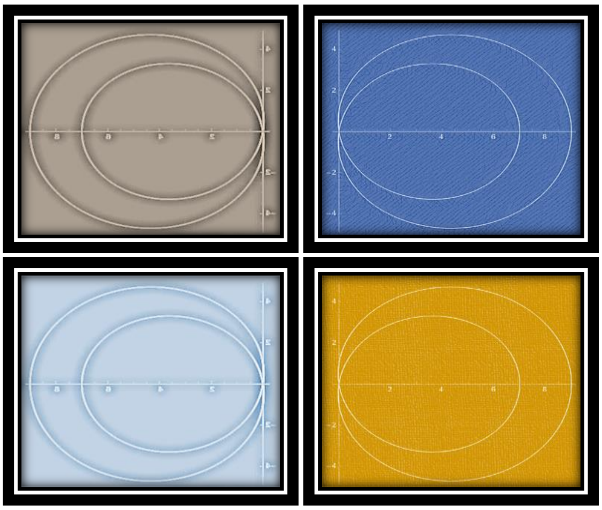


Figure 8.4(a)

8.4 (b) Design from Plot of $\frac{d}{d\theta}\left(1 + 4\ cos\ (\theta)\right)$- , $-2\pi \leq \theta \leq 2\pi$-, $-26 < \theta < 26$. In this case we find the same designs as figure 8.1(b), 8.2(b).

8.5 (a) Desigen from $r = 1 + 8\ cos\ (\theta)$- Choose polar plot of this function and develop more designs like previous cases we leave it for further development.

8.5 (b) Design from Plot of $\frac{d}{d\theta}\left(1 + 8\ cos\ (\theta)\right)$- , $-2\pi \leq \theta \leq 2\pi$-, $-26 < \theta < 26$. In this case we find the same designs as figure 8.1(b), 8.2(b).

8.6 **(a) Desigen from** $r = 1 + 16\ cos\ (\theta)$- Choose polar plot of this function and develop more designs like previous cases we leave it for further development.

8.6 **(b)** Design from Plot of $\frac{d}{d\theta}\big(1 + 16\ cos\ (\theta)\big)$- , $-2\pi \le \theta \le 2\pi$-, $-26 < \theta < 26$. In this case we find the same designs as figure 8.1(b), 8.2(b).

8.7 **(a) Desigen from** $r = 1 + 32\ cos\ (\theta)$- Choose polar plot of this function and develop more designs like previous cases we leave it for further development.

8.7 **(b)** Design from Plot of $\frac{d}{d\theta}\big(1 + 32\ cos\ (\theta)\big)$- , $-2\pi \le \theta \le 2\pi$-, $-26 < \theta < 26$. In this case we find the same designs as figure 8.1(b), 8.2(b).

Acknowledgement

I would like to thank all the team members at Wolfram Alpha for the amazing platform they have developed. So far, I have received an immediate response from Wolfram every time, thank you, Wolfram (https://www.wolframalpha.com/widgets/view.jsp?id=bad4781f9dadeb57bab5de78df22df91).

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