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Domination Problem In Triangular Grids

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Abstract:
Electric networks must be watched over constantly. Placing phase measuring units (PMUs) at certain network sites can effectively carry out this monitoring. The PMUs must be used in the smallest possible number due to their high cost. The goal of the power domination problem is to determine the bare minimum of PMUs required to monitor a specific electric power system. Despite the NP-hardness of the power domination problem, closed formulas for the power domination number of specific networks, like rectangular meshes [4], have been discovered. We apply these findings to triangular grids in this paper.
***Key Word**: Power domination; triangular grid graph.*

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I. Introduction

The issue of power domination occurred while monitoring electric power networks. A power network consists of nodes and the edges that connect them. It also includes a number of generators that produce electricity and a number of loads that receive the electricity. A power network has to have all of its state variables measured by measurement devices in order to be monitored. The voltage of a node and the current phase of the edges connected to the node can both be measured by a Phase Measurement Unit (PMU), a measurement tool that is mounted on a node.

The objective is to install the fewest number of PMUs necessary to monitor the entire system because PMUs are expensive equipment. These devices are equipped to monitor not only the nodes at which they are positioned but also their incident edges and neighboring nodes. PMUs are more advantageous because they do not require one measurement device per node like other measurement systems do.

Haynes et al. formulated this issue as a graph domination problem in [5]. But unlike the usual domination, this kind has a distinctive flavor. type problem, since the application of the domination rules can be iterated. Next, we give a formal description of the power domination problem in graph theory.

Let G = (V, E) be a graph representing an electric power system, where a vertex represents an electrical node and an edge represents a transmission line joining two electrical nodes. A PMU monitors, or dominates, the vertex at which it is placed and its incident edges and their end vertices. The other domination rules are as follows:
(1) Any vertex that is incident to a dominated edge is dominated.
(2) Any edge joining two dominated vertices is dominated.
(3) If a vertex is incident to $k > 1$ edges and if $k - 1$ of these edges are dominated, then all k of these edges are dominated.

A set S ⊆ V is defined to be a power dominating set of G if every vertex and every edge in G is dominated by S according to the previous domination rules. The power domination number of G, denoted by $\gamma p(G)$, is the minimum cardinality of a power dominating set of G. Notice that in the standard theory of domination, a set S ⊆ V is a dominating set in G if every vertex in V \S has at least one neighbor in S. The minimum cardinality of a dominating set of G is its domination number, denoted by $\gamma(G)$. Since any dominating set is also a power dominating set, we have $1 \leq \gamma p(G) \leq \gamma(G)$ for every graph G.

Consider a power network G = (V, E), in order to see the PDS problem and its graph theoretic description in further depth. Since the resistance of the power network's edges is a feature of the material used for its construction, it may be assumed that this resistance is known. We want to record electrical currents at the edges and voltages at all nodes. We can measure the voltage at a node v and the electrical current on each edge incident to v by installing a PMU at that node. Next, we may determine the voltage of any node around v by applying Ohm's law. Assume that v and all of its neighbors, with the exception of w, are known. We can calculate the current on all edges incident to v except the edge vw by using Ohm's law. The current on the edge vw is then computed using Kirchoff's law. Finally, applying Ohm's law on the edge vw gives us the voltage of w.

It has been demonstrated that the decision-making problem of determining if a given graph G has a power-dominating set of cardinality k is NP-complete even when restricted to bipartite graphs, chordal graphs, or even split graphs [9], a subclass of chordal graphs. In the case of interval graphs, Liao and Lee developed a linear approach addressing this issue, given that the interval ordering of the graph is known [9]. If the interval order is unknown, they provided an O(n logn) algorithm that they demonstrated to be asymptotically ideal. Other effective techniques for trees and, more generally, for graphs with bounded treewidth have been proposed [8].

Dorfling and Henning [4] were able to determine the grid graph's power domination number and minimal power dominating sets. When G is the lexicographic product of any two graphs or G is the direct product of paths, Dorbec et al. in [3] found γp(G). Later, Barrera and Ferrero obtained upper bounds for γp(G) when G is a cylinder, a torus, or a generalized Petersen graph,and identified instances in which their bounds overlap with the power domination number [2]. More generally, Zhao, Kang, and G.J. Chang [12] provided upper bounds for γp(G) for any arbitrary graph G. Upper bounds have been obtained for block graphs [11] and claw-free graphs [12].

The power domination number of the triangular grid graph is determined, in this study using a method akin to that used by Dorfling and Henning on the grid graph. All of the graphs taken into consideration here are undirected, finite, and simple, with V (G) and E(G) serving as the appropriate vertex set and edge set. We refer to [10] for all fundamental definitions and notations that are not covered in this paper.

II. Preliminaries

Triangular grids attract great attention due to its wide applications in interconnection networks. Various properties of triangular grids have been studied by many authors. The vertex bandwidth and the edge bandwidth of the triangular grid is obtained in [6] and [1], respectively. The number of triangular islands in triangular grids was obtained by Horvath et.al in [7]. In this section, we give the definitions and notations employed in this article.

Definition 1. [13] For any integer n, **the triangular grid with triangular boundary**, T_n, is the graph whose vertices are ordered triples of nonnegative integers summing to n, with an edge connecting two triples if they agree in one co-ordinate and differ by one in the other two.

Intuitively T_1, is a triangle. Then, T_2 is obtained from T_1 by adding three triangles to its bottom boundary. In general, T_n is obtained by adding 2n + 1 triangles to the bottom boundary of T_{n-1}. The dimension of T_n represents the number layers of triangles in it. An illustration of T_5 is given in Fig 1.

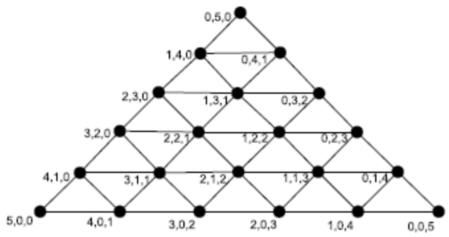

Fig 1: T_5

Let (i, j, k) be a vertex of a triangular grid T_n. We typically refer the first co-ordinate of such a triple as the i-coordinate, the second as the j-coordinate and the third as the k-coordinate. We use the notation i(v) to refer to the ith coordinate of v and so on. The following definition will enhance the geometric intuition behind our reasoning.

Definition 2. [3] For each integer c ∈ [0, n], the I-diagonal at c, I_c is defined as the subgraph induced by the vertices whose i-coordinate equals c. The J-diagonal at c, J_c and K-diagonal at c, K_c are defined similarly. A diagonal at zero is called a boundary of T_n. A vertex v is said to cover a diagonal if it belongs to that diagonal.

Note that there are n+1 I-diagonals, J-diagonals and K-diagonals, respectively in T_n. It is clear from the power domination rule (2) that in order to power dominate the whole graph it is enough to power dominate all the vertices. Hence, we can use the following simplified version of power domination introduced in [4] to obtain γ_p of triangular grids.

Definition 3. For a graph G and a set S ⊆ V (G), the closure of S in G is denoted by CG(S) is recursively defined as follows: Start with CG(S) = S. As long as exactly one of the neighbors of some element of CG(S) is not in CG(S), add that neighbor to CG(S).

If the graph G is clear from the context, we simply write C(S) rather than CG(S) . Note that the set of vertices power dominated by a set S is C(N[S]). In particular, if S ∈ V is power dominating set of G, then CG(N[S]) = V .

III. Triangular grid with triangular boundary

In this section we are going to prove that γ_P (Tn) $=\left\lceil\frac{n}{3}\right\rceil$for every positive integer n. We begin by showing that the previous expression gives an upper bound [13].

Lemma 3.1.[13] If G = T_n, then γ_P (G) $\leq\left\lceil\frac{n}{3}\right\rceil$

Proof. We consider three possibilities and give a power dominating set for each.

(i) If $n = 3m$:
$S = \{ \bigcup_{i=1}^{m} (3i-2,\ n-(3i-1),\ 1)\}$.

(ii) If $n = 3m+1$:
$S = \{ \bigcup_{i=1}^{m+1} (3i-3,\ n-(3i-2),\ 1)\}$.

(iii) If $n = 3m+2$
$S = \{ \bigcup_{i=1}^{m+1} (3i-3,\ n-(3i-2),\ 1)\}$.

In each case S is a power dominating set of order $\left\lceil\frac{n}{3}\right\rceil$. An illustration of a power dominating set for T_5 is given in Fig 2.

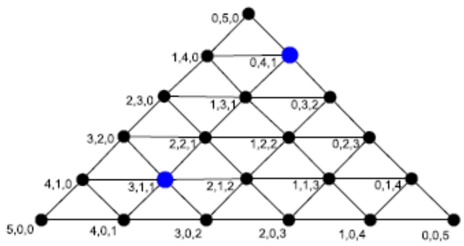

Fig 2: A power dominating set in T_5

To prove that the upper bound is also a lower bound we need the following result.

Lemma 3.2. [13] Let G = T_n. If S ⊆ G and $|S| < \frac{n}{3}$, then C(N[S]) covers at most 3 |S| diagonals.

Proof. Let G′ be the graph with V (G′) = V (G) and uv ∈ E(G′) if and only if $d_G(u, v)$ =2 and u and v do not cover a common diagonal or $d_G(u, v)$ = 2 and u and v cover a common boundary. It is clear that, for disjoint subsets S_1, S_2 ⊆ G, if no vertex of C(N[S_1]) is adjacent in G′ to any vertex of C(N[S_2]), then C (N[S_1]∪N[S_2]) = C (N[S_1])∪ C (N[S_2]). We may, therefore, assume that C(N[S]) is connected in G′.

An illustration of C(N[S]) is shown in Fig 3, where blue colored vertices form the set S. Note that in this case S is not a power dominating set. But, if we choose S to be the set {(0,4,1) , (3,1,1)}, then it power dominates the whole graph.

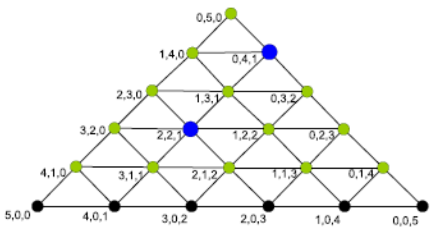

Fig 3: C(N[S]) in T_5

If |S| = 1, then it is clear that C(N[S]) can cover at most three diagonals. Let S ⊆ G with C(N[S]) connected in G′ and $|S| < \frac{n}{3}$. Assume the result for all S′ ⊆ S. Consider a maximal proper subset S′ ⊆ S such that C(N[S′]) is connected in G′. Now, since C(N[S]), is connected in G′, some vertex of C(N[S′]) is adjacent in G′ to some vertex of C(N[S\S′]). By maximality of S′, C(N[S\S′]) is connected. Since the inductive hypothesis also applies to S\S′, we have the following:

(1) The number of diagonals covered by $C(N[S'])\le 3\,|S'|$.
(2) The number of diagonals covered by $C((N[S\backslash S'])\le 3\,|S\backslash S'|$.
Therefore, from (1) and (2) we conclude that the number of diagonals covered by $C(N[S]) = C(N[S']) \cup C(N[S\backslash S'])$ is at most $3\,|S'| + 3\,|S\backslash S'| = 3\,|S|$.

Lemma 3.3. [13] If $G = T_n$, then $\gamma_P(G) \ge \left\lceil \frac{n}{3} \right\rceil$

Proof. By Lemma 3.2, if $S \subseteq G$ is such that $|S| < \frac{n}{3}$, then $C(N[S])$ covers at most n diagonals. But there are $n + 1$ diagonals in T_n and hence $\gamma_P(G) \ge \left\lceil \frac{n}{3} \right\rceil$

Theorem 3.1. [13] If $G = T_n$, then $\gamma_P(G) = \left\lceil \frac{n}{3} \right\rceil$

Proof. Clear from Lemma 3.2 and Lemma 3.3.

IV. Square triangular grids and rectangular triangular grids

In this section we extend the Theorem 3.1 to square triangular grid and also to rectangular triangular grids. The proofs are in similar lines. First, we slightly modify the definition of Tn to get a square triangular grid and then to get a rectangular triangular grid. We will use the following notation: for a given $n \in Z$,
$[m, n] = \{-m, -m + 1, \ldots, -1, 0, 1, 2, \ldots n\}$
$(n) = \{0, 1, 2, \ldots n\}$.

Definition 4. [13] A n × n triangular grid with square boundary , $ST_{n,n}$, has the vertex set,
$V(ST_{n,n}) = \{(i, j, k) : i \in [n, n], j, k \in (n) \text{ and } |i + j + k| = n\}$, with an edge connecting two triples if they agree in one co-ordinate and differ by one in the other two.

Theorem 4.1. [13]If $G = ST_{n,n}$, then $\gamma_P(G) = \left\lceil \frac{n}{3} \right\rceil$

Proof. For each of the possible case the set S given in Lemma 3.2 will be a power dominating set of order $\left\lceil \frac{n}{3} \right\rceil$for $ST_{n,n}$, which proves that the expression is an upperbound. To prove that it is an upperbound as well, the technique similar to that in Lemma 3.3 can be employed.

An illustration of a power dominating set in $ST_{5,5}$ is given in Fig 4.

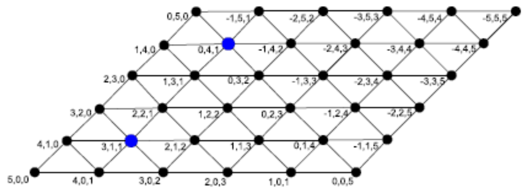

Fig 4: A power dominating set in $ST_{5,5}$

Definition 5. [13]Let n and m be integers such that $n < m$. A n×m rectangular triangular grid, $RT_{n,m}$, has the vertex set $V(RT_{n,m}) = \{(i, j, k) : i \in [m, n], j \in (m), k \in (n) \text{ and } |i + j + k| = n\}$, with an edge connecting two triples if they agree in one co-ordinate and differ by one in the other two.

Theorem 4.2. [13] If $G = RT_{n,m}$, then $\gamma_P(G) = \min\left\{\left\lceil \frac{m}{3} \right\rceil, \left\lceil \frac{n}{3} \right\rceil\right\}$

Proof. It is clear that, in order to power dominate the whole graph, we need to power dominate either all the I-diagonals or all the J-diagonals or all the K-diagonals. In $G = RT_{n,m}$, there are m+n+1 I-diagonals, m+1 J-diagonals and n+1 K-diagonals. Since the K-diagonals are minimum in number, it is desirable to power dominate all the K-diagonals, for which we need at least $\left\lceil \frac{n}{3} \right\rceil$ vertices. The proof follows in similar lines. An An illustration of a power dominating set in $RT_{5,6}$ is given in Fig 5.

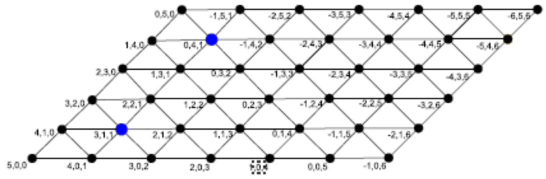

Fig 5: A power dominating set in $RT_{5,6}$

V. Conclusion

In this paper, we have computed the power domination number of triangular grids with triangular, square and rectangular borders.

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