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Analysis Of A Queuing System In A Restaurant (A Case Study Of TacoBell, Chandigarh)

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Abstract:
This paper has investigated application of single channel M/M/1 queuing model. The data has been collected from a busy restaurant located in Chandigarh. Arrival rate, Service rate and utilization rate were calculated.

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I. Introduction

The word queue comes from the Latin word 'cauda', meaning tail. Queuing theory is a mathematical study of queues or waiting lines. Service systems and waiting lines are an important part of our life. In queuing theory, a model is constructed and queue lengths and waiting times are predicted. This paper uses queuing theory to study waiting lines in Sagar Ratna restaurant situated in Chandigarh. Customers arrive in a random manner at the restaurant and are instructed to wait in a queue until it is their turn and once served, they leave the system. On average, 256 customers visit this restaurant daily during lunchtime. These numbers rise during weekends while decreasing during weekdays. There are many important factors which determine whether a restaurant is preferred by customers like the taste of food, cleanliness, the ambience of the place and management by staff. The results of queuing theory are often used by establishments like business groups and hospitals to make decisions about resources required for providing quality service.

II. History

AgnerKrarup Erlang, a Danish mathematician and engineer established queuing theory. He was employed by Copenhagen Telephone Exchange in 1903. He studied the congestion of telephone traffic. His main concern was that people should not wait too long on hold or in a phone queue. He found out how many telephone operators were required to handle a certain number of calls. His paper was published in the paper Telephone Waiting Times in 1920. Later on, many mathematicians and engineers started working on other queuing problems using probabilistic methods. Queuing models can be very useful in identifying appropriate levels of staff, equipment and beds as well as in making decisions about the allocation of resources.

III. Key Elements

- **Units or Customers -** Anything that arrives at a facility and requires service, for example machines, ships, trucks, packets and patients. These units or customers arrive at regular or irregular intervals of time at a point, called server or service center.
- **Server/Service Centre/Service Station-** Any server which provides the requested service, for example receptionist, doctor cashier, repair person, host, runways at airport etc. A system may have one or more servers. A queuing system is classified as 'Single Server Queuing System' if there is only one server and 'Multiple Server Queuing System'if there are multiple servers.
- **Leaving –** The customers leave the system once the system is completed. The act of departing from a queue position is called Leaving.

Customers Behaviour
Customers may behave in the following ways:
Balking – Customer decides not to join the queue, if it is long.
Reneging – Customer decides to leave the queue if waiting time is very long.
Jockeying – Customers switches from one queue to other if there are more than one queue.

IV. Basic characteristics of a queuing system

These characteristics completely specify a queuing model.

1. Arrival Process or Arrival Rate (Input) – Customers will directly go to the server if it is free but they will wait in the queue till it becomes free. Customers will come randomly. They will not come at a fixed regular

interval of time. The number of arrivals is computed using discrete probability distribution such as Poisson distribution.

2. Service (Departure) distribution – It is the pattern in which several customers leave the system. It may be constant or variable. It is described by an exponential probability distribution.
3. Service Mechanism/Service Channel – It includes several servers and an arrangement of servers (parallel or series). The waiting line system may have multiple or single servers. A queuing model is called a single-channel model when the system has one server only, for example in a doctor's clinic, and a multichannel model is when the system has several parallel servers, for example in a hospital or cinema ticket counter.
4. Queue Discipline – This is the order in which customers make a queue and are chosen for service.

Discipline		Examples
FCFS	First come,First serve	Restaurant, Doctor's clinic,ATM
LCFS	Last come, First serve	Files in Government office
SIRO	Service in random order	Emergency, Priority cases

FCFS is the most common discipline.

1. The Capacity of the System – A system may have the infinite capacity. But if the space is limited and it is filled, then arriving customers will not be allowed to join the system.

- Queue size – Numbers of customers waiting in line
- Queue length – Number of customers plus number of customers being served

Population – The source from which customers are generated may be finite or infinite

Kendall's Notation

Kendall proposed a notation for describing characteristics of a queuing situation

(a/b/c): (d/e/f)

where:

a – arrival fashion / arrival rate

b – service distribution (server time)

c- number of parallel servers

d – queue discipline

e – maximum number of units allowed in the system (finite or infinite)

f – size of the source (finite or infinite)

Little's Law

Little's Law developed by John Little states that expected number of customers (N) for a system in already statecan be determined using the following equation

$L_s = \lambda W_s$

λ – Mean arrival rate

W_s– Expected waiting time in the system

- L_s increases if λ or W_s increases
- λ increases if L_s increases or $W_s$decreases
- W_s increases if $L_s$increases or λ decreases

V. Queuing Model (M/M/1:FCFS/∞/∞)

We have obtained one week daily customer's data from the restaurant through observations at the restaurant. The queuing model that best illustrate operation of the restaurant at the restaurant is M/M/1:FCFS/∞/∞)

The restaurant has one server only. Arrivals are Poisson distributed and service time is exponentially distributed.

1) Customers come from an infinite population.
2) Arrivals follow the Poisson distribution
3) Customers behaviour is treated on 'FCFS' discipline. They do not balk or renege
4) Service time follows exponential distribution.
5) Utilization factor ρ<1 that is average service rate is faster than average arrival rate.

Following variables were investigated:

λ- The mean customer's arrival rate

μ - The mean service rate

$\rho = \frac{\lambda}{\mu}$; the utilization factor

Probability of zero customers in the restaurant

$P_0 = 1-\rho = 1-\frac{\lambda}{\mu}$

P_n = Probability of n customers in the restaurant

$P_n = P_0\rho^n = (1-\rho)\rho^n$

L_s= Average number of customers in the restaurant

$L_s = \frac{\rho}{1-\rho} = \frac{\lambda/\mu}{1-\lambda/\mu} = \frac{\lambda}{\mu-\lambda}$

L_q= The average number of customers in the queue

$L_q = L_s \times \rho = \frac{\rho^2}{1-\rho} = \frac{\rho\lambda}{\mu-\lambda}$

W_q = The average waiting time in the queue

$W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu-\lambda}$

W_s = The average waiting time spent in the restaurant

$W_S = \frac{L_s}{\lambda} = \frac{1}{\mu-\lambda}$

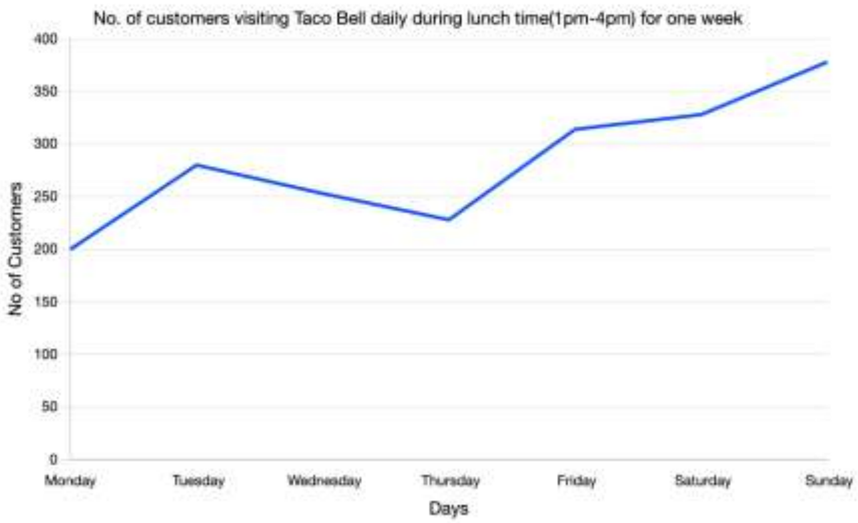
$L_S = L_q$ + No. of customers getting service

$W_s = W_q$ +waiting time in service

Observations

The customer's data was collected daily for one week by observations and discussion with manager.

Day	No.of customers(1pm-4pm)
Monday	200
Tuesday	280
Wednesday	253
Thursday	228
Friday	314
Saturday	328
Sunday	378



Calculations

Average Number of customers visiting restaurant in 3 hours = 283

Arrival rate $\lambda = \frac{283}{3h} = \frac{283}{180\ mins} = 1.57\ customers\ per\ minute(cpm)$

W_s= Average time spent in restaurant(estimated)= 50minutess

L_q = queue length(estimated) = 20 customers

W_q= waiting time (estimated) = 10 minutes

W_q(actual) = $\frac{20}{1.57}$= 12.7minutess = 13 minutes (approx)
$L_s = \lambda W_s$ = 1.57× 50 =78.5
Service rate = $\mu = \frac{\lambda(1+L_s)}{L_s}$

$$\mu = \frac{1.57(1+78.5)}{78.5} = 1.59$$

$\rho = \frac{\lambda}{\mu} = \frac{1.57}{1.59} = 0.987$

The utilization rate during period 1pm to 4pm is 0.987.
Probability of having zero customers in the restaurant $p_0 = 1-\rho$ = 1- 0.987 = 0.013

VI. Conclusions

M/M/1 queuing model has been applied in TacoBell. The customer's arrival rate is 1.57cpm and service rate is 1.59cpm. The probability of having zero customer in restaurant is quite less. The utilization factor is highi.e., 0.987.

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