IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 20, Issue 4 Ser. 2 (July. – August. 2023), PP 40-52
www.iosrjournals.org

Interval-Valued Neutrosophic Hypersoft Topological Spaces

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Abstract:
Aim of this paper is to introduce the concept of Interval-valued Neutrosophic Hypersoft Topological spaces. Some notions such as finer, coarser, discrete, indiscrete, neighbourhood, neighbourhood system, basis of an Interval-valued Neutrosophic Hypersoft Topological space are introduced and we have studied some properties with examples. Finally, Interval-valued Neutrosophic Hypersoft interior and closure are studied.
KeyWord*:Neutrosophic soft topology, Interval-valued neutrosophic soft topology, Hypersoft Topology*

Date of Submission: 04-08-2023 Date of Acceptance: 14-08-2023

I. Introduction

In order to deal with uncertainty, Lotfi A. Zadeh [1] in 1965 introduced the concept of Fuzzy logic and Fuzzy sets. In Fuzzy logic, it represents the degree of truth as an extension of valuation. To deal with imprecise and vague information K. Atanassov [2] in 1986 introduced the concept of Intuitionistic fuzzy sets and Intuitionistic fuzzy logic. Similarly Pythagorean fuzzy sets, Pythagorean fuzzy numbers and several other concepts and their applications in MCDM, MADM and MAGDM were proposed in [3] – [11]. Chang [16] in 1968 expanded the idea of Fuzzy sets to Fuzzy topology and developed many notions based on it. Compactness, Products and convergence of fuzzy topological spaces were proposed in [13]-[15]. In 2021, I. Zahan, et.al., developed the concept of fuzzy topological spaces, in which some of their properties and certain relationships among the closure of these spaces are discussed. Coker D. in 1997 introduced the concept of Intuitionistic fuzzy topological spaces. Later on in 2018, some notions based on Intuitionistic fuzzy topological spaces were given by Kim, et.al., [17]. The parametrization of the attributes is not discussed in any of the aforementioned studies.

Molodtsov [18] in 1999 generalized the concept of fuzzy set theory to soft set theory which helps to deal with uncertainty. Some basic properties of soft set theory was proposed by P. K. Maji, et. al., [19]. Later on several interesting results based on Soft set theory has been obtained by embedding the idea of Fuzzy set, Intuitionistic fuzzy set, Vague set, Rough set, Interval-valued intuitionistic fuzzy set and so on. Also, various applications of the above mentioned sets in decision making problems were developed in [20] – [26]. In 2011, Naim Cagman, et.al., [27] defined the concept of soft topology on a soft set and presented its related properties. Muhammad Shabir, et.al.,[28] introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. The notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms were introduced and their basic properties were investigated. The notion of a topology on soft subsets have been introduced by H. Hazra et.al.,[29]. And also they studied some basic properties of these topologies and the definition of continuity of soft mappings with their properties. Taha, et.al.,[30] introduced the concept of pointwise topology of soft topological spaces. Finally, they investigated the properties of soft mapping spaces and the relationships between some soft mapping spaces.

F. Samarandache [31] in 1995 introduced the concept of Neutrosophic sets and Neutrosophic logic with indeterminate data. Neutrosophic soft sets were introduced by Maji in [32]. He also gave an application on Neutrosophic soft sets through a decision making problem. The concept of Generalized Neutrosophic soft set theory was proposed by Said Broumi [33]. Similarly, several concepts based on Neutrosophic Soft set theory has been emerged in recent days. Salama A. A. et.al., [34] progressed a new concept called Neutrosophic topological spaces and defined some definitions based on it. In [35] Tuhin Bera, et.al., constructed the Neutrosophic soft Topology. Further the notions of Neutrosophic soft interior, closure, neighborhood, boundary are defined and also some of their basic properties are studied. In [36] the same authors introduced the concept of connectedness and compactness on neutrosophic soft topological spaces and studied several characteristics and related properties. They also studied neutrosophic soft continuous mappings on neutrosophic soft topological spaces. The separations axioms on Neutrosophic topological spaces were given by Cigdem, et.a.,[37].

In 2014 Irfan Deli[38] introduced the notion of interval valued neutrosophic soft sets which is a combination of an interval valued neutrosophic set and soft set. Later on Anjan Mukherjee, et.al., [39] developed interval valued neutrosophic soft topological spaces and studied some notions on it.

Smarandache [40] introduced Hypersoft sets which deals with multi-attribute functions. Further, Muhammad Saqlain, et.al., [41] progressed a new concept called Neutrosophic Hypersoft set and also studied some operations on it. Rana Muhammad Zulqarnain, et.al.,[42] developed the generalized version of aggregate operators on Neutrosophic Hypersoft sets. Sagvan Y. Musa, et.al.,[43] developed the concept of Hypersoft topological spaces and provided some basic notions on them. Further, certain new notions, connectedness and separation axioms of Hypersoft topological spaces have been explored by Sagvan Y. Musa, et.al.,[44]-[45]. Later on in 2022 Adem Yolce, et.al.,[46] introduced the concept of fuzzy hypersoft topological spaces and studied some basic notions on it. Taha Yasin Ozturk, et.al., [48] further extended the concept of Neutrosophic soft topological spaces to Neutrosophic Hypersoft topological spaces and presented some new notions.

In this study we introduce the concept of Interval-valued Neutrosophic Hypersoft Topological Spaces and establish some notions, properties and results, with examples and proofs.

II. Preliminaries

In this section we recalled some fundamentals such as Hypersoft topological spaces, Fuzzy hypersoft topological spaces, Neutrosophic Hypersoft topological spaces, etc., which would be helpful to introduce the concept of Interval-valued Neutrosophic Hypersoft topological spaces.

Definition 2.1. [43]
Let $\acute{T}_H$ be the collection of hypersoft sets over U, then $\acute{T}_H$ is said to be a hypersoft topology on U if

1. $(\phi, E), (\psi, E)$ belong to $\acute{T}_H$,
2. The intersection of any two hypersoft sets in $\acute{T}_H$ belongs to $\acute{T}_H$,
3. The union of any number of hypersoft sets in $\acute{T}_H$ belongs to $\acute{T}_H$.

Then $(U, \acute{T}_H, E)$ is called a hypersoft topological space over U.

Definition 2.2. [48]
Let $NHSS(\Delta, \Sigma)$ be the family of all neutrosophic hypersoft sets over the universe set Δ and $\acute{T} \subseteq NHSS(\Delta, \Sigma)$. Then $\acute{T}$ is said to be a neutrosophic hypersoft topology on Δ if

1. $0_{(\Delta_{NH}, \Sigma)}$ and $1_{(\Delta_{NH}, \Sigma)}$ belongs to $\acute{T}$
2. The union of any number of neutrosophic hypersoft sets in $\acute{T}$ belongs to $\acute{T}$
3. The intersection of finite number of neutrosophic hypersoft sets in $\acute{T}$ belongs to $\acute{T}$.

Then $(\Delta, \Sigma, \acute{T})$ is said to be a neutrosophic hypersoft topological space over Δ. Each members of $\acute{T}$ is said to be neutrosophic hypersoft open set.

Definition 2.3. [47]
Let U be a universal set and $P(U)$ be a power set of U and for $n \geq 1$, there are n distinct attributes such as $k_1, k_2, k_3, \dots, k_n$ and $K_1, K_2, K_3, \dots, K_n$ are sets for corresponding values attributes respectively with following conditions such as $K_i \cap K_j = \phi\ (i \neq j)$ and $i, j \in \{1,2,3 \dots n\}$. Then the pair (F, A) is said to be IVNHSS over U if there exists a relation $K_1 * K_2 * K_3 * \dots * K_n = A$. Where $F: K_1 * K_2 * K_3 * \dots * K_n \to (U)$ and $F(K_1 * K_2 * K_{\{3\}} * \dots * K_n) = \{< \vartheta, [u_A^L(\vartheta), u_A^U(\vartheta)], [v_A^L(\vartheta), v_A^U(\vartheta)], [w_A^L(\vartheta), w_A^U(\vartheta)] >: \vartheta \in U\}$, where $u_A^L(\vartheta), v_A^L(\vartheta)$ and $w_A^L(\vartheta)$ are lower and $u_A^U(\vartheta), v_A^U(\vartheta)$ and $w_A^U(\vartheta)$ are upper membership values for truthness, indeterminacy, and falsity respectively for A and $[u_A^L(\vartheta), u_A^U(\vartheta)], [v_A^L(\vartheta), v_A^U(\vartheta)], [w_A^L(\vartheta), w_A^U(\vartheta)] \subset [0,1]$ and $0 \leq \sup u_A(\vartheta) + \sup v_A(\vartheta) + \sup w_A(\vartheta) \leq 3$ for each $\vartheta \in U$.

Definition 2.4. [47]
Let F_A and $G_B \in IVNHSS$ over U, then $F_A \subseteq G_B$ if

1. $inf u_A(\vartheta) \leq inf u_B(\vartheta), sup\ u_A(\vartheta) \leq sup\ u_B(\vartheta)$
2. $inf v_A(\vartheta) \geq inf v_B(\vartheta), sup\ v_A(\vartheta) \geq sup\ v_B(\vartheta)$
3. $inf w_A(\vartheta) \geq inf w_B(\vartheta), sup\ w_A(\vartheta) \geq sup\ w_B(\vartheta)$

Definition 2.5. [47]
Let F_A over U, then

1. Empty IVNHSS can be represented as F_0, and defined as follows
   $F_0 = \{< \vartheta, [0,0], [1,1], [1,1] >: \vartheta \in U\}$
2. Universal IVNHSS can be represented as F_E, and defined as follows
   $F_E = \{< \vartheta, [1,1], [0,0], [0,0] >: \vartheta \in U\}$

3. The complement of IVNHSS can be defined as follows
$F_A^c = \{< \vartheta, [w_A^L(\vartheta), w_A^U(\vartheta)],\ [1 - v_A^U(\vartheta), 1 - v_A^L(\vartheta)],\ [u_A^L(\vartheta), u_A^U(\vartheta)] >: \vartheta \in U\}$.

Definition 2.6 [47]
Let F_A and $G_B \in IVNHSS$ over U, then

$$F_A \cap G_B = \begin{Bmatrix} < \vartheta, & [\min\{inf u_A(\vartheta), inf u_B(\vartheta)\}, \min\{sup\ u_A(\vartheta), sup\ u_B(\vartheta)\}], \\ < \vartheta, & [\max\{inf v_A(\vartheta), inf v_B(\vartheta)\}, \max\{sup\ v_A(\vartheta), sup\ v_B(\vartheta)\}], \\ < \vartheta, & [\max\{inf w_A(\vartheta), inf w_B(\vartheta)\}, \max\{sup\ w_A(\vartheta), sup\ w_B(\vartheta)\}] \end{Bmatrix}$$

Definition 2.6 [47]
Let F_A and $G_B \in IVNHSS$ over U, then

$$F_A \cup G_B = \begin{Bmatrix} < \vartheta, & [\max\{inf u_A(\vartheta), inf u_B(\vartheta)\}, \max\{sup\ u_A(\vartheta), sup\ u_B(\vartheta)\}], \\ < \vartheta, & [\min\{inf v_A(\vartheta), inf v_B(\vartheta)\}, \min\{sup\ v_A(\vartheta), sup\ v_B(\vartheta)\}], \\ < \vartheta, & [\min\{inf w_A(\vartheta), inf w_B(\vartheta)\}, \min\{sup\ w_A(\vartheta), sup\ w_B(\vartheta)\}] \end{Bmatrix}$$

III. Interval-valued Neutrosophic Hypersoft Topological Spaces

In this section we introduce the concept of Interval-valued Neutrosophic Hypersoft Topological Spaces and define some notions and properties on it. Throughout this paper Interval-valued Neutrosophic Hypersoft Set is denoted by IVNHSS.

Definition 3.1.
Let $IVNHSS(\mho, \xi)$ be the family of all IVNHSSs over the universe $\mho$ and $\acute{T} \subseteq IVNHSS(\mho, \xi)$. Then $\acute{T}$ is said to be an Interval-valued Neutrosophic Hypersoft Topology on $\mho$ if,

1. $0_{(\mho,\xi)}$ and $1_{(\mho,\xi)}$ are in $\acute{T}$, [where, $0_{(\mho,\xi)}$ denotes the empty IVNHSS and $1_{(\mho,\xi)}$ denotes the universal IVNHSS]
2. the union of any sub-collection of IVNHSSs of $\acute{T}$ is in $\acute{T}$,
3. the intersection of finite sub-collection of IVNHSSs of $\acute{T}$ is in $\acute{T}$,

Then the triplet $(\mho, \xi, \acute{T})$ is called an Interval-valued Neutrosophic Hypersoft Topological Space over $\mho$.

Definition 3.2.
Let $(\mho, \xi, \acute{T})$ be an Interval-valued Neutrosophic Hypersoft Topological Space, then every member of $\acute{T}$ is said to be an Interval-valued Neutrosophic Hypersoft open Set(IVNHS-open set) in $\mho$.

Example 3.3.
Let $\mho = \{u_1, u_2\}$ be the universal set and let $\xi = \{\xi_1, \xi_2, \xi_3\}$ be the parameter set with $\xi_1 = \{e_1, e_2\}, \xi_2 = \{e_3, e_4\}$ and $\xi_3 = \{e_5\}$, such that $\zeta_1 = (e_1, e_3, e_5), \zeta_2 = (e_1, e_4, e_5),\ \zeta_3 = (e_2, e_3, e_5)$ and $\zeta_4 = (e_2, e_4, e_5)$. Now IVNHSSs $0_{(\mho,\xi)}, 1_{(\mho,\xi)}, (\eta_1, \xi), (\eta_2, \xi), (\eta_3, \xi), (\eta_4, \xi)$ over $\mho$ are given by,

$$0_{(\mho,\xi)} = \begin{Bmatrix} \left((e_1, e_3, e_5), \dfrac{u_1}{([0,0],[1,1],[1,1])}, \dfrac{u_2}{([0,0],[1,1],[1,1])}\right), \\ \left((e_1, e_4, e_5), \dfrac{u_1}{([0,0],[1,1],[1,1])}, \dfrac{u_2}{([0,0],[1,1],[1,1])}\right), \\ \left((e_2, e_3, e_5), \dfrac{u_1}{([0,0],[1,1],[1,1])}, \dfrac{u_2}{([0,0],[1,1],[1,1])}\right), \\ \left((e_2, e_4, e_5), \dfrac{u_1}{([0,0],[1,1],[1,1])}, \dfrac{u_2}{([0,0],[1,1],[1,1])}\right) \end{Bmatrix}$$

$$1_{(\mho,\xi)} = \begin{Bmatrix} \left((e_1, e_3, e_5), \dfrac{u_1}{([1,1],[0,0],[0,0])}, \dfrac{u_2}{([1,1],[0,0],[0,0])}\right), \\ \left((e_1, e_4, e_5), \dfrac{u_1}{([1,1],[0,0],[0,0])}, \dfrac{u_2}{([1,1],[0,0],[0,0])}\right), \\ \left((e_2, e_3, e_5), \dfrac{u_1}{([1,1],[0,0],[0,0])}, \dfrac{u_2}{([1,1],[0,0],[0,0])}\right), \\ \left((e_2, e_4, e_5), \dfrac{u_1}{([1,1],[0,0],[0,0])}, \dfrac{u_2}{([1,1],[0,0],[0,0])}\right) \end{Bmatrix}$$

$$(\eta_1,\xi) = \left\{\begin{array}{l}\left((e_1,e_3,e_5), \quad \frac{u_1}{([0.4,0.8],[0.5,0.7],[0.3,0.5])}, \quad \frac{u_2}{([0.1,0.3],[0.4,0.6],[0.6,0.9])}\right),\\ \left((e_1,e_4,e_5), \quad \frac{u_1}{([0.6,0.8],[0.4,0.6],[0.1,0.3])}, \quad \frac{u_2}{([0.3,0.4],[0.4,0.8],[0.5,0.8])}\right),\\ \left((e_2,e_3,e_5), \quad \frac{u_1}{([0.6,0.7],[0.6,0.8],[0.2,0.4])}, \quad \frac{u_2}{([0.2,0.4],[0.5,0.7],[0.7,0.9])}\right),\\ \left((e_2,e_4,e_5), \quad \frac{u_1}{([0.8,1.0],[0.2,0.4],[0.1,0.2])}, \quad \frac{u_2}{([0.7,0.8],[0.1,0.2],[0.2,0.3])}\right)\end{array}\right\}$$

$$(\eta_2,\xi) = \left\{\begin{array}{l}\left((e_1,e_3,e_5), \quad \frac{u_1}{([0.4,0.6],[0.6,0.8],[0.5,0.6])}, \quad \frac{u_2}{([0.7,0.8],[0.5,0.7],[0.2,0.4])}\right),\\ \left((e_1,e_4,e_5), \quad \frac{u_1}{([0.2,0.5],[0.6,0.7],[0.7,0.9])}, \quad \frac{u_2}{([0.4,0.7],[0.6,0.8],[0.6,0.9])}\right),\\ \left((e_2,e_3,e_5), \quad \frac{u_1}{([0.5,0.9],[0.4,0.8],[0.1,0.4])}, \quad \frac{u_2}{([0.6,0.9],[0.3,0.5],[0.2,0.6])}\right),\\ \left((e_2,e_4,e_5), \quad \frac{u_1}{([0.4,0.7],[0.4,0.8],[0.6,0.9])}, \quad \frac{u_2}{([0.1,0.4],[0.4,0.7],[0.6,1.0])}\right)\end{array}\right\}$$

$$(\eta_3,\xi) = \left\{\begin{array}{l}\left((e_1,e_3,e_5), \quad \frac{u_1}{([0.4,0.8],[0.5,0.7],[0.3,0.5])}, \quad \frac{u_2}{([0.7,0.8],[0.4,0.6],[0.2,0.4])}\right),\\ \left((e_1,e_4,e_5), \quad \frac{u_1}{([0.6,0.8],[0.4,0.6],[0.1,0.3])}, \quad \frac{u_2}{([0.4,0.7],[0.4,0.8],[0.5,0.8])}\right),\\ \left((e_2,e_3,e_5), \quad \frac{u_1}{([0.6,0.9],[0.4,0.8],[0.1,0.4])}, \quad \frac{u_2}{([0.6,0.9],[0.3,0.5],[0.2,0.6])}\right),\\ \left((e_2,e_4,e_5), \quad \frac{u_1}{([0.8,1.0],[0.2,0.4],[0.1,0.2])}, \quad \frac{u_2}{([0.7,0.8],[0.1,0.2],[0.2,0.3])}\right)\end{array}\right\}$$

$$(\eta_4,\xi) = \left\{\begin{array}{l}\left((e_1,e_3,e_5), \quad \frac{u_1}{([0.4,0.6],[0.6,0.8],[0.5,0.6])}, \quad \frac{u_2}{([0.1,0.3],[0.5,0.7],[0.6,0.9])}\right),\\ \left((e_1,e_4,e_5), \quad \frac{u_1}{([0.2,0.5],[0.6,0.7],[0.7,0.9])}, \quad \frac{u_2}{([0.3,0.4],[0.6,0.8],[0.6,0.9])}\right),\\ \left((e_2,e_3,e_5), \quad \frac{u_1}{([0.5,0.7],[0.6,0.8],[0.2,0.4])}, \quad \frac{u_2}{([0.2,0.4],[0.5,0.7],[0.7,0.9])}\right),\\ \left((e_2,e_4,e_5), \quad \frac{u_1}{([0.4,0.7],[0.4,0.8],[0.6,0.9])}, \quad \frac{u_2}{([0.1,0.4],[0.4,0.7],[0.6,1.0])}\right)\end{array}\right\}$$

Here, $\acute{T}_1 = \{0_{(\mho,\xi)}, 1_{(\mho,\xi)}, (\eta_1,\xi), (\eta_2,\xi), (\eta_3,\xi), (\eta_4,\xi)\}$ is an Interval-valued Neutrosophic Hypersoft Topology over $(\mho,\xi)$ and also $(\mho,\xi,\acute{T}_1)$ is an Interval-valued Neutrosophic Hypersoft Topological Space.

But, $\acute{T}_2 = \{0_{(\mho,\xi)}, 1_{(\mho,\xi)}, (\eta_1,\xi), (\eta_2,\xi)\}$ is not an Interval-valued Neutrosophic Hypersoft Topology over $(\mho,\xi)$, since the union and intersection of two-IVNHSSs (η_1,ξ) and (η_2,ξ) of $\acute{T}_2$ does not belongs to $\acute{T}_2$.
(i.e.,) $(\eta_1,\xi) \cup (\eta_2,\xi) \notin \acute{T}_2$ and $(\eta_1,\xi) \cap (\eta_2,\xi) \notin \acute{T}_2$.

Definition 3.4.
Let $(\mho, \xi, \acute{T})$ be an Interval-valued Neutrosophic Hypersoft topological space over $\mho$ and let (η,ξ) be an IVNHSS in $\mho$. Then, (η,ξ) is said to be an Interval-valued Neutrosophic Hypersoft closed set(IVNHS-closed set) of $\mho$ if its complement is an Interval-valued Neutrosophic Hypersoft open set in $\mho$.

Theorem 3.5.
Let $IVNHSS(\mho,\xi,\acute{T})$ be an Interval-valued Neutrosophic Hypersoft Topological space over $\mho$. Then,

1. $0_{(\mho,\xi)}$ and $1_{(\mho,\xi)}$ are IVNHS-closed sets in $\mho$,
2. the union of finite sub-collection of IVNHS-closed sets is an IVNHS-closed set over $\mho$,
3. the intersection of any sub-collection of IVNHS-closed sets is an IVNHS-closed set over $\mho$.

Proof. This is obvious from Definition 3.4.

Example 3.6.
Consider the same universe and parameters as of Example 3.3. IVNHSSs $0_{(\mho,\xi)}$, $1_{(\mho,\xi)}$, (α_1,ξ), (α_2,ξ), (α_3,ξ) and (α_4,ξ) over $\mho$ are given by;

$$0_{(\mho,\xi)} = \left\{\begin{array}{l} \left((e_1,e_3,e_5), \quad \frac{u_1}{([0,0],[1,1],[1,1])}, \quad \frac{u_2}{([0,0],[1,1],[1,1])}\right), \\ \left((e_1,e_4,e_5), \quad \frac{u_1}{([0,0],[1,1],[1,1])}, \quad \frac{u_2}{([0,0],[1,1],[1,1])}\right), \\ \left((e_2,e_3,e_5), \quad \frac{u_1}{([0,0],[1,1],[1,1])}, \quad \frac{u_2}{([0,0],[1,1],[1,1])}\right), \\ \left((e_2,e_4,e_5), \quad \frac{u_1}{([0,0],[1,1],[1,1])}, \quad \frac{u_2}{([0,0],[1,1],[1,1])}\right) \end{array}\right\}$$

$$1_{(\mho,\xi)} = \left\{\begin{array}{l} \left((e_1,e_3,e_5), \quad \frac{u_1}{([1,1],[0,0],[0,0])}, \quad \frac{u_2}{([1,1],[0,0],[0,0])}\right), \\ \left((e_1,e_4,e_5), \quad \frac{u_1}{([1,1],[0,0],[0,0])}, \quad \frac{u_2}{([1,1],[0,0],[0,0])}\right), \\ \left((e_2,e_3,e_5), \quad \frac{u_1}{([1,1],[0,0],[0,0])}, \quad \frac{u_2}{([1,1],[0,0],[0,0])}\right), \\ \left((e_2,e_4,e_5), \quad \frac{u_1}{([1,1],[0,0],[0,0])}, \quad \frac{u_2}{([1,1],[0,0],[0,0])}\right) \end{array}\right\}$$

$$(\alpha_1,\xi) = \left\{\begin{array}{l} \left((e_1,e_3,e_5), \quad \frac{u_1}{([0.3,0.5],[0.3,0.5],[0.4,0.8])}, \quad \frac{u_2}{([0.6,0.9],[0.4,0.6],[0.1,0.3])}\right), \\ \left((e_1,e_4,e_5), \quad \frac{u_1}{([0.1,0.3],[0.4,0.6],[0.6,0.8])}, \quad \frac{u_2}{([0.5,0.8],[0.2,0.6],[0.3,0.4])}\right), \\ \left((e_2,e_3,e_5), \quad \frac{u_1}{([0.2,0.4],[0.2,0.4],[0.6,0.7])}, \quad \frac{u_2}{([0.7,0.9],[0.3,0.5],[0.2,0.4])}\right), \\ \left((e_2,e_4,e_5), \quad \frac{u_1}{([0.1,0.2],[0.6,0.8],[0.8,1.0])}, \quad \frac{u_2}{([0.2,0.3],[0.8,0.9],[0.7,0.8])}\right) \end{array}\right\}$$

$$(\alpha_2,\xi) = \left\{\begin{array}{l} \left((e_1,e_3,e_5), \quad \frac{u_1}{([0.5,0.6],[0.2,0.4],[0.4,0.6])}, \quad \frac{u_2}{([0.2,0.4],[0.3,0.5],[0.7,0.8])}\right), \\ \left((e_1,e_4,e_5), \quad \frac{u_1}{([0.7,0.9],[0.3,0.4],[0.2,0.5])}, \quad \frac{u_2}{([0.6,0.9],[0.2,0.4],[0.4,0.7])}\right), \\ \left((e_2,e_3,e_5), \quad \frac{u_1}{([0.1,0.4],[0.2,0.6],[0.5,0.9])}, \quad \frac{u_2}{([0.2,0.6],[0.5,0.7],[0.6,0.9])}\right), \\ \left((e_2,e_4,e_5), \quad \frac{u_1}{([0.6,0.9],[0.2,0.6],[0.4,0.7])}, \quad \frac{u_2}{([0.6,1.0],[0.3,0.6],[0.1,0.4])}\right) \end{array}\right\}$$

$$(\alpha_3,\xi) = \left\{\begin{array}{l} \left((e_1,e_3,e_5), \quad \frac{u_1}{([0.3,0.5],[0.3,0.5],[0.4,0.8])}, \quad \frac{u_2}{([0.2,0.4],[0.4,0.6],[0.7,0.8])}\right), \\ \left((e_1,e_4,e_5), \quad \frac{u_1}{([0.1,0.3],[0.4,0.6],[0.6,0.8])}, \quad \frac{u_2}{([0.5,0.8],[0.2,0.6],[0.4,0.7])}\right), \\ \left((e_2,e_3,e_5), \quad \frac{u_1}{([0.1,0.4],[0.2,0.6],[0.6,0.9])}, \quad \frac{u_2}{([0.2,0.6],[0.5,0.7],[0.6,0.9])}\right), \\ \left((e_2,e_4,e_5), \quad \frac{u_1}{([0.1,0.2],[0.6,0.8],[0.8,1.0])}, \quad \frac{u_2}{([0.2,0.3],[0.8,0.9],[0.7,0.8])}\right) \end{array}\right\}$$

$$(\alpha_4,\xi) = \left\{\begin{array}{l} \left((e_1,e_3,e_5), \quad \frac{u_1}{([0.5,0.6],[0.2,0.4],[0.4,0.6])}, \quad \frac{u_2}{([0.6,0.9],[0.3,0.5],[0.1,0.3])}\right), \\ \left((e_1,e_4,e_5), \quad \frac{u_1}{([0.7,0.9],[0.3,0.4],[0.2,0.5])}, \quad \frac{u_2}{([0.6,0.9],[0.2,0.4],[0.3,0.4])}\right), \\ \left((e_2,e_3,e_5), \quad \frac{u_1}{([0.2,0.4],[0.2,0.4],[0.5,0.7])}, \quad \frac{u_2}{([0.7,0.9],[0.3,0.5],[0.2,0.4])}\right), \\ \left((e_2,e_4,e_5), \quad \frac{u_1}{([0.6,0.9],[0.2,0.6],[0.4,0.7])}, \quad \frac{u_2}{([0.6,1.0],[0.3,0.6],[0.1,0.4])}\right) \end{array}\right\}$$

Here, $0_{(\mho,\xi)}, 1_{(\mho,\xi)}, (\alpha_1,\xi), (\alpha_2,\xi), (\alpha_3,\xi), (\alpha_4,\xi)$ are IVNHS-closed sets in $\mho$ as their complements are IVNHS-open sets in $\mho$ over $(\mho,\xi,\acute{T})$.

Definition 3.7.
Let IVNHSS$(\mho,\xi)$ be the family of all IVNHSSs over the universe $\mho$.

1. If $\acute{T} = \{0_{(\mho,\xi)}, 1_{(\mho,\xi)}\}$, then $\acute{T}$ is said to be an Interval-valued Neutrosophic Hypersoft indiscrete topology on $(\mho,\xi)$ and the triplet $(\mho,\xi,\acute{T})$ is said to be an Interval-valued Neutrosophic Hypersoft indiscrete topological space over $\mho$.
2. If $\acute{T} =$ IVNHSS$(\mho,\xi)$, then $\acute{T}$ is said to be an Interval-valued Neutrosophic Hypersoft discrete topology on $(\mho,\xi)$ and the triplet $(\mho,\xi,\acute{T})$ is said to be an Interval-valued Neutrosophic Hypersoft discrete topological space over $\mho$.

Definition 3.8.
Let $(\mho,\xi,\acute{T_1})$ and $(\mho,\xi,\acute{T_2})$ be two Interval-valued Neutrosophic Hypersoft topological spaces over $\mho$.

1. If $\acute{T_1} \subseteq \acute{T_2}$, then $\acute{T_1}$ is said to be coraser than $\acute{T_2}$ and $\acute{T_2}$ is said to be finer than $\acute{T_1}$.
2. If $\acute{T_1} \subseteq \acute{T_2}$ or $\acute{T_2} \subseteq \acute{T_1}$, then $\acute{T_1}$ and $\acute{T_2}$ are said to be comparable over $\mho$.

Example 3.9.
Consider Example 3.3. In which,
$\acute{T_1} = \{0_{(\mho,\xi)}, 1_{(\mho,\xi)}, (\eta_1,\xi), (\eta_2,\xi), (\eta_3,\xi), (\eta_4,\xi)\}$,
$\acute{T_2} = \{0_{(\mho,\xi)}, 1_{(\mho,\xi)}\}$.
Here, $\acute{T_2} \subseteq \acute{T_1}$, then $\acute{T_2}$ is finer than $\acute{T_1}$, $\acute{T_1}$ is coarser than $\acute{T_2}$ and also $\acute{T_1}$ and $\acute{T_2}$ are comparable.

Proposition 3.10.
If $(\mho,\xi,\acute{T_1})$ and $(\mho,\xi,\acute{T_2})$ be two Interval-valued Neutrosophic Hypersoft topological spaces over $\mho$, then $\acute{T_1} \cap \acute{T_2}$ is also an Interval-valued Neutrosophic Hypersoft topology on $(\mho,\xi)$.

Proof.
Assume $(\eta_1,\xi), (\eta_2,\xi) \in IVNHSS(\mho,\xi)$.

1. Plainly, $0_{(\mho,\xi)}, 1_{(\mho,\xi)} \in \acute{T_1} \cap \acute{T_2}$,
2. Let $(\eta_1,\xi), (\eta_2,\xi) \in \acute{T_1} \cap \acute{T_2}$,
   $\Rightarrow (\eta_1,\xi), (\eta_2,\xi) \in \acute{T_1}$ *and* $(\eta_1,\xi), (\eta_2,\xi) \in \acute{T_2}$
   $\Rightarrow (\eta_1,\xi) \cap (\eta_2,\xi) \in \acute{T_1}$ *and* $(\eta_1,\xi) \cap (\eta_2,\xi) \in \acute{T_2}$
   $\Rightarrow (\eta_1,\xi) \cap (\eta_2,\xi) \in \acute{T_1} \cap \acute{T_2}$.
3. Let $\{(\eta_i,\xi): i \in I\} \in \acute{T_1} \cap \acute{T_2}$,
   $\Rightarrow (\eta_i,\xi) \in \acute{T_1}$ *and* $(\eta_i,\xi) \in \acute{T_2}$
   $\Rightarrow \cup_{i\in I} (\eta_i,\xi) \in \acute{T_1}$ *and* $\cup_{i\in I} (\eta_i,\xi) \in \acute{T_2}$
   $\Rightarrow \cup_{i\in I} (\eta_i,\xi) \in \acute{T_1} \cap \acute{T_2}$.

Here, $\acute{T_1} \cap \acute{T_2}$ is an Interval-valued Neutrosophic Hypersoft topology on $\mho$ and the triplet $(\mho,\xi,\acute{T_1} \cap \acute{T_2})$ is an Interval-valued Neutrosophic Hypersoft topological space over $\mho$.

Remark 3.11.
The union of two Interval-valued Neutrosophic Hypersoft topologies on $\mho$ may not be an Interval-valued Neutrosophic Hypersoft topology on $\mho$.

Example 3.12.
Consider Example 3.3. Let $\acute{T_1}$ and $\acute{T_2}$ be two Interval-valued Neutrosophic Hypersoft topologies on $\mho$, which is given by,
$\acute{T_1} = \{0_{(\mho,\xi)}, 1_{(\mho,\xi)}, (\eta_1,\xi)\}$, $\acute{T_2} = \{0_{(\mho,\xi)}, 1_{(\mho,\xi)}, (\eta_2,\xi)\}$.
Here, $\acute{T_1} \cap \acute{T_2} = \{0_{(\mho,\xi)}, 1_{(\mho,\xi)}\}$ which is an Interval-valued Neutrosophic Hypersoft topology on $\mho$.
But, $\acute{T_1} \cup \acute{T_2} = \{0_{(\mho,\xi)}, 1_{(\mho,\xi)}, (\eta_1,\xi), (\eta_2,\xi)\}$ which is not an Interval-valued Neutrosophic Hypersoft topology on $\mho$, since $(\eta_1,\xi) \cup (\eta_2,\xi) \notin \acute{T_1} \cup \acute{T_2}$ and $(\eta_1,\xi) \cap (\eta_2,\xi) \notin \acute{T_1} \cup \acute{T_2}$.

Definition 3.13.

Let $\acute{T}$ be an Interval-valued Neutrosophic Hypersoft topology on $(\mho, \xi)$ and let $(\eta_4, \xi), (\eta_3, \xi) \in$ IVNHSS on $\mho$. Then, (η_3, ξ) is said to be an Interval-valued Neutrosophic Hypersoft neighbourhood(IVNHS-neighbourhood) of (η_4, ξ) if there exists an IVNHS-open set (η_1, ξ) [(i.e.,) $(\eta_1, \xi) \in \acute{T}$] such that $(\eta_4, \xi) \subseteq (\eta_1, \xi) \subseteq (\eta_3, \xi)$.

Example 3.14.

Consider Example 3.3, in which $(\eta_4, \xi) \subseteq (\eta_1, \xi) \subseteq (\eta_3, \xi)$, hence (η_3, ξ) is an Interval-valued Neutrosophic Hypersoft neighbourhood of (η_4, ξ), since $(\eta_4, \xi), (\eta_3, \xi)$ are IVNHSSs in $(\mho, \xi)$ and (η_1, ξ) is an IVNHS-open set in $\mho$.

Proposition 3.15.

Let $(\mho, \xi, \acute{T})$ be an Interval-valued Neutrosophic Hypersoft topological space over $\mho$.

1. If (η_1, ξ) and (η_2, ξ) are two Interval-valued Neutrosophic Hypersoft neighbourhoods of some IVNHSS(η_4, ξ) on $\mho$, then $(\eta_1, \xi) \cap (\eta_2, \xi)$ is also an IVNHS-neighbourhood of IVNHSS(η_4, ξ) on $\mho$.
2. If (η_1, ξ) is an IVNHS-neighbourhood of IVNHSS (η_4, ξ) on $\mho$ and if $(\eta_1, \xi) \subseteq (\eta_2, \xi)$, then (η_2, ξ) is also an IVNHS-neighbourhood of IVNHSS(η_4, ξ) on $\mho$.

Proof.

1. Let (η_1, ξ) and (η_2, ξ) be two Interval-valued Neutrosophic Hypersoft neighbourhoods of some IVNHSS(η_4, ξ) on $\mho$. Then by Definition 3.13 there exist IVNHS-open sets (η_3, ξ), $(\eta_5, \xi) \in \acute{T}$ on $\mho$, such that $(\eta_4, \xi) \subseteq (\eta_3, \xi) \subseteq (\eta_1, \xi)$ and $(\eta_4, \xi) \subseteq (\eta_5, \xi) \subseteq (\eta_2, \xi)$. Now, $(\eta_4, \xi) \subseteq (\eta_3, \xi)$ and $(\eta_4, \xi) \subseteq (\eta_5, \xi)$ implies that $(\eta_4, \xi) \subseteq [(\eta_3, \xi) \cap (\eta_5, \xi)]$ and also $[(\eta_3, \xi) \cap (\eta_5, \xi)] \in \acute{T}$. From which we have, $(\eta_4, \xi) \subseteq [(\eta_3, \xi) \cap (\eta_5, \xi)] \subseteq [(\eta_1, \xi) \cap (\eta_2, \xi)]$. Thus, $(\eta_1, \xi) \cap (\eta_2, \xi)$ is an IVNHS-neighbourhood of IVNHSS(η_4, ξ).
2. Let (η_1, ξ) be an Interval-valued Neutrosophic Hypersoft neighbourhood of IVNHSS(η_4, ξ) on $\mho$ and also assume that $(\eta_1, \xi) \subseteq (\eta_2, \xi)$. By Definition 3.13, there exist an IVNHS-open set (η_3, ξ) such that $(\eta_4, \xi) \subseteq (\eta_3, \xi) \subseteq [(\eta_1, \xi) \cap (\eta_2, \xi)]$. Thus, (η_2, ξ) is an IVNHS-neighbourhood of IVNHSS(η_4, ξ).

Definition 3.16.

Let $(\mho, \xi, \acute{T})$ be an Interval-valued Neutrosophic Hypersoft topological space over $\mho$. If $(\eta, \xi) \in$ IVNHSS$(\mho, \xi)$, then the family of all IVNHS-neighbourhoods of (η, ξ) is said to be an IVNHS-neighbourhood system of (η, ξ) on topology $\acute{T}$ and is denoted by Nbd(η, ξ).

Theorem 3.17.

If Nbd(η_4, ξ) is an IVNHS-neighbourhood system of the IVNHSS(η_4, ξ) on $\mho$ then,

1. finite intersection of the members of Nbd(η_4, ξ) belongs to Nbd(η_4, ξ),
2. each IVNHSS containing a member of Nbd(η_4, ξ) belongs to Nbd(η_4, ξ).

Proof.

1. Assume $(\eta_1, \xi), (\eta_2, \xi) \in$ IVNHSS and also let $(\eta_1, \xi), (\eta_2, \xi) \in Nbd(\eta_4, \xi)$. Then by Definition 3.13 there exists an IVNHS-open sets $(\eta_3, \xi), (\eta_5, \xi) \in \acute{T}$ on $\mho$ such that $(\eta_4, \xi) \subseteq (\eta_3, \xi) \subseteq (\eta_1, \xi)$ and $(\eta_4, \xi) \subseteq (\eta_5, \xi) \subseteq (\eta_2, \xi)$. Since we have $(\eta_3, \xi) \cap (\eta_5, \xi) \in \acute{T}$ implies that $(\eta_4, \xi) \subseteq [(\eta_3, \xi) \cap (\eta_5, \xi)] \subseteq [(\eta_2, \xi) \cap (\eta_1, \xi)]$. Thus $[(\eta_2, \xi) \cap (\eta_1, \xi)] \in$ Nbd(η_4, ξ).
2. Assume $(\eta_1, \xi) \in Nbd(\eta_4, \xi)$ on $\mho$ and if (η_2, ξ) is an IVNHSS containing (η_4, ξ) then, by Definition 3.13 there exist an IVNHS-open set $(\eta_3, \xi) \in \acute{T}$ on $\mho$ such that $(\eta_4, \xi) \subseteq (\eta_3, \xi) \subseteq (\eta_1, \xi) \subseteq (\eta_2, \xi)$, which implies $(\eta_4, \xi) \subseteq (\eta_3, \xi) \subseteq (\eta_2, \xi)$. Hence, $(\eta_2, \xi) \in Nbd(\eta_4, \xi)$.

Definition 3.18.

Let $(\mho, \xi, \acute{T})$ be an Interval-valued Neutrosophic Hypersoft space over $\mho$ and if Υ is a non-empty subset of $\mho$, then,

$$\acute{T}_{\Upsilon} = \{(\eta_{\Upsilon}, \xi) \mid (\eta, \xi) \in \acute{T}\}$$

is said to be an Interval-valued Neutrosophic Hypersoft relative topology on Υ and the triplet $(\Upsilon, \xi, \acute{T}_{\Upsilon})$ is said to be an Interval-valued Neutrosophic Hypersoft subspace(IVNHS-subspace) of $(\mho, \xi, \acute{T})$.

Corollary 3.19.

The following conditions hold for an IVNHS-subspace,

1. any IVNHS-subspace of an Interval-valued Neutrosophic Hypersoft indiscrete topological space is an Interval-valued Neutrosophic Hypersoft indiscrete topological space.
2. any IVNHS-subspace of an Interval-valued Neutrosophic Hypersoft discrete topological space is an Interval-valued Neutrosophic Hypersoft discrete topological space.

Proof. Follows from Definition 3.18.

Proposition 3.20.
Let $(\Upsilon, \xi, \acute{T}_\Upsilon)$ be an IVNHS-subspace of an Interval-valued Neutrosophic Hypersoft topological space $(\mho, \xi, \acute{T})$ and also (η_Υ, ξ) be an IVNHS-open set in Υ. If $(\Upsilon, \xi) \in \acute{T}$ then, $(\eta_\Upsilon, \xi) \in \acute{T}$.

Proof.
Let (η_Υ, ξ) be an IVNHS-open set in Υ, then by Definition 3.18 there exists an IVNHS-open set (η, ξ) in $\mho$ such that
$(\eta_\Upsilon, \xi) = (\Upsilon, \xi) \cap (\eta, \xi)$. By our assumption, if $(\Upsilon, \xi) \in \acute{T}$, then we have $(\Upsilon, \xi) \cap (\eta, \xi) \in \acute{T}$ which implies $(\eta_\Upsilon, \xi) \in \acute{T}$.

Proposition 3.21.
If $(\Upsilon, \xi, \acute{T}_\Upsilon)$ and $(\delta, \xi, \acute{T}_\delta)$ are two IVNHS-subspace of an Interval-valued Neutrosophic Hypersoft topological space $(\mho, \xi, \acute{T})$ and if $\Upsilon \subseteq \delta$ then, $(\Upsilon, \xi, \acute{T}_\Upsilon)$ is an IVNHS-subspace of $(\delta, \xi, \acute{T}_\delta)$.

Proof.
Straight forward from Definition 3.18.

Definition 3.22.
For an IVNHSS(η, ξ),

1. an Interval-valued Neutrosophic Hypersoft point(IVNHS-point) is given by an element $(\zeta, \eta(\zeta))$ of (η, ξ) such that,
   $\eta(\zeta) \notin 0_{(\mho,\xi)}$ and $\eta(\zeta') \in 0_{(\mho,\xi)}$, for all $\zeta \in \xi$ and $\zeta' \in \xi - \{e\}$ and is denoted by ζ_η.
2. the complement of an IVNHS-point ζ_η is another IVNHS-point ζ_η^c, which is given by $\eta^c(\zeta) = (\eta(\zeta))^c$.
3. an IVNHS-point $\zeta_{n_1} \in (\eta_2, \xi)$ only if for the element $\zeta \in \xi$, $\eta_1(\zeta) \leq \eta_2(\zeta)$, for $(\eta_2, \xi) \in$ IVNHSS on $(\mho, \xi)$.

Example 3.23.
Consider Example 3.3. An IVNHS-point of (η_1, ξ) for the parameter $\zeta_1 = (e_1, e_3, e_5)$ is given by,
$(e_1, e_3, e_5)_{\eta_1} = \{(u_1, ([0.4,0.8], [0.5,0.7], [0.3,0.5])), u_2, ([0.1,0.3], [0.4,0.6], [0.6,0.9]))\}$
and its complement is given by,
$(e_1, e_3, e_5)_{\eta_1}^c = \{(u_1, ([0.3,0.5], [0.3,0.5], [0.4,0.8])), u_2, ([0.6,0.9], [0.4,0.6], [0.1,0.3]))\}$
Let $(\eta_2, \xi) \in$ IVNHSS in $(\mho, \xi)$ such that,
$(e_1, e_3, e_5)_{\eta_2} = \{(u_1, ([0.5,0.9], [0.4,0.6], [0.2,0.4])), u_2, ([0.5,0.8], [0.1,0.3], [0.3,0.7]))\}$
in which $\zeta_{1\eta_1} \in (\eta_2, \xi)$ because for the element $\zeta_1 \in \xi$, $\eta_1(\zeta_1) \leq \eta_2(\zeta_1)$ in $(\mho, \xi)$.

IV. Some properties of Interval-valued Neutrosophic Hypersoft Topological Spaces

In this section, we define Interval-valued Neutrosophic Hypersoft base, interior, closure and discuss some of their properties.

Definition 4.1.
Let $(\mho, \xi, \acute{T})$ be an Interval-valued Neutrosophic Hypersoft topological space over $\mho$. Then, the sub-collection $\acute{B}$ of $\acute{T}$ is an Interval-valued Neutrosophic Hypersoft base(IVNHS-base) for $\acute{T}$ (called Interval-valued Neutrosophic Hypersoft base element (IVNHS-base element)), if every member of $\acute{T}$ can be expressed as an union of some elements of $\acute{B}$.

Example 4.2.
Consider Example 3.3.
$\acute{T}_1 = \{0_{(\mho,\xi)}, 1_{(\mho,\xi)}, (\eta_1, \xi), (\eta_2, \xi), (\eta_3, \xi), (\eta_4, \xi)\}$ then the sub-collection,
$\acute{B} = \{0_{(\mho,\xi)}, 1_{(\mho,\xi)}, (\eta_1, \xi), (\eta_2, \xi), (\eta_4, \xi)\}$, is an IVNHS-base for $\acute{T}_1$ in $(\mho, \xi)$.

Theorem 4.3.

Let $(\mho, \xi, \acute{T})$ be an Interval-valued Neutrosophic Hypersoft Topological space over $\mho$ and if $\acute{B} \subseteq \acute{T}$, then the following conditions are equivalent,

1. $\acute{B}$ is an IVNHS-basis of $\acute{T}$.
2. For every IVNHSS$(\eta, \xi) \in \acute{T}$ and for ζ_η in (η, ξ) there exists $\acute{B}_i \in \acute{B}$ such that $\zeta_\eta \in \acute{B}_i \subseteq (\eta, \xi)$, for all $\zeta \in \xi$.

Proof.

Straight forward from Definition 4.1.

Theorem 4.4.

Let $(\mho, \xi)$ be an IVNHSS and let $\acute{B}$ be an IVNHS-basis for Interval-valued Neutrosophic Hypersoft topology $\acute{T}$ on $(\mho, \xi)$. Then,

1. $\acute{T}$ equals the collection of all union of elements of $\acute{B}$.
2. $\acute{T}$ is the smallest Interval-valued Neutrosophic Hypersoft topology containing $\acute{B}$.

Proof.

1. Given a collection of elements of $\acute{B}$, which are also members of $\acute{T}$. Since $\acute{T}$ is a topology, the union of members of $\acute{T}$ is also in $\acute{T}$.

   Conversely, given $(\eta, \xi) \in \acute{T}$, choose for each $\zeta_{i_\eta} \in (\eta, \xi)$ an element $\acute{B}_{i_\eta}$ of $\acute{B}$ such that $\zeta_{i_\eta} \in \acute{B}_{i_\eta} \subseteq (\eta, \xi)$. Therefore, $(\eta, \xi) = \cup_{\zeta_{i_\eta} \in (\eta,\xi)} \acute{B}_{i_\eta}$, for all $\zeta_i \in \xi$.
2. This is obvious.

Theorem 4.5.

Let $(\mho, \xi, \acute{T})$ be an Interval-valued Neutrosophic Hypersoft topological space over $\mho$ and let $\acute{B} \in$ IVNHSS on $(\mho, \xi)$. Then, $\acute{B}$ is an IVNHS-basis for an Interval-valued Neutrosophic Hypersoft topology $\acute{T}$ on $(\mho, \xi)$ iff for any arbitrary IVNHSS(η, ξ) in $\acute{T}$ the following conditions hold,

1. for each $\zeta_\eta \in (\eta, \xi)$ there exist atleast one IVNHS-basis element $\acute{B}_i \in \acute{B}$ such that $\zeta_\eta \in \acute{B}_i$, for $\zeta \in \xi$.
2. if ζ_η belongs to the intersection of two IVNHS-basis elements $\acute{B}_1$ and $\acute{B}_2$ [(i.e.,) $\zeta_\eta \in \acute{B}_1 \cap \acute{B}_2$], then there is an IVNHS-basis element $\acute{B}_3 \in \acute{B}$, such that $\zeta_\eta \in \acute{B}_3 \subseteq [\acute{B}_1 \cap \acute{B}_2]$, for $\zeta \in \xi$.

Proof.

Assume that $\acute{B}$ is an IVNHS-basis for Interval-valued Neutrosophic Hypersoft topology $\acute{T}$ on $(\mho, \xi)$. Then, by Definition 3.24, $(\eta, \xi) = \cup_i \acute{B}_i$ for $\acute{B}_i \in \acute{B}$. Therefore, for $\zeta_\eta \in (\eta, \xi) \Longrightarrow \zeta_\eta \in \acute{B}_i$, for some i.

Next, assume that $\acute{B}_1, \acute{B}_2 \in \acute{B}$. Then by Definition 3.24, $\acute{B}_1, \acute{B}_2$ are IVNHS-basis elements in $(\mho, \xi)$. Hence, $\acute{B}_1 \cap \acute{B}_2$ is also an IVNHS-open set in $(\mho, \xi)$. Hence, for $\zeta_\eta \in [\acute{B}_1 \cap \acute{B}_2]$, there exist another IVNHS-basis element $\acute{B}_3 \in \acute{B}$ such that $\zeta_\eta \in \acute{B}_3 \subseteq [\acute{B}_1 \cap \acute{B}_2]$.

Conversely, assume 1. and 2. hold. We have to show that $\acute{B}$ is an IVNHS-basis for Interval-valued Neutrosophic Hypersoft topology $\acute{T}$ on $(\mho, \xi)$.

By Theorem 3.27, $\acute{T}$ can be expressed as a collection of union of elements of $\acute{B}$. Since 1. is true, we have $(\eta, \xi) = \cup_i \acute{B}_i$ for some $\acute{B}_i \in \acute{B}$, then $(\eta, \xi) \in \acute{T}$. Now, if $(\eta_3, \xi), (\eta_2, \xi)$ belongs to Interval-valued Neutrosophic Hypersoft topology and if $\zeta_\eta \in [(\eta_3, \xi) \cap (\eta_2, \xi)]$ there exist IVNHS-basis elements $\acute{B}_1, \acute{B}_2 \in \acute{B}$ such that

$\zeta_\eta \in \acute{B}_1 \subseteq (\eta_3, \xi)$ and $\zeta_\eta \in \acute{B}_2 \subseteq (\eta_2, \xi)$, which implies $\zeta_\eta \in [\acute{B}_1 \cap \acute{B}_2] \subseteq [(\eta_3, \xi) \cap (\eta_2, \xi)]$. Now by 2., there exists another IVNHS-basis element $\acute{B}_3 \in \acute{B}$ such that $\zeta_\eta \in \acute{B}_3 \subseteq [\acute{B}_1 \cap \acute{B}_2] \subseteq [(\eta_3, \xi) \cap (\eta_2, \xi)]$ which implies that the IVNHSS-$(\eta_3, \xi) \cap (\eta_2, \xi)$ can be expressed as the union of the elements in $\acute{B}$. (i.e.,) $[(\eta_3, \xi) \cap (\eta_2, \xi)] \in \acute{T}$. Suppose if (η_3, ξ) and (η_2, ξ) are disjoint, then $[(\eta_3, \xi) \cap (\eta_2, \xi)] = 0_{(\mho,\xi)} \in \acute{T}$. Therefore, $\acute{B}$ is an IVNHS-basis for the Interval-valued Neutrosophic Hypersoft topology on $(\mho, \xi)$.

Theorem 4.6.

Let (η, ξ) be an IVNHSS on $(\mho, \xi)$ and let $\acute{B}_1$ and $\acute{B}_2$ be two IVNHS-basis for two Interval-valued Neutrosophic Hypersoft topologies $\acute{T}_1$ and $\acute{T}_2$ on $(\mho, \xi)$. Then, the following conditions are equivalent,

1. $\acute{T}_2$ is finer than $\acute{T}_1$, (i.e.,) $\acute{T}_1 \subseteq \acute{T}_2$,

2. for each $\zeta_\eta \in (\eta, \xi)$ and each IVNHS-basis element $b_1 \in \acute{B}_1$ containing ζ_η, there exist IVNHS-basis element $\acute{b}_2 \in \acute{B}_2$ such that $\zeta_\eta \in \acute{B}_2 \subseteq \acute{B}_1$.

Proof.
Follows from Definition 4.1.

Definition 4.8.
Let $(\mho, \xi, \acute{T})$ be an IVNHS-topological space and (η_1, ξ) be an arbitrary IVNHSS on $(\mho, \xi)$. Then, the Interval-valued Neutrosophic Hypersoft interior(IVNHS-interior) of (η_1, ξ) is denoted by $(\eta_1, \xi)^\circ$ and is given by,

$$(\eta_1, \xi)^\circ = \cup \{(\eta_2, \xi) | (\eta_2, \xi) \in \acute{T}, (\eta_2, \xi) \subseteq (\eta_1, \xi)\}$$

(i.e.,) union of all IVNHS-open sets contained in (η_1, ξ) over $(\mho, \xi)$.

Definition 4.9.
Let $(\mho, \xi, \acute{T})$ be an IVNHS-topological space and (η_1, ξ) be an arbitrary IVNHSS on $(\mho, \xi)$. Then, the Interval-valued Neutrosophic Hypersoft closure(IVNHS-closure) of (η_1, ξ) is denoted by $\overline{(\eta_1, \xi)}$ and is given by,

$$\overline{(\eta_1, \xi)} = \cap \{(\eta_2, \xi) | (\eta_2, \xi)^c \in \acute{T}, (\eta_1, \xi) \subseteq (\eta_2, \xi)\}$$

(i.e.,) intersection of all IVNHS-closed supersets of (η_1, ξ).

Example 4.9.
Consider Example 3.3. Let $(\eta_5, \xi) \in IVNHSS$ on $(\mho, \xi)$ and is given by,

$$(\eta_5, \xi) = \left\{ \begin{array}{l} \left((e_1, e_3, e_5), \dfrac{u_1}{([0.5,0.7],[0.5,0.7],[0.3,0.5])}, \dfrac{u_2}{([0.8,0.9],[0.3,0.5],[0.1,0.3])} \right), \\ \left((e_1, e_4, e_5), \dfrac{u_1}{([0.4,0.8],[0.3,0.5],[0.4,0.8])}, \dfrac{u_2}{([0.5,0.8],[0.2,0.4],[0.3,0.5])} \right), \\ \left((e_2, e_3, e_5), \dfrac{u_1}{([0.7,0.9],[0.1,0.3],[0.1,0.3])}, \dfrac{u_2}{([0.7,1.0],[0.1,0.2],[0.2,0.3])} \right), \\ \left((e_2, e_4, e_5), \dfrac{u_1}{([0.5,0.8],[0.2,0.5],[0.3,0.4])}, \dfrac{u_2}{([0.3,0.4],[0.2,0.3],[0.5,0.9])} \right) \end{array} \right\}$$

Here,
IVNHS-interior$(\eta_5, \xi) = (\eta_5, \xi)^\circ = \cup \{0_{(\mho,\xi)}, (\eta_2, \xi), (\eta_4, \xi)\} = (\eta_2, \xi)$,
IVNHS-closure$(\eta_5, \xi) = \overline{(\eta_1, \xi)} = 1_{(\mho,\xi)} = 0^c_{(\mho,\xi)} \in \acute{T}$.

Proposition 4.10.
Let $(\mho, \xi, \acute{T})$ be an Interval-valued Neutrosophic Hypersoft topological space over $\mho$ and let (η_1, ξ) and (η_2, ξ) be two IVNHSS on $(\mho, \xi)$. Then,

1. $(\eta_1, \xi)^\circ$ is the largest IVNHS-open set contained in (η_1, ξ),
2. $(\eta_1, \xi)^\circ \subseteq (\eta_1, \xi)$,
3. $(\eta_1, \xi) \subseteq (\eta_2, \xi) \Rightarrow (\eta_1, \xi)^\circ \subseteq (\eta_2, \xi)^\circ$,
4. $(\eta_1, \xi)^\circ \subseteq \acute{T}$,
5. $(\eta_1, \xi) \in \acute{T} \Leftrightarrow (\eta_1, \xi)^\circ = (\eta_1, \xi)$,
6. $((\eta_1, \xi)^\circ)^\circ = (\eta_1, \xi)^\circ$,
7. $(0_{(\mho,\xi)})^\circ = 0_{(\mho,\xi)}, (1_{(\mho,\xi)})^\circ = 1_{(\mho,\xi)}$,
8. $[(\eta_1, \xi) \cap (\eta_2, \xi)]^\circ = (\eta_1, \xi)^\circ \cap (\eta_2, \xi)^\circ$,
9. $[(\eta_1, \xi) \cup (\eta_2, \xi)]^\circ \supseteq (\eta_1, \xi)^\circ \cup (\eta_2, \xi)^\circ$,
10. $(\eta_1, \xi)^\circ$ is an IVNHS-open set. (i.e.,) $(\eta_1, \xi)^\circ \in \acute{T}$.

Proof

1. Straight forward.

2. Straight forward.
3. $(\eta_1, \xi) \subseteq (\eta_2, \xi) \Rightarrow (\eta_1, \xi)^\circ \subseteq (\eta_2, \xi)^\circ$.
   Since, $(\eta_1, \xi)^\circ \subseteq (\eta_1, \xi) \subseteq (\eta_2, \xi)$ and $(\eta_2, \xi)^\circ$ is the largest open subset contained in (η_2, ξ),
   We have $(\eta_1, \xi)^\circ \subseteq (\eta_2, \xi)^\circ$.
4. This is obvious.
5. Assume $(\eta_1, \xi) \in \acute{T}$, then by 2. we have, $(\eta_1, \xi)^\circ \subseteq (\eta_1, \xi)$. -----------------(1)
   Since, $(\eta_1, \xi) \subseteq (\eta_1, \xi)$

$(\eta_1,\xi) \subseteq \cup\{(\eta_2,\xi) \mid (\eta_2,\xi) \in \acute{T}, (\eta_2,\xi) \subseteq (\eta_1,\xi)\} = (\eta_1,\xi)^{\circ}$.
(i.e.,) $(\eta_1,\xi) \subseteq (\eta_1,\xi)^{\circ}$. -----------------(2)
From (1) and (2), $(\eta_1,\xi) = (\eta_1,\xi)^{\circ}$.
Conversely, let $(\eta_1,\xi) = (\eta_1,\xi)^{\circ}$, by 4. We have $(\eta_1,\xi)^{\circ} \in \acute{T}$ which implies $(\eta_1,\xi) \in \acute{T}$.

6. Obvious from 1. and 2.
7. As $0_{(\mho,\xi)}, 1_{(\mho,\xi)} \in \acute{T}$, by 5. We have $0^{\circ}_{(\mho,\xi)} = 0_{(\mho,\xi)}$ and $1^{\circ}_{(\mho,\xi)} = 1_{(\mho,\xi)}$.
8. By 2. We have,
$(\eta_1,\xi)^{\circ} \subseteq (\eta_1,\xi)$ and $(\eta_2,\xi)^{\circ} \subseteq (\eta_2,\xi)$. Thus,
$[(\eta_1,\xi)^{\circ} \cap (\eta_2,\xi)^{\circ}] \subseteq [(\eta_1,\xi) \cap (\eta_2,\xi)]$. Hence,
$[(\eta_1,\xi)^{\circ} \cap (\eta_2,\xi)^{\circ}]^{\circ} \subseteq [(\eta_1,\xi) \cap (\eta_2,\xi)]^{\circ}$
$\Rightarrow [(\eta_1,\xi)^{\circ} \cap (\eta_2,\xi)^{\circ}] \subseteq [(\eta_1,\xi) \cap (\eta_2,\xi)]^{\circ}$ -------------------------(1)
Since, $[(\eta_1,\xi) \cap (\eta_2,\xi)] \subseteq (\eta_1,\xi)$ and $[(\eta_1,\xi) \cap (\eta_2,\xi)] \subseteq (\eta_2,\xi)$, by 3. we get,
$[(\eta_1,\xi) \cap (\eta_2,\xi)]^{\circ} \subseteq (\eta_1,\xi)^{\circ}$ and $[(\eta_1,\xi) \cap (\eta_2,\xi)]^{\circ} \subseteq (\eta_2,\xi)^{\circ}$,
$\Rightarrow [(\eta_1,\xi) \cap (\eta_2,\xi)]^{\circ} \subseteq [(\eta_1,\xi)^{\circ} \cap (\eta_2,\xi)^{\circ}]$ -------------------------(2)
From (1) and (2), we have $[(\eta_1,\xi) \cap (\eta_2,\xi)]^{\circ} = (\eta_1,\xi)^{\circ} \cap (\eta_2,\xi)^{\circ}$
9. Since $(\eta_1,\xi) \subseteq [(\eta_1,\xi) \cup (\eta_2,\xi)]$, by 3. we have,
$(\eta_1,\xi)^{\circ} \subseteq [(\eta_1,\xi) \cup (\eta_2,\xi)]^{\circ}$. Also since
$(\eta_2,\xi) \subseteq [(\eta_1,\xi) \cup (\eta_2,\xi)]$, again by 3. we have,
$(\eta_2,\xi)^{\circ} \subseteq [(\eta_1,\xi) \cup (\eta_2,\xi)]^{\circ}$. Hence,
$(\eta_1,\xi)^{\circ} \cup (\eta_2,\xi)^{\circ} \subseteq [(\eta_1,\xi) \cup (\eta_2,\xi)]^{\circ}$.
10. This is obvious.

Proposition 4.11
Let $(\mho,\xi,\acute{T})$ be an Interval-valued Neutrosophic Hypersoft topological space over $\mho$ and let (η_1,ξ) and (η_2,ξ) be two IVNHSS on $(\mho,\xi)$. Then,

1. $\overline{(\eta_1,\xi)}$ is the smallest IVNHS-closed set that containing (η_1,ξ),
2. $(\eta_1,\xi) \subseteq \overline{(\eta_1,\xi)}$,
3. $(\eta_1,\xi) \subseteq (\eta_2,\xi) \Rightarrow \overline{(\eta_1,\xi)} \subseteq \overline{(\eta_2,\xi)}$,
4. $\left[\overline{(\eta_1,\xi)}\right]^c \subseteq \acute{T}$,
5. (η_1,ξ) is an IVNHS-closed set iff $(\eta_1,\xi) = \overline{(\eta_1,\xi)}$,
6. $\overline{[\overline{(\eta_1,\xi)}]} = \overline{(\eta_1,\xi)}$,
7. $\overline{0_{(\mho,\xi)}} = 0_{(\mho,\xi)}, \overline{1_{(\mho,\xi)}} = 1_{(\mho,\xi)}$,
8. $\overline{[(\eta_1,\xi) \cap (\eta_2,\xi)]} \subseteq \overline{(\eta_1,\xi)} \cap \overline{(\eta_2,\xi)}$,
9. $\overline{[(\eta_1,\xi) \cup (\eta_2,\xi)]} = \overline{(\eta_1,\xi)} \cup \overline{(\eta_2,\xi)}$,
10. $\overline{(\eta_1,\xi)}$ is an IVNHS-closed set. (i.e.,) $\overline{(\eta_1,\xi)} \in \acute{T}^c$.

Proof

1. Straight forward.

2. Straight forward.

3. Assume $(\eta_1,\xi) \subseteq (\eta_2,\xi)$. From 2. $(\eta_2,\xi) \subseteq \overline{(\eta_2,\xi)}$ which implies $(\eta_1,\xi) \subseteq \overline{(\eta_2,\xi)}$. But $\overline{(\eta_2,\xi)}$ is an IVNHS-closed set. Thus, $\overline{(\eta_2,\xi)}$ is an IVNHS-closed set containing (η_1,ξ). By 1., $\overline{(\eta_1,\xi)}$ is an smallest IVNHS-closed set in $(\mho,\xi)$ containing (η_1,ξ), hence $\overline{(\eta_1,\xi)} \subseteq \overline{(\eta_2,\xi)}$.

4. This is obvious.

5. Suppose (η_1,ξ) is closed.
Then (η_1,ξ) is a closed set that contains (η_1,ξ),
So the intersection of all the closed sets that contain (η_1,ξ) is $\overline{(\eta_1,\xi)}$.
Hence, $(\eta_1,\xi) = \overline{(\eta_1,\xi)}$.
Suppose $(\eta_1,\xi) = \overline{(\eta_1,\xi)}$, since $\overline{(\eta_1,\xi)}$ is a closed set, (η_1,ξ) is closed.

6. This is obvious from 1. and 2.

7. Since, $0_{(\mho,\xi)}, 1_{(\mho,\xi)}$ are both IVNHS-open sets as well as IVNHS-closed sets, by 5. we have $\overline{0_{(\mho,\xi)}} = 0_{(\mho,\xi)}$ and $\overline{1_{(\mho,\xi)}} = 1_{(\mho,\xi)}$.

8. Since, $[(\eta_1,\xi) \cap (\eta_2,\xi)] \subseteq (\eta_1,\xi)$ and $[(\eta_1,\xi) \cap (\eta_2,\xi)] \subseteq (\eta_2,\xi)$
$\Rightarrow \overline{[(\eta_1,\xi) \cap (\eta_2,\xi)]} \subseteq \overline{(\eta_1,\xi)}$ and $\overline{[(\eta_1,\xi) \cap (\eta_2,\xi)]} \subseteq \overline{(\eta_2,\xi)}$
$\Rightarrow \overline{[(\eta_1,\xi) \cap (\eta_2,\xi)]} \subseteq \left[\overline{(\eta_1,\xi)} \cap \overline{(\eta_2,\xi)}\right]$.

9. By 2., we have $(\eta_1,\xi) \subseteq \overline{(\eta_1,\xi)}$ and $(\eta_2,\xi) \subseteq \overline{(\eta_2,\xi)}$.
Thus, $[(\eta_1,\xi) \cup (\eta_2,\xi)] \subseteq \left[\overline{(\eta_1,\xi)} \cup \overline{(\eta_2,\xi)}\right]$.
$\overline{(\eta_1,\xi)}, \overline{(\eta_2,\xi)}$ are closed sets.
$\Rightarrow \overline{(\eta_1,\xi)} \cup \overline{(\eta_2,\xi)}$ is also closed.
Now, $\overline{(\eta_1,\xi)} \cup \overline{(\eta_2,\xi)}$ is a closed set containing $(\eta_1,\xi) \cup (\eta_2,\xi)$.
But $\overline{[(\eta_1,\xi) \cup (\eta_2,\xi)]}$ is the smallest closed set containing $(\eta_1,\xi) \cup (\eta_2,\xi)$.
Consequently, $\overline{[(\eta_1,\xi) \cup (\eta_2,\xi)]} \subseteq \left[\overline{(\eta_1,\xi)} \cup \overline{(\eta_2,\xi)}\right]$. ---------------------(1)

Since, $(\eta_1,\xi) \subseteq [(\eta_1,\xi) \cup (\eta_2,\xi)]$ and $(\eta_2,\xi) \subseteq [(\eta_1,\xi) \cup (\eta_2,\xi)]$
$\Rightarrow \overline{(\eta_1,\xi)} \subseteq \overline{[(\eta_1,\xi) \cup (\eta_2,\xi)]}$ and $\overline{(\eta_2,\xi)} \subseteq \overline{[(\eta_1,\xi) \cup (\eta_2,\xi)]}$
$\Rightarrow \overline{(\eta_1,\xi)} \cup \overline{(\eta_2,\xi)} \subseteq \overline{[(\eta_1,\xi) \cup (\eta_2,\xi)]}$ --------------------(2)
From (1) and (2), we get $\overline{(\eta_1,\xi)} \cup \overline{(\eta_2,\xi)} = \overline{[(\eta_1,\xi) \cup (\eta_2,\xi)]}$.

Proposition 4.12.
Let $(\mho, \xi, \acute{T})$ be an Interval-valued Neutrosophic Hypersoft topological space over $\mho$ and let $(\eta,\xi), (\eta_1,\xi)$ and (η_2,ξ) be an IVNHSS on $\mho$. Then,

1. $[(\eta,\xi)^{\circ}]^c = \overline{((\eta,\xi)^c)}$
2. $(\overline{(\eta,\xi)})^c = [(\eta,\xi)^c]^{\circ}$
3. $\overline{(\eta,\xi)} = ([(\eta,\xi)^c]^{\circ})^c$
4. $(\eta,\xi)^{\circ} = (\overline{((\eta,\xi)^c)})^c$

Proof

1. By Definition 4.7, 4.8
$(\eta_1,\xi)^{\circ} = \cup\{(\eta_2,\xi) | (\eta_2,\xi) \in \acute{T}, (\eta_2,\xi) \subseteq (\eta_1,\xi)\}$
$[(\eta,\xi)^{\circ}]^c = [\cup\{(\eta_2,\xi) | (\eta_2,\xi) \in \acute{T}, (\eta_2,\xi) \subseteq (\eta_1,\xi)\}]^c$
$= \cap\{(\eta_2,\xi)^c \mid (\eta_2,\xi) \in \acute{T}, (\eta_1,\xi)^c \subseteq (\eta_2,\xi)^c\}$
$[(\eta,\xi)^{\circ}]^c = \overline{((\eta,\xi)^c)}$

2. By Definition 4.7, 4.8
$\overline{(\eta_1,\xi)} = \cap\{(\eta_2,\xi) | (\eta_2,\xi)^c \in \acute{T}, (\eta_1,\xi) \subseteq (\eta_2,\xi)\}$
$[\overline{(\eta_1,\xi)}]^c = [\cap\{(\eta_2,\xi) | (\eta_2,\xi)^c \in \acute{T}, (\eta_1,\xi) \subseteq (\eta_2,\xi)\}]^c$
$= \cup\{(\eta_2,\xi)^c \mid (\eta_2,\xi)^c \in \acute{T}, (\eta_2,\xi)^c \subseteq (\eta_1,\xi)^c\}$
$(\overline{(\eta,\xi)})^c = [(\eta,\xi)^c]^{\circ}$

3. This is obvious by considering complement of 2.
4. This is obvious by considering complement of 1.

Proposition 4.13.
Let Let $(\mho, \xi, \acute{T})$ be an Interval-valued Neutrosophic Hypersoft topological space over $\mho$ and let (η,ξ) be an IVNHSS on $\mho$. Then, $(\eta,\xi)^{\circ} \subseteq (\eta,\xi) \subseteq \overline{(\eta,\xi)}$.

Proof.
Straight forward from Proposition 3.32 2. and Proposition 3.33 2.

V. Conclusion

In this paper, we have introduced the concept of Interval-valued Neutrosophic Hypersoft Topology. Further, some notions such as finer, coarser, discrete, indiscrete, neighbourhood, neighbourhood system, basis of an Interval-valued Neutrosophic Hypersoft Topology are introduced along with their properties and examples. Later, Interval-valued Neutrosophic Hypersoft interior and closure are studied and also some of their relationships are investigated. In future, connectedness, compactness, separation axioms and similarity measures can be studied.

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