

Reimann's Hypothesis Solved

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Abstract:

This paper solves the perpetual problem of Reimann's hypothesis that has remained unsolved till date and has intrigued scientists for ages, centuries. Concrete proof along with mathematical equations is used to show that the hypothesis is wrong and that the values and assumptions suggested by Reimann is incorrect.

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I. Introduction:

The Riemann Hypothesis, one of the most famous unsolved problems in mathematics, was proposed by the German mathematician Bernhard Riemann in 1859. It relates to the distribution of prime numbers and zeros of the Riemann zeta function. The hypothesis states that all non-trivial zeros of the zeta function lie on a vertical line in the complex plane, called the critical line. It is of great interest in number theory because it implies results about the distribution of prime numbers. While there has been significant progress in understanding the hypothesis, it remains an open problem in mathematics that has intrigued generations of mathematicians.

II. Observations:

The Riemann zeta function $\zeta(s)$ is a function whose argument s may be any complex number other than 1, and whose values are also complex. It has zeros at the negative even integers; that is, $\zeta(s) = 0$ when s is one of $-2, -4, -6, \dots$, called its trivial zeros. The zeta function is also zero for other values of s , which are called nontrivial zeros. The Riemann hypothesis is concerned with the locations of these nontrivial zeros, and states that:

The real part of every nontrivial zero of the Riemann zeta function is $1/2$.

Thus as per the hypothesis all the nontrivial zeros lie on the critical line consisting of the complex numbers $1/2 + it$, where t is a real number and i is the imaginary unit.

The Riemann zeta function is defined for complex s with real part greater than 1 by the absolutely convergent infinite series as shown below:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \dots$$

III. Trivial Zero:

For trivial zero, where $s = -4$ (for example), $\zeta(s) = 0$

Or,

$$\sum_{n=1}^{\infty} \frac{1}{n^{-4}} = 0$$

$$\sum_{n=1}^{\infty} n^4 = 0$$

The above equation implies that n can have only one and one value for the series sum to be 0, i.e. 0

In other words,

$$n = 0$$

As you see that n is zero and not starting from 1 as from the original zeta function (for the trivial zeros to be possible for negative even integers in the series).

IV. Nontrivial Zero:

For nontrivial zero, where $s = 1/2 + it$, $\zeta(s) = 0$

Or,
$$\sum_{n=1}^{\infty} \frac{1}{n^{1/2+it}} = 0$$

Again for the above equation to be nontrivial zero, the value of n has to be zero and s or $1/2 + it$ has to be less than zero (otherwise the value is infinity not 0).

Implies, $n = 0$ and $s = 1/2 + it < 0$

As you see that n is zero and not starting from 1 and s is negative and not greater than 1 as assumed from the original zeta function for the nontrivial zeros to be possible in the critical line.

V. Infinity:

For $\zeta(s) = \infty$ implies that s has to be less than zero or $s < 0$ otherwise the absolute sum is a fraction or a very small number.

As you see that s is negative and not greater than 1 as assumed from the original zeta function for the infinity poles to be possible in the series.

VI. Discussions:

It can be easily seen from the observation section that the value of n is zero for trivial and nontrivial zero's cases with the value of s being less than zero for the last two cases of nontrivial and infinity. The original premise of Reimann hypothesis is that the value of s is greater than 1 and the value of n ranging from 1 to infinity for the convergent series. This means only one thing that the hypothesis is wrong and that the values of trivial, nontrivial zero's and infinity is not possible within the convergent series, taking into consideration the assumptions of the Reimann's hypothesis.

VII. Final Conclusion:

The analysis proves that the basic assumptions in the Reimann's hypothesis is wrong and that the hypothesis is not true which means that the real value of nontrivial zero's is not $1/2$ for the zeta function. This thus solves the enigmatic problem in pure mathematics of Reimann's dilemma by proving it wrong.

References

- [1]. Bombieri, Enrico (2000), The Riemann Hypothesis – Official Problem Description (PDF), Clay Mathematics Institute, Retrieved 2008-10-25 Reprinted In (Borwein Et Al. 2008).
- [2]. Borwein, Peter; Choi, Stephen; Rooney, Brendan; Weirathmueller, Andrea, Eds. (2008), The Riemann Hypothesis: A Resource For The Afficionado And Virtuoso Alike, CMS Books In Mathematics, New York: Springer, Doi:10.1007/978-0-387-72126-2, ISBN 978-0-387-72125-5
- [3]. Borwein, Peter; Ferguson, Ron; Mossinghoff, Michael J. (2008), "Sign Changes In Sums Of The Liouville Function", Mathematics Of Computation, 77 (263): 1681–1694, Bibcode:2008macom..77.1681B, Doi:10.1090/S0025-5718-08-02036-X, MR 2398787
- [4]. Connes, Alain (2000), "Noncommutative Geometry And The Riemann Zeta Function", Mathematics: Frontiers And Perspectives, Providence, R.I.: American Mathematical Society, Pp. 35–54, MR 1754766
- [5]. Connes, Alain (2016), "An Essay On The Riemann Hypothesis", In Nash, J. F.; Rassias, Michael (Eds.), Open Problems In Mathematics, New York: Springer, Pp. 225–257, Arxiv:1509.05576, Doi:10.1007/978-3-319-32162-2_5
- [6]. Platt, Dave; Trudgian, Tim (January 2021), "The Riemann Hypothesis Is True Up To $3 \cdot 10^{12}$ ", Bulletin Of The London Mathematical Society, Wiley, 53 (3): 792–797, Arxiv:2004.09765, Doi:10.1112/Bllms.12460, S2CID 234355998
- [7]. https://En.Wikipedia.Org/Wiki/Riemann_Hypothesis