IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 19, Issue 4 Ser. 1 (July. – August. 2023), PP 24-29
www.iosrjournals.org

Kamal Transform Technique To Coupled Systems Of Linear Ordinary Differential Equations

Onuoha, N. O.
Department Of Mathematics, Imo State University, Owerri, Imo State, Nigeria

Abstract
The solutions of differential equations can be obtained using different analytical methods. Many transform methods have been applied to differential equations and results were obtained. Coupled systems of linear ordinary differential equations often occur in context where two or three different variables are expected to interact. Context such as in population models, mechanical systems, and in electrical engineering. This study showcases the applicability of the new integral transform, Kamal transform, to coupled systems of linear ordinary differential equations by solving three different coupled systems of linear ordinary differential equations.

Keywords: *Kamal transform, Ordinary differential equation, Coupled systems, Mechanical systems, Intergral transform*

Date of Submission: 09-07-2023 Date of Acceptance: 19-07-2023

I. Introduction

Real life problems are represented mathematically in form of differential equations, integral equations, difference equations and so on. In all the models, differential equations seem to have great applications than the other models especially in sciences and engineering; mechanical, civil and electrical engineering. The qualitative and quantitative study of differential equations has received more patronage due to its broad applications. This increases the interest of researchers on solution methods. The approach of using one fold integral transforms to solve differential equations has proved its great applicability which Kamal transform is not an exceptional.

Kamal transform was introduced by Abdehilal Kamal Hassan Sedeeg in 2016 [1]. He derived the transform from the classical Fourier integral. Abdehilal, targeted facilitating solutions of ordinary and partial differential equations in the time domain. Abdehilal and Zahra [3], derived Kamal transform of partial derivatives and applied it to four different partial differential equations, and obtained particular solutions to the four differential equations.

Sudhanshu and Gyanvendra [21], found the Kamal transform of error and complementary error functions to demonstrate the usefulness of Kamal transform of error functions. Sudhanshu and swarg Deep [22],applied Kamal transform to Abel's integral equation and some numerical applications in his application section to explain the effectiveness of the transform to Abel's integral equation. In 2018, Anjana et al [3] used Kamal transform to solve linear partial integro-differential equations. They described and illustrated the technique with applications. This new integral transform and congruence modulo operator had been used to encrypt and decrypt a message [19]. Rachama et al [20] introduced fractional calculus for Kamal transform and some nonhomogenous fractional ordinary differential equation by kamal transform. The combination of Kamal transform and Adomian decomposition has been applied to nonlinear differential equations. This combination is proposed by A. Emimal and C. Dhinesh [6] and is called Kamal decomposition. Kamal transform is gradually, gaining interest in cryptography. Ayush and Ravindra [8] used Kamal transform and congruence modulo operator involving ASCII value for encryption and decryption of message. In fluid mechanics, Kamal transform is proving its effectiveness. Johnson et al [12] considered the flow of a viscous incompressible fluid between two parallel plates due to the normal motion of the plates for two dimension. The solution of the governing nonlinear equations and their associated boundary conditions were obtained by Kamal transform decomposition method. The verify the solution by approximate analytic results obtained by homotopy perturbation method. Sudhanshu [23] determined the Kamal transform of Bessel's functions. Some applications of Kamal transform of Bessel's function for evaluating the integral, which contain Bessel's function, are given.

N. T Katre and R.T Katre [17] did a comparative study of Kamal and Laplace transform. They solved linear differential equations using the two methods and concluded that the integral transforms are in close connection to each other. Zainab and Nejmaddin [24], presented solution of the linear system of Volterra integro-differential equations of the second kind by Kamal transform. In their research, they concluded that

Kamal transform is very effective for obtaining the exact solution of the linear system of Volterra integro-differential equations of the second kind. Muhammad et al [15], solved heat and temperature problems using Kamal transform where the concluded that descriptive properties examples show the efficiency of its suitability in solving differential equations. Ghanwat and Gaikwad [7] solved linear volterra equation of second kind and integro-differential equations using Kamal transform. Combination of Adomian decomposition and Kamal integral transformation is applied to solve differential equations of fractional order [10]. Their result shows that the combination is very accurate in solving differential equations of fractional order. Chander and Hemlata [9] considered a generalized fractional kinetic equation which contain generalized Mittag-Leffler function and the solution is obtained by the method of Laplace transform and Kamal transform. Antony et al [14] proposed a new method for investigating the Ulam stability of linear differential equations by using Kamal transform. Pandama and Yogesh [18] considered the Kamal transform of derived function and demonstrated that the Kamal transform of derived functions can be shown by an infinite arrangement or Heaviside function.

Coupled differential equations have been analyzed using different technique. [5], [10], [11], [13], and [14] present solutions to coupled systems of differential equations using different solution methods.

II. Definition of Kamal Transform

Kamal transform is defined for function of exponential order [1]. The Kamal transform of a function $f(t)$ in a given set A is defined by

$$k\left[f(t)\right] = G(v) = \int_0^\infty f(t) e^{-\frac{t}{v}} dt \quad t \geq 0,\ k_1 \leq v \leq k_2$$

(1a)

where the set A is defined by

$$A = \left\{ f(t) : \exists M, k_1, k_2 > 0, \left| f(t) \right| < M e^{\frac{|t|}{k_j}}, if\ t \in (-1)^j \times [0, \infty) \right\}$$

(1b)

Kamal transform has a good correlation trend with the Laplace, Elzaki and other transforms [1].

This new integral transform will be used here to solve coupled systems of linear ordinary differential equations.

First Coupled System of Linear Ordinary Differential Equations

$$x_1' = 3x_1 - 3x_2 + 2$$
$$x_2' = -6x_1 - t \qquad (2a)$$
$$x_1(0) = 1,\ x_2(0) = -1 \qquad (2b)$$

To solve equation (2a), let the Kamal transform of $x_1(t)$ and $x_2(t)$ be as follows

$$K\left\{x_1(t)\right\} = \int_0^\infty e^{-\frac{t}{v}} x_1(t) dt = X_1(v) \qquad (3)$$

$$K\left\{x_2(t)\right\} = \int_0^\infty e^{-\frac{t}{v}} x_2(t) dt = X_2(v)$$

(4)

Kamal transform of both differential equations using equation (3) and (4) is

$$\frac{1}{v} X_1(v) - x_1(0) = 3X_1(v) - 3X_2(v) + 2V$$

$$\frac{1}{v} X_2(v) - x_2(0) = -6X_1(v) - V^2$$

(5)

Applying equation (2b) on equation (5) and solving equation (5) simultaneously to find $X_1(v)$, it gives

$$\left(\frac{1}{v^2}-\frac{3}{v}-18\right)X_1(v)=\frac{2v+1}{v}+3v^2+3$$

(6)

From equation (6),

$$X_1(v)=\frac{\frac{5}{v^2}+\frac{1}{v^3}+3}{\frac{1}{v^2}\left(\frac{1}{v}-6\right)\left(\frac{1}{v}+3\right)} \tag{7}$$

By partial fraction decomposition, equation (7) becomes

$$X_1(v)=\frac{1}{108}\left(\frac{133v}{1-6v}-\frac{28v}{1+3v}+3v-18v^2\right)$$

(8)

Taking the inverse Kamal transform of $X_1(v)$, equation (8) gives

$$x_1(t)=\frac{1}{108}\left(133e^{6t}-28e^{-3t}-18t+3\right)$$

(9)

Next is to find $x_2(t)$. Substitute equation (9) into any of the equations in the coupled equations of equation (2a).

$$x_2(t)=-\frac{1}{108}\left(133e^{6t}+56e^{3t}+18t\right)+\frac{3}{4}$$

(10)

Second Coupled System of Linear Ordinary Differential Equations

$$\begin{aligned} &x''-2x'+3y'+2y=4\\ &2y'-x'+3y=0 \end{aligned} \tag{11a}$$

$$x(0)=x'(0)=y(0)=0 \tag{11b}$$

Applying Kamal transform, equation (1) to equation (9a), we define the Kamal transform of $x(t)$ and $y(t)$ respectively as

$$K\{x(t)\}=\int_0^\infty e^{-\frac{t}{v}}x(t)\,dt=X(v) \tag{12}$$

$$K\{y(t)\}=\int_0^\infty e^{-\frac{t}{v}}y(t)\,dt=Y(v) \tag{13}$$

Taking the Kamal transform of equation (11a) using equations, (12), (13), and (11b), we have

$$\frac{1}{v^2}X(v)-\frac{2}{v}X(v)+\frac{3}{v}Y(v)+2Y(v)=4v$$

$$\frac{2}{v}Y(v)-\frac{1}{v}X(v)+3Y(v)=0$$

(14)

On solving equation (14) simultaneously, we have

$$X(v)=\frac{\frac{4}{v}+6}{\frac{1}{v^2}\left(\frac{1}{v}+2\right)\left(\frac{1}{v}-1\right)} \tag{15}$$

By partial fraction decomposition, equation (15) becomes

$$X(v) = -\frac{7v}{2} - 3v^2 + \frac{v}{6(1+2v)} + \frac{10v}{v(1-v)}$$

(16)

Taking the inverse Kamal transform of equation (16),

$$x(t) - \frac{7}{2} - 3t + \frac{1}{6}e^{-2t} + \frac{10}{3}e^{t} \tag{17}$$

To find the second variable, $y(t)$, substitute for $X(v)$ in equation (14) using equation (15), we get

$$Y(v) = \frac{2}{\frac{1}{v}\left(\frac{1}{v}+2\right)\left(\frac{1}{v}-1\right)} \tag{18}$$

By partial fraction decomposition, equation (18) becomes

$$Y(v) = -v + \frac{v}{3(1+2v)} + \frac{2v}{3(1-v)} \tag{19}$$

Finding the inverse Kamal transform of equation (19), we have

$$y(t) = -1 + \frac{1}{3}e^{-2t} + \frac{2}{3}e^{-t} \tag{20}$$

Third Coupled System of Linear Ordinary Differential Equations

$$\dot{x} = 3x + y + z$$

$$\dot{y} = x + 5y + z \tag{21a}$$

$$\dot{z} = x + y + 3z$$

$$x(0) = 3,\ y(0) = z(0) = 0 \tag{21b}$$

Solving equation (21a) using Kamal transform, we take the Kamal transform of the equation.
Taking the Kamal transform, we get

$$\left(\frac{1}{v}-3\right)X(v) - Y(v) - Z(v) = 3$$

$$X(v) - \left(\frac{1}{v}-5\right)Y(v) + Z(v) = 0 \tag{22}$$

$$X(v) + Y(v) - \left(\frac{1}{v}-3\right)Z(v) = 0$$

Equation (22) can be written in the form

$$\begin{pmatrix} \frac{1}{v}-3 & -1 & -1 \\ 1 & -\frac{1}{v}+5 & 1 \\ 1 & 1 & -\frac{1}{v}+3 \end{pmatrix} \begin{pmatrix} X(v) \\ Y(v) \\ Z(v) \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$

(23)

On solving equation (23), we obtain

$$X(v)=\frac{3v}{2(1-2v)}+\frac{v}{1-3v}+\frac{v}{2(1-6v)}$$
$$Y(v)=-\frac{v}{1-3v}+\frac{v}{1-6v}$$
$$Z(v)=-\frac{3v}{2(1-2v)}+\frac{v}{1-3v}+\frac{v}{2(1-6v)}$$

(24)

Taking the inverse Kamal transform of equation (24), we have

$$x(t)=\frac{3}{2}e^{2t}+e^{3t}+\frac{1}{2}e^{6t}$$
$$y(t)=-e^{3t}+e^{6t}$$
$$z(t)=-\frac{3}{2}e^{2t}+e^{3t}+\frac{1}{2}e^{6t} \qquad (26)$$

Which is the solution of equation (21a) and (21b).

III. Conclusion

The new one fold integral transform, Kamal transform has been applied to coupled systems of linear ordinary differential equations. The process shows that Kamal transform is less computational and effective. The results obtained, proves its applicability to coupled systems of linear ordinary differential equations.

References

[1]. Abdelilah Kamal H. Sedeeg (2016). The New Integral Transform “Kamal Transform”.

[2]. Advances in Theoretical and Applied Mathematics. ISSN 0973 – 4554 Volume 11, Number 4, pp 45 – 48

[3]. Abdelilah Hassan Sedeeg and Zahra I. Adam Mahamoud (2017). The Use of Kamal Transform for Solving Partial Differential equations. Advances in Theoritical and Applied Mathematics. ISSN 0973 – 4554 Volume 12, No 1, pp 7-13

[4]. Anjana Gupta, Sudhaushu Aggarwal and Deeksha Agrawal (2018). Solution of Linear Partial Integro-Differential Equations Using Kamal Transform.

[5]. Antony Raj Aruldass, Divyakumari Pachaiyappan, and Choonkil Park (2021). Kamal Transform and Ulam Stability of Differential Equations. Journal of Applied Analysis & Computation. 11(3): 1631-1639.

[6]. A.C. Allison (1970). The Numerical Solution of Couple Differential Equations Arising from the Schrodinger Equation. Journal of Computational Physics. Volume 6, Issue 3, pages 378-391.

A. Emimal Kanaga Pushpam and C. Dhiesh Kumar (2019): Kamal Decomposition Method for Solving Nonlinear Delay Differential Equations. Bulletin of Pur and Apllied Sciences.
Vol.38E(Math & stat), No.1, p. 231-234 ISSN 2320-3226. DOI: 10.5958/2320-3226

[7]. A.J. Ghanwat, and S.B. Gaikwad (2022): Application of Kamal Transform for Solving Linear Volterra Integral Equations of Second Kind. Journal of Emerging Technology and Innovative Research (JETIR). Volume 9, Issue 4, ISSN: 2349-5162, pp 17-14.

[8]. Ayush Mittal and Ravindra Gupta (2019). Kamal Transformation Based Cryptographic Technique in Network Security Involving ASCII Value. International Journal of Innovative Technology and Exploring Engineering (IJITEE) ISSN: 2278-3075, volume-8 , Issue-12.

[9]. Chander Prakash Samar and Dr. Hemlata Sexena (2022). Solution of Generalized Fractional Kinetic equation by Laplace and Kamal Transformation. International Journal of Mathematics Trends and Technology, 67. 6, 38-43.

[10]. C X Chuan (1988). Generalisation of the Theory of Coupled Differential Equations. Journal of PhysicsA: Mathematical and Genaral, Volume 21, Number 3.

[11]. Eugene D. Denman (1972). A Numerical Methods for Coupled Differential Equations.International Journal for Numerical Methods in Engineering. Volume 4, issue 4/p. 587-596

[12]. Johnson Adekunle Owolabi, Olufemi Elijah Ige and Emmanuel Idowu akinola (2019).

[13]. Applications of Kamal decomposition Transform Method in Solving Two Dimensional Unsteady Flow. International Journal of Difference Equations (IJDE) ISSN: 0973-6069, Volume 14, Number 2, 207-214.

[14]. John Newman (1968). Numerical Solution of Coupled Ordinary Differential Equations.

[15]. Industrial & Engineering Cemistry Fundamentals. Vol 7, Issue 3, p 514-517.

[16]. Keqin Gu, Yashum Zhang and Shengyuan Xu (2011). Small Gain Problem in Coupled Differential-Difference Equations, Time-Varying delays, and Direct Lyapnuov Method.

[17]. International Journal of Robust and Nonlineat Control/Volume 21, Issue 4/p. 492-451.

[18]. Muhammad waqas, Khurrem Shehzad, Ali Moazzam, and Alizay Batool (2022).

[19]. Applications of Kamal Transformation in Temprature Problems. Scholars Journal of Engineering and Technology.
ISSN 2321 – 4351 DOI: 10. 36347/sjet. V10i02. 001

[20]. Muhamad Deni johansyah, Asep K. Supriatna, Endang Rusyaman, Jumadil Saputra (2022).

[21]. Solving Differential Equations of Fractional Order using Combined Adomian Decomposition Method with Kamal Integral Transformation. Mathematics and Statistics Vol. 10(1), pp 187 – 194 DOI: 10.13189/ms. 2022. 100117

[22]. N T Katre and R T Katre (2021). A Comparative Study of Laplace and Kamal Transform. Journal of Physics: Confrernce Series, volume 1913. DOI: 10. 1088/1742 – 6596/1913/1/012115

[23]. Padama Kumawat and Yogesh Khandelwal (2021).
The Kamal Transform of Derived Function Demonstrated by HeavisideFunction. Indian Journal of Science and Research. 1(1),17 - 19

[24]. P. Senthil Kumar and S. Vasuki (2018). Application of 'Kamal' Transform in Cryptography.
International Journal of Interdisciplinary Research and Innovations. Vol 6, Issue 3, pp. (182 – 186)

[25]. Rachama Khandelwal, Priyaanka Chondhary and Yogesh Khandelwal (2018). Solution of Fractional Ordinary Differential Equation by Kamal Transform. International Journal of Statistics and Applied Mathematics. 3(2): 279 – 284.

[26]. Sudhanshu Aggarwal and Gyanvendra Pratap Singh (2019). Kamal Transform of Error Function. Journal of Applied Science and Computations. ISSN NO: 1076 – 5131, pp 2223 –2235, volume IV, Issue V.

[27]. Sudhanshu aggarwal and Swag Deep Sharma (2019). Application of Kamal Transformation for Solving Abel's Integral equations. Global Journal of Engineering Science and Researches. Doi. 10. 5281/zewdo. 2593989 ISSN 2348 - 8034

[28]. Sudhanshu Aggarwal (2018). Kamal Transform of Bessel's Functions. International Journal of Research and Innovation in Applied Science (IJRIAS) | Volume III, Issue VII |ISSN 2454 – 6194.

[29]. Zainab Rustam and Nejmaddin Sulaiman (2021). Kamal Transform Technique for Solving Systems of Linear Volterra Integro-Differential equations of the Second Kind. International Journal of Nonlinear Analysis and Applications. Doi: 10. 22078/IJNAA. 25605.3065.Volume 14, Issue 1, pp 185 – 192.