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Optimal Supply Policy For Items With Modified Three-Parameter Weibull Deterioration, Inventory-Dependent Demand, And Partial Backlogging.

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ABSTRACT
The inventory model (IM) for products with inventory-dependent demand levels (IDDL) was considered in the study. In this model, wear time was assumed to follow a modified three-parameter Weibull distribution while the demand rate (DR) was a quadratic function (QF) with shortages permitted and no backlog. The model was examined under the presumption that the deficit is manageable and has only partially accrued. To reduce the cost of inventory every cycle, the study addresses the ideal replenishment strategy in terms of the number of goods and the duration of storage. The three-parameter Weibull Hazard Index illustrates the effect of both obsolete and potentially degrading inventory items. Numerical example using the Sangote Restaurant (Catfish unit) was used to illustrate the applicability in practice. This study's findings are distinct from prior findings in that they model consumption time using a modified three-parameter Weibull distribution with a quadratic stock-dependent withdrawal rate and partial lag (PL). The concept was discovered to be appropriate for goods whose inventory levels (IL) are aesthetically presented to draw customers and consequently boost sales. The effects of outmoded items entering inventory as well as units that may start to deteriorate in the future are captured by a modified three-parameter Weibull instantaneous rate that was incorporated in the model.

Keywords: *Inventory; Optimal, Stock-dependent; Modified Three-parameter Weibull, EOQ.*

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I. Introduction

Every management decision is made according to the basic principle to maximize profit in the shortest possible time with minimum cost. Management decisions of every profit-oriented organization include, but are not limited to, the quantity and quality of goods to be produced or purchased; when too many goods are produced or purchased, resulting in overspending over a period of time, there is always a cost burden associated with it. In addition, producing or purchasing too few goods so that inventory is held and customer demands are not met results in other costs. The costs associated with producing or purchasing too much or too little of an item has made controlling and maintaining physical inventory difficult for corporate enterprises. To establish and maintain a cost-effective inventory policy, many various issue situations must be mathematically handled in the form of models, taking into account business organization elements such as system architecture, market features, product classifications, and expenses. After a certain amount of time, especially after they arrive in stock, products begin to deteriorate. Several inventory models have been developed in an attempt to have profound methods of inventory management that can address the inventory concerns as pointed by [1] which are: (i) how much to order (produce or purchase), and (ii) when to order to optimize the total cost. Demand is usually one of the main factors considered in inventory research. The assumption that the level of demand is constant may not generally be appropriate for most items of inventory, because in reality, the demand for physical goods may depend on time or on inventory, price, etc. For this reason, different research results have been devoted to the study of different types of requirements related to wearable inventory items. Such works include [2], who considered an exponentially declining level of demand; [3] studied ramp-type velocity; [4] studied time-dependent demand; [5] found that demand depends on inventory; [1] studied quadratic demand rates etc. The selling price of goods can significantly affect the demand for those goods [6]. One of the main criteria for a customer who visits the market to buy a product is the price of that product. Generally, the lower the price of a product, the higher the sales level of that product, while a higher price has the opposite effect. Subsequent research focused on different price-dependent DR for deteriorating items. The study by [7-9] assumed a linear price-dependent DR when modeling the economic order quantity (EOQ) of deteriorating inventory units. Also, [10] considered EOQ models for exponential demand levels of item deterioration. While the study by [11] presented different inventory models for deteriorating goods with a quadratic sales price-dependent demand rate. Deterioration rate is an-other important factor in inventory research for perishable items. It describes the deterioration of items. In the classic EOQ model developed by [12], the rate of deterioration was assumed to be constant. This led most re-searchers in the early stages of the study of stock deterioration to assume a constant rate of deterioration. Although it simplifies the issue, a constant pace does not accurately represent the state of deterioration in reality. The pace of decline should ideally change throughout time. The study by [13] made the assumption that the rate of deterioration was non-constant extended the fundamental EOQ model. They took into account the inventory system's exponential degradation in their investigation. However, the literature suggests that the exponential distribution suits better to model the behavior of units that have a constant failure rate or units that do not deteriorate over time. It does not consider whether the pace of deterioration is accelerating or slowing down. The work by [6] supports the empirical observation that a wide range of outcomes can be expressed using the Weibull distribution. The deterioration process starts after a specific period has passed since the goods arrived in the warehouse. A partial delay is present when the model is being constructed. The study's objectives are to give a perfect model replenishment policy, develop an analytical framework for the accepted inventory system, and carry out a sensitivity assessment to determine the optimal inventory policy. These methods include the modified three-parameter Weibull hazard rate, linear IDDL, and partial backlogging rate (PBR).

The study by [14] argued that a three-parameter extension of the Weibull distribution addresses common circumstances in modelling survival processes with a variety of forms in the hazard function. This argument served as the basis for the usage of the three-parameter Weibull distribution for modelling deteriorating inventory. The usage of the three-parameter Weibull distribution has also been justified by authors such as [15-16]. The benefits of the three-parameter Weibull distribution will be used in this study to characterize the rate at which inventory items on trade credit with shortages deteriorate. A significant barrier to applying the model is the basic EOQ model's assumption of a constant demand rate. Although this presumption simplifies the issue, it does not accurately represent the actual situation because demand is dynamic. The assumption of a constant demand rate, according to [2], was not always relevant in many inventory systems (for instance, for electronic items, trendy clothing, etc.), as their demand rates may fluctuate. Additionally, throughout the product's growth phase, many products enjoy a period of increased demand. On the other side, the arrival of more desirable products may cause consumers' preferences to shift, resulting in a drop in the demand for particular products (that is, substitute goods). The majority of EOQ models for depreciating inventory items taken into account include linear demand [17], exponential demand, stock-dependent demand [18-19], time-dependent demand [20], ramp-type demand, constant demand [21], trapezoidal-type demand [22] emphasized that a product's demand, which is influenced by its price, determines the overall quantity that a store orders. The selling price of a product and the demand for it are typically found to be inversely related. As a result, the price of the items controls the demand for the majority of inventory items. This served as the foundation for the study's usage of quadratic price-dependent and quadratic time-dependent demand rates. Some authors provided a deterministic inventory model for degrading commodities whose demand rate is a function of both price and time that decreases exponentially. Many scholars have created EOQ models for degrading items with price-dependent demand rates. Presumably, the rate of depreciation is a constant percentage of the inventory in stock. The deterioration rate and holding cost of the EOQ inventory model created by [23] are linearly time-dependent, but the demand rate is price-dependent. In a deterministic inventory model created by [24], the demand rate is depicted as a linear function of price under profit maximization. The rate of deterioration follows the Weibull distribution, and shortages are permitted. An EOQ inventory model was presented by [25] for goods with two-parameter Weibull-distributed degradation. Prices affect the rate of demand, thus shortages are permitted. A model for a two-echelon inventory system with price-dependent demand and allowable payment delays was put forth by [26].

Using a two-parameter Weibull degradation, a time-varying holding cost, and a demand rate that depends on price and advertising cost, [27] created a deterministic inventory model for a depreciating commodity. There can be shortages, and they are somewhat backlogged. To find the best ordering strategy for inventory goods with an exponentially distributed deterioration rate under trade credit, [8] developed an inventory model that depends on the order amount. The selling price, which increases over time, is thought to be a function of the demand rate. There are no restrictions on shortages, and they are entirely backlogged. However, as was previously stated, the exponential distribution is not the best choice for characterizing the rate of item deterioration. It is better to use the Weibull distribution. By modifying the two-parameter Weibull distribution to characterize the rate of deterioration of the inventory items in light of this, [28] expanded [8] work. In their research, shortages are both permitted and fully backlogged, and the demand rate is price-dependent. The study by [29] developed an economic order quantity model for decaying items in a single warehouse using quadratic price-dependent demand.

The study by [30] developed an economic order quantity (EOQ) inventory model in their study for a deteriorating item with the following features: (i). the demand rate was predictable and two-staged; in the first half of the cycle, it is constant, and in the second, it follows a linear function of time. (ii) The rate of degradation is proportional to time. Shortages are not permitted (iii). Minimizing the total average cost leads to the optimal cycle time and order quantity. Using a genetic algorithm and the impact of inflation, the study by [31] provided a two-warehouse inventory model where demand is dependent on supplies. In their study, [32] examined the effects of partial backlogs on an inventory model for deteriorating goods with ramp-type demand and price discounts. The study by [33] suggested a two-warehouse inventory model where the degradation of goods under the influence of inflation follows a two-parameter Weibull distribution and the demand rate varies exponentially with time. An inventory model for degrading products with a stock-dependent and ramp-type demand taking into account reserve money and carbon emission was described by [34]. According to the study by [35], a two-warehouse inventory model for perishable goods was developed with ramp-type demand, in which shortages are permitted and partially backlogged. The proposed model was taken into account for two scenarios: Before the inventory level in the rented warehouse drops to zero, the demand rate stabilizes. ii) When the rented warehouse is empty and the need is satisfied by the owner's warehouse, the demand rate stabilizes. Two different inventory models were looked at in the study by [36] for a depreciating item under the conditions of frequent advertising and price-sensitive market aggregate demand where the depreciation percentage follows the Weibull distribution. One model does not consider the stock-out scenario, while another somewhat backorders items to deal with the stock-out issue based on how long customers will have to wait. It can be shown from a comparison of the relevant findings that the inventory model with shortages is more cost-effective from a profit-maximizing perspective. It was discovered that the decision-maker may significantly boost profit by putting the right marketing strategies into practice to boost market demand. This study by [37] provided a practical and cost-effective multi-criteria inventory strategy for deteriorating products in a supply chain with nonlinear ramp functions and allowable payment delays under inflation. Two requirements considered for the purpose of developing models were Shortages Followed by Inventory (SFI) and Inventory Followed by Shortages (IFS). were taken into consideration. The goal of developing these models was to reduce the overall average cost per unit of time. The study's findings show that as quadratic function parameter values are increased while maintaining cost demand, the cost per unit time of the IFS model reduces. In contrast, the cost per unit of time for the SFI model drops as the exponential function's parameter values are increased while keeping the cost per unit of time constant. Yet it was discovered that the cost per unit time of the IFS model increased when the ordering cost, shortfall cost, and parameter values for the exponential functions increased. On the other hand, it was discovered to grow when taking into account the SFI condition, if the quadratic function's parameter values as well as the ordering cost and shortage cost increase. The study found that, up to a point, the model including the IFS situation performs better than the SFI model. In order to determine whether any financial techniques have been studied for supporting the management of this class of products and to confirm the existing literature gap, the study by [38] investigated inventory management models for products that change value over time, as well as financial techniques and strategies to support companies in inventory management. According to the study, one buyer-based finance method that offers a potential answer was reverse factoring, in which the buyer arranges for early payments to its supplier so that the money may be used to improve the products' quality. In addition, it was found that shared investment can be taken into account to enhance the effectiveness of the supply chain and ensure that the necessary environment is provided for those items. In their work, [39] proposed the two-warehouse issue of the supply chain system for perishable goods, where demand is assumed to be the function of stock with the exponent function of time. Due to the inventory problem in the supply chain system, which occurs so frequently in day-to-day life, the practical condition of shortage also includes that. To make the program more cognizant of actual life circumstances, backlog cases of scarcity were also taken into consideration. In order to account for the impact of inflation, the study built an inflation component into the model. Under certain assumptions, the analysis discovered the best value for the total variable cost, with inflation having a significant impact on the inventory problem.

From the review of relevant literature, it was found that various inventory functions have been developed for modelling deteriorating items using Weibull distribution. The need for an efficient deterioration item model for Economic Order Quantity (EOQ) inventory cannot be overemphasized. This is due to the fact that it can be helpful in precisely predicting the demand for goods with a short shelf life. In EOQ inventory, this is crucial since it helps prevent both overstocking and under-stocking of goods. A deteriorating item model is essential to EOQ inventory's goal of minimizing the cost of maintaining inventory. The model assists in lowering the cost of retaining inventory by ensuring that the appropriate quantity of stock is ordered by precisely estimating consumer demand for products. A degradation item model assists in guaranteeing that consumers can have the things they need when they need them by precisely anticipating demand and ensuring that the appropriate quantity of stock is available. Customer loyalty and satisfaction, therefore, rise as a result of this. The inventory management process may be automated, which reduces labour costs and boosts productivity. This may result in cost savings and higher productivity, both of which are necessary for keeping a competitive edge in the market today. Therefore, proposing a modified three-parameter Weibull deterioration item model for economic order quantity inventory will aid in effectively determining how much of a given item should be stocked and when allowing for the reasonable and prompt fulfilment of customer orders from stock until the next replenishment. Being aware that the on-hand inventory decreases due to the combined impacts of degradation and demand has a significant impact on these decisions. The proposed Weibull hazard rate is expected to provide a good estimate of the real instantaneous rate of decay of practical inventory items, which is how most practical inventory items deteriorate over time.

II. Material and Methods

The methodology for this study was developed based on the following assumptions and notations:

Notations:

The following notations will be used in the development of the EOQ models:

$\phi(T,t_1)$ = Profit Function
T = Order Cycle Length
T^* = Optimal Value of T
Q = Order Quantity (OQ)
Q^* = Optimal Order Quantity (OOQ)
$I(t)$ = Inventory Level at Time t
$\theta(t)$ = Deterioration Rate (DR)
p = Unit Selling Price
C_1 = Inventory Holding Cost per Unit Time
C_2 = Shortage Cost per Unit Time
C_3 = Ordering Cost per Order
C_4 = Unit Purchasing Cost
I_0 = Initial Inventory Level
t_1 = Time at which there is no Shortage
t_1^* = Optimal Value of t_1
M = Permissible period of delay in payment.

Model Assumptions:

(a) The DR for the product is quadratic and is described by (1) a non-linear function of time, $D(t)=a+bt+ct^2$, $a\geq 0$, $b\neq 0$ and $c\neq 0$, and (2) a non-linear function of price, $D(p)=a+bp+cp^2$, where $a\geq 0$ is the initial rate of demand, $b\neq 0$ is the rate with which the DR grows and $c\neq 0$ is the rate at which the DR itself changes.

(b) During the interval [t_1, T], the shortages are permitted; however all of them are entirely backlogged and are replaced during the subsequent cycle.

(c) Only one component is taken into account.

(d) Items continue to deteriorate over time, making up a constant percentage ($0 < \theta < 1$) of the inventory that is currently at hand. The rate of item deterioration follows a modified three-parameter Weibull distribution model. That is

$$f(t)=(\gamma\beta t-\alpha)^{\beta-1}\ell^{\gamma(\gamma\beta t-\alpha)^{\beta}}\ ,\ t>0 \qquad (1)$$

Where, the instantaneous rate function is

$$\theta(t)=(\gamma\beta t-\alpha)^{\beta-1} \qquad (2)$$

and t is the time of deterioration, $\alpha>0$ is the shape parameter, $\beta>0$ is the scale parameter and γ is the location parameter.

(e) The selling price of the product and all associated costs are known and unaffected by the passage of time.

The study takes into account an inventory system that starts with Q units of each item and in which the quantity of inventory on display has an impact on the rate at which items are withdrawn from the system. Inventory depletes solely as a result of demand in the period $[0,\gamma]$, and as a result of demand and some degree of deterioration in the time interval $[\gamma,t_1]$. At time t_1, there is no inventory and all upcoming demand (i.e., time T >

t_1) is partially stocked. The next replenishment replaces all of the products that were not ordered. Items' degradation times follow a modified Weibull distribution with three parameters.

Using the assumptions of the model, we have:

$$\frac{d\ \mathrm{I(t)}}{dt} + \mathrm{I(t)}\ (\gamma\beta \mathrm{t}\text{-}\alpha)^{\beta-1} = -(\mathrm{a+bt+ct^2}),\ 0 \le \mathrm{t} \le \mathrm{t_1} \qquad (3)$$

$$\frac{d\ \mathrm{I(t)}}{dt} = -(\mathrm{a+bt+ct^2}),\ \mathrm{t_1} \le \mathrm{t} \le \mathrm{T} \qquad (4)$$

The equation (4) is a first order differential equation and their integrating factor is:

$$\exp\left\{\int(\gamma\beta t-\alpha)^{\beta-1}dt\right\} = \ell^{\gamma\beta(\gamma\beta t-\alpha)^{\beta-1}} \qquad (5)$$

Suppose we multiply both sides of (4) with (5), we shall obtain:

$$\frac{d}{dt}\left[\mathrm{I(t)}\ell^{\gamma\beta(\gamma\beta t-\alpha)^{\beta-1}}\right] = -(\mathrm{a+bt+ct^2})\ell^{\gamma\beta(\gamma\beta t-\alpha)^{\beta-1}} \qquad (6)$$

$$\left[\mathrm{I(t)}\ell^{\gamma(\gamma\beta t-\alpha)^{\beta}}\right]\Bigg|_{\mathrm{t}}^{\mathrm{t_1}} = -\int_{\mathrm{t}}^{\mathrm{t_1}}(\mathrm{a+bt+ct^2})\ell^{\gamma(\gamma\beta t-\alpha)^{\beta}}\ \mathrm{dt} \qquad (7)$$

The total cost per unit time, $\phi\left[T,\ t_1\right]$, of the inventory system consist of the deterioration cost (DC), the holding cost (HC), the shortage cost (SC) and the ordering cost. Suppose we put differently, the total cost per unit time will be:

$$\phi\left[T,\ t_1\right] = \frac{1}{T}\left\{\text{deterioration cost + holding cost +shortage cost + ordering cost}\right\}.$$

The components of the total relevant cost can be derived as follows:
The total quantity of deteriorated items during is given [0, t_1] is given by
D= Initial inventory – Total demand during [0, t_1]

$$= I_0 - \int_0^{t_1}(a+bt+ct^2)\mathrm{d}t = I_0 - at_1 - \frac{b}{2}t_1^2\ - \frac{c}{3}t_1^3 \qquad (8)$$

Where,

$$I_0 = a\left[t_1+\frac{\gamma}{\beta+1}\left((\gamma\beta t_1-\alpha)^{\beta+1}-(-\alpha)^{\beta+1}\right)\right]+$$

$$b\left[\frac{t_1^2}{2}+\gamma\left[\frac{1}{\beta+1}\left[t_1(\gamma\beta t_1-\alpha)^{\beta+1}\right]\right.\right.$$

$$\left.\left.-\frac{1}{(\beta+1)(\beta+2)}\ (\gamma\beta t_1-\alpha)^{\beta+2}-(-\alpha)^{\beta+2}\right]\right]$$

$$+c\left[\frac{t_1^3}{3}+\gamma\left[\frac{1}{\beta+1}\left[t_1^2(\gamma\beta t_1-\alpha)^{\beta+1}\right]\right.\right.$$

$$-\frac{2}{(\beta+1)}\left(\frac{1}{(\beta+2)}\left[t_1(\gamma\beta t_1-\alpha)^{\beta+2}\right]-\frac{1}{(\beta+2)(\beta+3)}\right.$$

$$\left.\left.(\gamma\beta t_1-\alpha)^{\beta+3}-(-\alpha)^{\beta+3}\right)\right]$$

$$\left.-\gamma\left[at_1+\frac{1}{2}bt_1^2+\frac{1}{3}ct_1^3\right](-\alpha)^{\beta}\right]$$

Thus, the deterioration cost per unit time is

$$\mathrm{DC}=\frac{C_4}{T}\left(I_0-at_1-\frac{b}{2}t_1^2-\frac{c}{3}t_1^3\right) \qquad (9)$$

The average shortage cost during [t_1, T] is

$$SC=\frac{C_2}{T}-\int_{t_1}^{T}(a+bt+ct^2)(T-t)\mathrm{d}t$$

$$=\frac{C_2}{12T}\left[(T-t_1)^2\left\{6a+2b(T-2t_1)+c(T^2+2Tt_1+3t_1^2)\right\}\right]$$

The average inventory holding cost accumulated over the period [0, t_1] is:

$$HC=\frac{C_1}{T}-\int_{0}^{t_1}I(t)\mathrm{d}t \qquad (11)$$

The equation (11) can be approximated with $\frac{1}{2}\frac{C_1}{T}\mathrm{I}_0 t_1$ to simplify the derivation of the cost function. Similar treatment of the inventory depletion curve with linear approximation can be found in [40-42].

Thus, the average inventory holding cost is approximately:

$$HC=\frac{1}{2}\frac{C_1}{T}\mathrm{I}_0 t_1.$$

The total inventory cost per unit time is:

$$\phi(T,\ t_1)=\frac{C_4}{T}\left(I_0-at_1-\frac{b}{2}t_1^2\ -\frac{c}{3}t_1^3\right)+\frac{C_1}{2T}I_0t_1$$

$$+\frac{C_2}{12T}\left[(T-t_1)^2\left\{6a+2b(T+2t_1)+c(T^2+2Tt_1+3t_1^2)\right\}\right]$$

$$+\frac{C_3}{T} \qquad (12)$$

We assume t_1 = KT, 0 < K < 1. This assumption seems reasonable because the length of the shortage interval is part of the cycle time.

Substituting t_1 = KT in equation (12), we shall obtain:

$$\phi(T,\ K)=\left(\frac{C_4}{T}+\frac{C_1K}{2}\right)\mathrm{I}_0^K-\mathrm{C}_4\mathrm{a}K-\frac{1}{2}\mathrm{C}_4\mathrm{b}K^2\mathrm{T}$$

$$-\frac{1}{3}\mathrm{C}_4\mathrm{c}K^3\mathrm{T}^2+\frac{1}{2}\mathrm{C}_2(1-K)^2aT+$$

$$\frac{1}{6}\mathrm{C}_2(1-K)^2(1+2K)bT^2+$$

$$\frac{1}{12}\mathrm{C}_2(1-K)^2(1+2K+2K^2)cT^3+\frac{C_3}{T} \qquad (13)$$

Where,

$$\mathrm{I}_0^K=\mathrm{a}\left[KT+\frac{\gamma}{\beta+1}\left(\left(\gamma\beta KT-\alpha\right)^{\beta+1}-\left(-\alpha^{\beta+1}\right)\right)\right]+$$

$$\mathrm{b}\left[\frac{K^2T^2}{2}+\gamma\left(\frac{1}{\beta+1}\left(KT\left(\gamma\beta KT-\alpha\right)^{\beta+1}\right)\right.\right.$$

$$\left.\left.-\frac{1}{(\beta+1)(\beta+2)}\left(\left(\gamma\beta KT-\alpha\right)^{\beta+2}-\left(-\alpha^{\beta+2}\right)\right)\right)\right]+$$

$$c\left[\frac{K^3T^3}{3}+\gamma\left(\frac{1}{\beta+1}\left(K^2T^2\left(\gamma\beta KT-\alpha\right)^{\beta+1}\right)\right.\right.$$

$$-\frac{1}{\beta+1}\left(\frac{1}{\beta+2}\left(KT\left(\gamma\beta KT-\alpha\right)^{\beta+2}\right)\right)-$$

$$\left.\left.\frac{1}{(\beta+2)(\beta+3)}\left(\left(\gamma\beta KT-\alpha\right)^{\beta+3}-\left(-\alpha^{\beta+3}\right)\right)\right)\right]$$

$$-\gamma\left[aKT+\frac{1}{2}bK^2T^2+\frac{1}{3}cK^3T^3\right]\left(-\alpha^{\beta}\right) \qquad (14)$$

The optimal T and K can then be determined. The total average cost per unit time $\phi\left[T,\ K\right]$ being a function of two variable T and K, has to be partially differentiated with respect to T and K separately and then put equal to zero

The total back-order quantity for the cycle is:

$$a(T^{*}-t_1^{*})+\frac{b}{2}(T^{*2}-t_1^{*2})+\frac{c}{3}(T^{*3}-t_1^{*3}) \qquad (15)$$

The optimal order quantity cab then be expressed as:

$$I^{*}=I_0^{*}+a(T^{*}-t_1^{*})+\frac{b}{2}(T^{*2}-t_1^{*2})+\frac{c}{3}(T^{*3}-t_1^{*3}) \qquad (16)$$

The optimal inventory policy of the proposed model is:
To order I^{*} units for every T* time unit, we use the expression:

$[a(T^{*}-t_1^{*})+\frac{b}{2}(T^{*2}-t_1^{*2})+\frac{c}{3}(T^{*3}-t_1^{*3})]$ units to offset the back ordered quantity (BOQ) and begin a new cycle with I_0^{K} [42].

III. Data Analysis and Result

Taking into account the illustration provided in [42], which used an inventory issue at a restaurant to show how the model was used. Let's say the Sangote Restaurant's inventory manager wishes to create an inventory policy for the restaurant's catfish (also known as "point-and-kill" fish) unit. The quantity of catfish held, the amount consumed each day, the length of time the catfish are stored before they die, and the costs associated with maintaining the component inventory all seem to be related to the sales of "point-and-kill." It is anticipated that 450 catfish orders are placed each month and that after 24 hours of delivery at Sangote Restaurant, no deaths were reported (catfish producers typically replace dead fish on the first day of delivery). Thereafter, 25 deaths were recorded. It should be noted that Sangote Restaurant offers live catfish for customers to choose from by pointing it and thereafter it is killed and prepared.

C_1 = NGN (3000/ (450×30)) = NGN 0.22 per catfish per day,
C_2 = NGN 70 per catfish per day,
C_3 = NGN 1600 per order,
C_4 = NGN 250 per catfish,

The catfish consumption data was analyzed using the MINITAB statistical software to obtain parameters a=15.11, b=0.43 and c=0.47. R-programming software was used to estimate the parameters α, β and γ for the time-to-death data and obtain the following: α = 0.92, β = 2,03 and γ = 1.54 (See the R code for estimating parameters of Weibull distribution at the appendix). Hence, the data for the inventory problem was summarized as: C1=NGN 0.22 per catfish per day, C2 = NGN 70 per catfish per day, C3 = NGN 1600 per order, C4 = NGN 250 per catfish, a=15.11, b=0.43 and c=0.47, α = 0.92, β = 2,03 and γ = 1.54.

The result of the analysis was as follows:

Table 1. Comparison of result for the inventory problem of Sangote Restaurant

/N0.	**Parameters**	**Result of [30]**	**Result of the Proposed Model**
	Instantaneous rate function	$\alpha\beta(t-\gamma)^{\beta-1}$	$(\gamma\beta t-\alpha)^{\beta-1}$
	Optimum cycle time T^{*}	1.3200 months	0.5877 months
	Optimum value of K, K^{*}	0.8201	0.1191
	Optimum stock-period t_1^{*}	1.0825 months	0.07 months
	Optimum initial inventory quantity I_0^{*}	601.7 catfish	7.9839 catfish
	Economic order quantity I^{*}	604.2 catfish	15.9109 catfish
	Optimum total cost per unit time $\varphi(T,K)^{*}$	NGN 5,897.44 per cycle	NGN 5,923.62 per cycle

The Sensitivity Analysis was performed to evaluate how the ideal total inventory cost, stock level, and cycle time are affected by variations in the values of the demand parameters, deterioration parameters, and cost parameters. The sensitivity analysis is carried out by altering the values of each parameter by +50%, +25%, -25%, and -50% while leaving the values of the other nine parameters intact. Here, we've assumed that the imply % changes for insensitive, moderately sensitive, and very sensitive are, respectively, -3 to +3, -20 to +20, and "more" (Samanta and Bhowmick, 2010).

Table 2. Sensitivity analysis of the inventory model parameters.

Parameter	% of Change	% of Change in		
		T^*	I^*	Φ^*
a	+50	55.113153	28.25862	80.87453
	+25	38.301855	19.55138	70.92538
	-25	-12.063978	-6.1241	41.58805
	-50	-28.858261	-14.5667	31.94128
b	+50	21.473541	10.90762	61.04889
	+25	21.507572	10.92521	61.06899
	-25	21.575634	10.96041	61.1092
	-50	21.558618	10.95161	61.09916
c	+50	21.507572	10.92521	61.06899
	+25	21.507572	10.92521	61.06899
	-25	21.524587	10.93401	61.07905
	-50	21.524587	10.93401	61.07905
C1				
	+25	21.507572	10.92521	61.06899
	-25	21.507572	10.92521	61.06899
	-50	21.507572	10.92521	61.06899
C2	+50	19.057342	9.672614	59.63774
	+25	20.282457	10.2986	60.35301
	-25	25.199932	12.8163	63.22987
	-50	23.974817	12.1885	62.51245
C3	+50	48.732346	24.94453	77.08783
	+25	35.119959	17.91162	69.05186
	-25	-5.7172027	-2.91938	45.24983

TABLE 2 CONTINUED

C1	+50	21.507572	10.92521	61.06899
	+25	21.507572	10.92521	61.06899
	-25	21.507572	10.92521	61.06899
	-50	21.507572	10.92521	61.06899
C2	+50	19.057342	9.672614	59.63774
	+25	20.282457	10.2986	60.35301
	-25	25.199932	12.8163	63.22987
	-50	23.974817	12.1885	62.51245
C3	+50	48.732346	24.94453	77.08783
	+25	35.119959	17.91162	69.05186
	-25	-5.7172027	-2.91938	45.24983
	-50	-19.32959	-9.78323	37.40704
C4	+50	57.512336	29.50744	82.30153
	+25	39.509954	20.17485	71.63778
	-25	-32.482559	-16.3825	29.86658
	-50	-14.480177	-7.34214	40.19628
α	+50	21.490556	10.91642	61.05893

	+25	21.507572	10.92521	61.06899
	-25	21.524587	10.93401	61.07905
	-50	21.524587	10.93401	61.07905
β	+50	21.507572	10.92521	61.06899
	+25	21.507572	10.92521	61.06899
	-25	21.507572	10.92521	61.06899
	-50	21.507572	10.92521	61.06899
γ	+50	21.507572	10.92521	61.06899
	+25	21.507572	10.92521	61.06899
	-25	21.524587	10.93401	61.07905
	-50	21.524587	10.93401	61.07905

IV. Conclusion

In this study, a three-parameter modified Weibull distribution was used to analyze the wear duration of products in an inventory model for items having an inventory-dependent demand levels (IDDL). The model was examined on the presumption that the deficit was largely incurred and under control. In order to reduce the cost of inventory per cycle, this study concentrated on the optimal replenishment plan in terms of the number of items and the amount of time they should be held.

The findings of the inventory policy for Sangote Restaurant (Catfish unit) showed that the proposed model was to order 16 catfish every 0.5877 months and use 8 catfish to offset the backorder quantity.

The inventory cost associated with the proposed policy was found as NGN 5,923.62 per cycle. This result compares well with result obtained by [42] even though it was found that the optimal order quantity decreased by 97.4%, the optimal cycle length was found to decrease by 55.5%, and there was an approximate 0.44% increase in the total inventory cost per unit time. The significant percent decrease in the optimal order quantity was attributed to the choice of using a modified three parameter Weibull distribution for the model

Hence, the findings of the present study are distinct from those of past studies in that: (i) wear time was modelled using a modified three-parameter Weibull distribution model; (ii) a quadratic rate of return that was dependent on inventory; and (iii) PL was used. Due to the simultaneous considerations, the model was found to be more flexible in replicating a genuine situation. The methodology described in this study was shown to be suitable for a product whose inventory levels (IL) was exhibited aesthetically to draw clients and consequently boost sales. A modified three-parameter Weibull instantaneous rate was used in the model to represent the impact of both units that are currently deteriorating in inventory as well as those that could do so in the future.

References

[1]. Sanni, S. S. And Chukwu, W.I.E. (2016). An Inventory Model With Three-Parameter Weibull Deterioration, Quadratic Demand Rate And Short-Ages, American Journal Of Mathematical And Management Sciences, 35(2): 159-170.

[2]. Ouyang, L-Y., Wu, K-S., Cheng, M-C. (2005).An Inventory Model For Deteriorating Items With Exponential Declining Demand And Partial Backlogging, Yugoslav Journal Of Operations Re-Search, 15(2): 277-288.

[3]. Sanni, S. S. And Chukwu, W.I.E. (2013). An Economic Order Quantity Model For Items With Three-Parameter Weibull Distribution Deterioration, Ramp-Type Demand And Shortages, Applied Mathematical Modeling, 37: 9698-9706.

[4]. Kumar, S. And Rajput, U.S. (2014). A Two Warehouse Inventory Model For Non-Instantaneous Deteriorating Items With Time-Dependent Demand And Partial Backlogging, International Journal Of Applied Engineering Re-Search, 9(11): 1347-1356.

[5]. Tyagi, R. And Chouhan, P. (2015). An Inventory Model For Non-Instantaneous Deteriorating Items With Partial Backlogging And Stock-Dependent Demand, International Journal Of Mathematics And Computer Applications Research, 5(1): 1-6.

[6]. Begum, R., Sahoo, R. R., Sahu, S. K. And Mishra, M. (2010). An EOQ Model For Varying Items With Weibull Distribution Deterioration And Price-Dependent Demand, Journal Of Scientific Research, 2(1): 24-36.

[7]. Burwell, T.H., Dave, D.S., Fitzpatrick, K.E. And Roy, M.R., (1997), "Economic Lot Size Model For Price-Dependent Demand Under Quantity And Freight Discounts", International Journal Of Production Economics, 48(2): 141-155.

[8]. Annadurai, K. (2013). Integrated Inventory Model For Deteriorating Items With Price-Dependent Demand Under Quantity-Dependent Trade Credit, International Journal Of Manufacturing Engineering, (1): 1– 8.

[9]. Kumar, M., Chauhan, A. And Kumar, R. (2012). A Deterministic Inventory Model For Deteriorating Items With Price-Dependent Demand And Time Varying Holding Cost Under Trade Credit, International Journal Of Soft Computing And Engineering, 2(1): 99-105.

[10]. Singh, C. And Singh, S.R. (2011). "Imperfect Production Process With Exponential Demand Rate, Weibull Deterioration Under Inflation", International Journal Of Operational Research, 12(4): 430-445.

[11]. Rai, V. And Sharma, B. K. (2017). An EOQ Model For Generalized Weibull Distribution Deterioration With Selling Price Sensitive Demand And Varying Holding Cost, Applied Mathematics And Information Science Letters, 5(1): 27-32.

[12]. Harris, F.W. (1913). How Much Stocks To Keepon-Hand, Factory: The Magazine Of Management, 10(3): 240-241 And 280-284.

[13]. Ghare, P.M. And Schrader, G.F. (1963). A Model For Exponentially Decaying Inventories, Journal Of Industrial Engineering, 14: 238-243.

[14]. Rinne, H. (2009).The Weibull Distribution: A Handbook, Chapman & Hall/CRC, Florida.

[15]. Philip, G.C. (1974). A Generalized EOQ Model For Items With Weibull Distribution Deterioration, AIIE Transactions, 6: 159-162.

[16]. Chakrabarty, T., Giri, B.C. And Chaudhuri, K.S. (1998), An EOQ Model For Items Withweibull Distribution Deterioration, Shortages And Trended Demand: An Extension Of Philip's Model, Computers Operations Research, 25(8): 649-657.
[17]. Chung, K. J. (2012). A Note On The Article "Economic Order Quantity With Trade Credit Financing For Non-Decreasing Demand", Omega, Vol. 40, 328–335.
[18]. Min, J., Zhou, Y. And Zhao, J. (2010). An Inventory Model For Deteriorating Items Under Stock-Dependent Demand And Two-Level Trade Credit,Applied Mathematical Modeling, Vol. 34, 3273–3285.
[19]. Jiangtao, M., Guimei, C. Ting, F. And Hong, M. (2014). Optimal Ordering Policies For Perishable Multi-Itemunder Stock-Dependent Demand And Two-Level Trade Credit, Applied Mathematical Modelling, Vol. 38, 2522–2532.
[20]. Chang, H.-J. And Dye, C.-Y. (1999). An EOQ Model For Deteriorating Items With Time Varying Demand And Partial Backlogging, The Journal Of The Operational Research Society, Vol. 50(11): 1176-1182.
[21]. Taleizadeh, A.A. (2014). An Economic Order Quantity Model For Deteriorating Itemin A Purchasing System With Multiple Prepayments, Applied Mathematical Modelling, 38: 5357–5366.
[22]. Zhao, L. (2014). An Inventory Model Under Trapezoidal Type Demand,Weibull-Distributed Deterioration, And Partial Backlogging, Journal Of Applied Mathematics, Vol. 2014, Article ID 747419, 10 Pages, DOI: Http://Dx.Doi.Org/10.1155/2014/747419.
[23]. Kumar, M., Chauhan, A. And Kumar, R. (2012). A Deterministic Inventory Model For Deteriorating Items With Price-Dependent Demand And Time Varying Holding Cost Under Trade Credit, International Journal Of Soft Computing And Engineering, 2(1): 99-105.
[24]. Krishnaraj, R. B. And Ramasamy, K. (2013). An Inventory Model For Items With Two Parameter Weibull Distribution Deterioration And Price-Dependent Demand, International Journal Of Mathematics And Computer Applications Research, 3(2): 147-154.
[25]. Geetha, K. And Anusheela, N. (2015). An Inventory Model For Deteriorating Items With Two Parameter Weibull Deterioration And Price Dependent Demand, International Journal Of Engineering Research And General Science, 3(5): 895- 899.
[26]. Krugon, S. And Nagaraju, D. (2016). Optimality Of Cycle Time And Inventory Decisions In A Two Echelon Inventory System With Price Dependent Demand Under Credit Period, Indian Journal Of Science And Technology, Vol. 9(36), 1-9.
[27]. Dhandapani, J. And Uthayakumar, R. (2017). An EOQ Model For Two-Parameter Weibull Distribution Deteriorating Item, Advertisement And Selling Price Depended Demand, Linearly Time-Dependent Holding Costwith Salvage Value And Shortages, International Journal Of Advanced Research In Basic Engineering Sciences And Technology, 3(24): 387 – 393.
[28]. Chigbu, P.E., Ukaegbu, E.C. And Dike, O.C. (2020). An EOQ Model For Items With Two-Parameter Weibull-Distributed Deterioration, Trade Credit And Price-Dependent Demand, Preprint.
[29]. Patra, S.K., Lenka, T.K. And Ratha, P.C. (2010), An Order Level EOQ Model For Deteriorating Items In A Single Warehouse System With Price Depended Demand In Non-Linear (Quadratic)Form, International Journal Of Computational And Applied Mathematics, 5 (3): 277 – 285.
[30]. Singh, T., Mishra, P. J., And Pattanayak, H. (2017). An Optimal Policy For Deteriorating Items With Time-Proportional Deterioration Rate And Constant And Time-Dependent Linear Demand Rate. Journal Of Industrial Engineering International, Springer, Heidelberg, 13(4): 455-463. Https://Doi.Org/10.1007/S40092-017-0198-6
[31]. Chauhan, N. And Yadav, A.S. (2020). An Inventory Model For Deteriorating Items With Two-Warehouse And Dependent Demand Using Genetic Algorithm, International Journal Of Advanced Science And Technology, 29(5): 1152-1162.
[32]. Saha, S., Sen, N. And Nath, B.K. (2018). Inventory Model With Ramp-Type Demand And Price Discount On Back Order For Deteriorating Items Under Partial Backlogging, Applications And Applied Mathematics, 13(1): 472-483.
[33]. Singh, S., Sharma, S. And Singh, S.R. (2018). Two-Warehouse Inventory Model For Deteriorating Items With Time Dependent Demand And Partial Backlogging Under Inflation, International Journal Of Mathematical Modelling And Computations, 8(2): 73-88,
Yadav, D., Singh, S.R. And Sarin, M. (2020). Inventory Model Considering Deterioration, Stock-Dependent And Ramp-Type Demand With Reserve Money And Carbon Emission, International Journal Of Recent Technology And Engineering, 8(5): 5330-5337.
[34]. Garg, G., Sanjay Singh, S., And Singh, V. (2020). A Two Warehouse Inventory Model For Perishable Items With Ramp Type Demand And Partial Backlogging. International Journal Of Engineering Research & Technology (IJERT), 9(6): 1504-1520
[35]. Alshanbari, H. M., El-Bagoury, A. H., Khan, M. A., Mondal, S., Shaikh, A. A., And Rashid, A. (2021). Economic Order Quantity Model With Weibull Distributed Deterioration Under A Mixed Cash And Prepayment Scheme. Computational Intelligence And Neuroscience, 9588685: 1-16. Https://Doi.Org/10.1155/2021/9588685
[36]. Iqbal, K. I. B., Rafique, A. And Uddin, M. F. (2022). Inventory Optimization Model Of Deteriorating Items With Nonlinear Ramped Type Demand Function. Journal Of Naval Architecture And Marine Engineering, 19(2022): 13-30.
[37]. Marchi, B., Zavanella, L. E., And Zanoni, S. (2022). Supply Chain Finance For Ameliorating And Deteriorating Products: A Systematic Literature Review. Journal Of Business Economics, 21(7): 1-30. Https://Doi.Org/10.1007/S11573-022-01108-6
[38]. Rizwanullah, M., Verma, S. K. And Junnaidi, M. H. (2021). Supply-Chain Two-Warehouse Inventory Model For Deteriorating Items On Exponential Time Function With Shortage And Partial Backlogging In Inflationary Environment. Journal Of Physics: Conference Series 1913 (2021) 012130: 1-14. Doi:10.1088/1742-6596/1913/1/012130
[39]. Covert, R.P. And Philip, G.C. (1973): An EOQ Model For Items With Weibull Distribution Deterioration, AIIE Transactions, 5: 323-326.
[40]. Chakrabarty, T., Giri, B.C. And Chaudhuri, K.S. (1998), An EOQ Model For Items With Weibull Distribution Deterioration, Shortages And Trended Demand: An Extension Of Philip's Model, Computers Operations Research, 25(8): 649-657.
[41]. Ghosh, S.K. And Chaudhuri, K.S. (2004). An Order-Level Inventory Model For A Deteriorating Item With Weibull Distribution Deterioration, Time-Quadratic Demand And Short- Ages. Advanced Modeling And Optimization, 6(1): 21-35.
[42]. Sanni, S.S. (2012). An Economic Order Quan-Tity Inventory Model With Time Dependent Weibull Deterioration And Trended Demand, M.Sc. Project Report, University Of Nigeria, Nsukka.