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spgωα-Continuous Functions in Topological Spaces

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Abstract:
In this paper, we apply spgωα-open sets to present and study a new class of functions called spgωα-continuous functions and spgωα-irresolute functions in topological spaces. Some characterizations and several properties are obtained.
Keywords: *spgωα-closed sets, spgωα-continuous functions, spgωα-irresolute maps.*

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I. Introduction:

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed, a significant theme in General Topology and Real Analysis concern the variously modified forms of continuity, separation axioms etc by utilizing generalized closed sets. The notion of generalized closed sets has been extensively studied by many researchers in recent years [9], [10].

Continuous functions stand among the most fundamental point in the whole of the Mathematical Science. Different forms of stronger and weaker forms of functions have been introduced over the years. Balachandran et al [2] introduced the concept of generalized continuous maps and generalized irresolute maps in topological spaces. In the year 2017, M.M. Holliyavar et al. [8], introduced and studied new weaker form of closed set called spgωα-closed sets in topological spaces.

The purpose of this paper is to introduce and investigate the properties of weaker forms of continuous functions called spgωα-continuous functions and spgωα-irresolute in topological spaces. Some new characterizations and several fundamental properties of these functions are investigated.

II. Preliminaries:

Throughout this paper spaces (X, τ) and (Y, σ) (or simply X and Y) always denote topological spaces on which no separation axioms are assumed unless explicitly stated.

For the convenience of the reader we first review some basic concepts, most of them are very well-known from the literature.

Definition 2.1: A subset A of a topological space X is called
(i) semi-open set [9] if A ⊆ cl(int(A)) and semi-closed set if int(cl(A))⊆A.
(ii) α-open set [12] if A⊆int(cl(int(A))) and α-closed set if cl(int(cl(A)))⊆A.
(iii) regular-open set [16] if A = int(cl(A)) and regular-closed if A = cl(int(A)).
(iv) semi-preopen (=β-open) [1], if A ⊆ cl(int(cl(A))) and semi-preclosed (β-closed) if int(l(int(A))) ⊆ A.

Definition 2.2: Let X be a topological space. A subset A of X is said to be
(i) g-closed [10] (respectively αg-closed [7]) if cl(A) ⊆ U (respectively αcl(A) ⊆ U) whenever A ⊆ U and U is open in X.
(ii) ω-closed [15] if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
(iii) ωα-closed [3] (resp. gωα-closed [5], g*ωα-closed [13]) if α-cl(A) (resp. α-cl(A), cl(A)) ⊆ U whenever A ⊆ U and U is ω-open (resp. ωα-open) in X.
(iv) spgωα-closed [8] (resp. spωα-closed [14]) if spcl(A) (resp. scl(A)) ⊆ U whenever A ⊆ U and U is ωα-open in X.

Definition 2.3: A function f: X → Y is called

(i) g-continuous [2] (respectively α-continuous [11], ω-continuous [15], αg-continuous [7]) if $f^{-1}(G)$ is g-closed (respectively α-closed, ω-closed, αg-closed) set in X for every closed set G of Y.

(ii) gωα-continuous [5] if $f^{-1}(G)$ is gωα-closed set in X for every closed set G of Y.

(iii) ω-irresolute [15] (resp. ωα-irresolute [3]) if $f^{-1}(G)$ is ω-closed (resp. ωα-closed) in X for each ω-closed (resp. ωα-closed) set G of Y.

Definition 2.4 [8]: If A is spgωα-closed, then spgωα-cl(A) = A. If A is spgωα-open then spgωα-int(A) = A.

Definition 2.5 [8]: Let x ∈ X and V ⊂ X, then V is called spgωα-neighbourhood of x in X if there exists spgωα-open set U of X such that x ∈ U ⊆ V.

Theorem 2.6. [8]: Let A be a subset of X. Then x ∈ spgωαcl(A) if and only if for any spgωα-nbd N_x of x in X such that $N_x \cap A \neq \varphi$.

III. spgωα - Continuous Functions

This section deals with the concept of semi pre generalized ωα-continuous (briefly spgωα-continuous) functions and some of their basic properties in topological spaces.

Definition 3.1: A function f: X →Y is called spgωα-continuous, for every V ∈C(Y), $f^{-1}(V)$ ∈ spgωα-C(X).

Example 3.2: Let X = Y = {1, 2, 3}, τ = {X, τ, {1}, {2, 3}} and σ = {Y, φ, {1}, {2, 3}}. Then f : X → Y be the identity function. We can observe that f is spgωα-continuous.

Theorem 3.3: Every continuous function is spgωα-continuous function.

However, the converse of the above theorem need not be true as seen from the following example.

Example 3.4: Let X = Y = {1, 2, 3}, τ = {X, φ, {1}, {2}, {1, 2}} and σ = {Y, φ, {1}, {2, 3}}. Consider the identity function f : X → Y. Then f is spgωα-continuous but not a continuous function. The closed set {1} in Y, $f^{-1}(\{1\})$ = {1} is not closed in X but it is spgωα-closed in X.

Theorem 3.5: A function f: X → Y is spgωα-continuous if and only if the inverse image of every open set in Y is spgωα-open in X.

Proof: The proof is obvious.

Theorem 3.6: Following statements are equivalent for the function f : X → Y :

(i) f is spgωα-continuous.
(ii) the inverse image of each open set in Y is spgωα-open in X.
(iii) the inverse image of each closed set in Y is spgωα-closed in X.
(iv) for each x in X, the inverse image of every neighborhood of f(x) is a spgωα-neighborhood of x.
(v) for each x in X and each neighborhood N of f(x) there is a spgωα-neighborhood W of x such that f(W) ⊆ N.
(vi) f(spgωα-cl(A)) ⊆ cl(f(A)) holds for each A ⊂ X.
(vii) sgωαcl($f^{-1}(B)$) ⊆ f^{-1}(cl(B)) holds for each B ⊂ Y .

Proof: (i) → (ii): Follows from the Theorem 3.5.

(ii) → (iii): Follows from $f^{-1}(A^c) = (f^{-1}(A))^c$.

(ii) → (iv): Let x ∈ X and let N be a neighborhood of f(x). Then for each V ∈ O(Y) such that f(x) ∈ V ⊆ N and so $f^{-1}(V)$ ∈ spgωα-O(X) and x ∈ $f^{-1}(V) \subseteq f^{-1}(N)$. Thus f−1 (N) is spgωα neighborhood of f(x). (iv) → (v): Let x ∈ X and let N be a neighborhood of f(x). From assumption W = $f^{-1}(N)$ which is a spgωα neighborhood of x and f(W) = $f(f^{-1}(N))$ ⊆ N.

(v) → (vi): Let y ∈ f(spgωα-cl(A)) and let N be any neighborhood of y. So ∃ x ∈ X and spgωα neighborhood W of x such that f(x) = y, x ∈ W. Thus x ∈ spgωα-cl(A) and f(W) ⊆ N. But from Theorem 2.7, W ∩ A≠ φ, so f(A) ∩ N ≠ φ. Thus y ∈ f(x) ∈ cl(f(A)). Thus f(spgωα-cl(A)) ⊆ cl(f(A)).

(vi) → (vii): Let B ⊂Y . Then replacing A by $f^{-1}(B)$ in (vi), we get f(spgωα-cl($f^{-1}(B)$) ⊆ cl(f($f^{-1}(B)$)) ⊆ cl(B), that is spgωα-cl($f^{-1}(B)$) ⊆ f^{-1}(cl(B)).

(vii) → (i): Let G ∈O(Y), then Y \ G ∈ C(Y). Thus, $f^{-1}(Y \setminus G) = f^{-1}(cl(Y \setminus G))$ ⊆ spgωα-cl($f^{-1}(Y \setminus G)$) = X\ (spgωα-int($f^{-1}(G)$), that is spgωα-int($f^{-1}(G)$) ⊆ X \$f^{-1}(Y \setminus G)$) = $f^{-1}(G)$. Thus, spgωα-int($f^{-1}(G)$) ⊆ $f^{-1}(G)$. But, we have $f^{-1}(G)$ ⊆ spgωα-int($f^{-1}(G)$) is always true. Thus $f^{-1}(G)$ = spgωα-int($f^{-1}(G)$), that is $f^{-1}(G)$ ∈ spgωα-(X). Thus f is spgωα-continuous.

IV. spgωα-Irresolute Maps

In this section, authors introduced and studied spgωα-irresolute maps and some of their basic properties in topological spaces.

Definition 4.1: A map f : X → Y is called spgωα-irresolute, for every V ∈ spgωα-C(Y), $f^{-1}(V)$ ∈ spgωα-C(X).

Theorem 4.2: A map f : X → Y is spgωα-irresolute if and only if for every A ∈ spgωα-O(Y), $f^{-1}(A)$ ∈ spgωα-O(X).

Proof: Let G ∈ spgωα-O(Y) and so Y \G ∈ spgωα-C(Y). From hypothesis f is spgωα-irresolute, so $f^{-1}(Y \setminus G)$ ∈ spgωα-C(X) and hence $f^{-1}(G)$ ∈ spgωα-O(X).

On the other hand, let K ∈ spgωα-C(Y) and so Y \ K ∈ spgωα-O(Y). Again, from hypothesis, f^{-1}(Y \ K) ∈ spgωα-O(X) and hence f^{-1}(K) ∈ spgωα-C(X). Thus, f is spgωα-irresolute map.

Theorem 4.3: Let A ⊂ X and f: X → Y is spgωα-irresolute map. Then f(spgωα-cl(A)) ⊆ cl(f(A)).

Proof: Let A ⊂ X. Then cl(f(A)) ∈ spgωα-C(Y). Since f is spgωα-irresolute, f^{-1}(cl(f(A))) ∈ spgωα-C(X). As A ⊂ f^{-1}(f(A)) ⊆ f^{-1}(cl(f(A))). By the property of spgωα-closure, spgωα-cl(A) ⊆ f^{-1} (cl(f(A))). Consequently, f(spgωα-cl(A)) ⊆ f(f^{-1}(cl(f(A)))) ⊆ cl(f(A)).

Theorem 4.4: Every spgωα-irresolute map is spgωα-continuous.

Proof: Let f be spgωα-irresolute map and A be any closed set in Y and so V is spgωα-closed set in Y. As f is spgωα-irresolute, f^{-1}(V) ∈ spgωα-C(Y). Thus f is spgωα-continuous.

The converse of the above theorem need not be true follows from the following example.

Example 4.5: X = Y = {1, 2, 3 }, τ = {X, φ, {1}, {2}, {1, 2 }, { 1, 3 }} and σ = {Y, φ, {1}, {2}, {1, 2}}. A function f : X → Y be the identity function. Then f is spgωα-continuous but not spgωα-irresolute, since for the set A = {1} in Y, f^{-1}({1}) = {1} is not spgωα-closed in X.

Theorem 4.6: If f: X → Y is bijective, closed and ωα-irresolute, then the inverse map f^{-1}: Y → X is also spgωα-irresolute.

Proof: Let A ∈ spgωα-C(X). Let $(f^{-1})^{-1}$(A) = f(A) ⊆ U ∈spgωα-O(Y). Then A ⊆ f^{-1}(U). But f^{-1}(U) ∈ spgωα-O(X) with cl(A) ⊆ f^{-1}(U) and so f(cl(A)) ⊆ U. Again, f is closed and so f(cl(A)) is closed and hence f(cl(A)) ∈ spgωα-C(Y). Thus cl(f(cl(A)))) ⊆ U, so that cl(f(A)) ⊆ U. Thus f(A) ∈ spgωα-C(Y). Thus the inverse map f^{-1} : Y → X is spgωα-irresolute.

Theorem 4.7: Composition of two spgωα-irresolute map is again spgωα-irresolute.

Proof: Let f : X → Y and g : Y → Z be any two spgωα-irresolute maps. Let A ∈ spgωα-O(Z). Then g^{-1}(A) ∈ spgωα-O(Y) as g is spgωα-irresolute. Since f is spgωα-irresolute, then f^{-1}(g^{-1}(A)) ∈ spgωα-O(X). But, f^{-1}(g^{-1}(A)) = $(g \circ f)^{-1}$(A) ∈ spgωα-O(X). Thus g ∘ f : X → Z is spgωα-irresolute map.

Theorem 4.8: Let f : X → Y be spgωα-irresolute and g : Y → Z be spgωα-continuous maps. Then g ∘ f : X → Z be spgωα continuous.

Proof: Let V be an open set in Z. Then g^{-1}(V)∈ spgωα-O(Y) as g is spgωα-continuous. Since f is spgωα-irresolute, then f^{-1}(g^{-1}(V)) ∈ spgωα-O(X). But f^{-1}(g^{-1} (V)) = $(g \circ f)^{-1}$(V) ∈ spgωα-O(X). Thus g ∘ f is spgωα-continuous.

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