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Applying Intuitionistic Fuzzy Linear Programming In The Analysis Of Project Networks".

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Abstract
In this paper, a new fuzzy linear programming model is proposed to find fuzzy critical path and fuzzy completion time of a fuzzy project. All the activities in the project network are represented by trapezoidal Intuitionistic fuzzy numbers(TFI). A new representation of trapezoidal Intuitionistic fuzzy number is introduced to reduce the constraints in the fuzzy linear programming model. Further, an example is illustrated which shows the advantages of using the proposed representation over the existing representation of trapezoidal Intuitionistic fuzzy number and will present with great clarity the proposed technique and illustrate its application for fuzzy critical path of the project network problems.
Key words: *Trapezoidal Intuitionistic fuzzy number(TFI), Fuzzy linear Programming,Centroid, Ranking function, Fuzzy critical path .*

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I. Introduction and Literature review

The Critical Path Method is a network-based method designed for scheduling and managing complex projects in real-world applications. By identifying the critical path, decision makers can manage project time and improve resource allocation to ensure project quality. However, in some cases, projects may be too complex to manage, and activity times may not be offered in a specific manner. To deal with this vagueness, fuzzy set theory can be employed, which allows for the representation of activity times using fuzzy sets.Fuzzy sets were proposed by Zadeh in 1965 and have played an important role in solving management problems. The concept of intuitionistic fuzzy sets was introduced by Atanassov in 1986, and Deng-Feng Li defined arithmetic operations of Trapezoidal Institute Fuzzy Numbers (TIFN) using membership and non-membership values. Basic arithmetic operations of TIFNs, such as addition, subtraction, and multiplication, were defined by S. Mahapatra and T.K. Roy in 2009. L. Shen et al. (2010) defined ordering methods based on probabilities and hesitations, while Deng-Feng-Li (2010) developed the ratio ranking method of TIFN. A.Nagoorgani and K.Ponnalagu (2012) defined ordering of TIFNs using integral value by considering six tuple TIFNs. X.F. Wang (2008) defined the scoring function of a fuzzy number intuitionistic fuzzy value.Fuzzy set theory has been widely used to capture vagueness in decision-making problems. Lei Yang et al. (2008) proposed a normalization technique for ascertaining non-membership functions of IFS. The fuzzy shortest path problem is an extension of fuzzy numbers and has many real-life applications in the field of communication, robotics, scheduling, and transportation. Lisy Cherian and Sunny Kuriakose (2009) proposed an IFLPP with a non-membership function. R. Sophia et al. (2014) modified the approach on the shortest path in an intuitionistic fuzzy environment. Wang CY, Chen SM (2017) proposed a multiple attribute decision-making based on interval-valued intuitionistic fuzzy sets, linear programming methodology, and the extended TOPSIS method. Nasseri SH, Goli M, Bavandi S. (2018) discussed an approach for solving linear programming problems with intuitionistic fuzzy objective coefficients. Afzali A, Rafsanjani MK, Saeid AB (2016) proposed a fuzzy multi-objective linear programming model based on interval-valued intuitionistic fuzzy sets for supplier selection. Wan, S.P. et al. (2015) proposed an intuitionistic fuzzy linear programming method for logistics outsourcing provider selection.In section 2 of the document, basic definitions such as arithmetic operations of intuitionistic fuzzy sets and intuitionistic numbers are discussed. The representation of Trapezoidal Intuitionistic Fuzzy Numbers and the new Representation of Trapezoidal Intuitionistic Fuzzy Numbers are also discussed. In section 3, crisp linear programming formulation of crisp critical path and fuzzy critical path problems are presented. Section 4 describes a method to find the fuzzy critical path and fuzzy completion time of a fuzzy project network using Trapezoidal Intuitionistic Fuzzy Numbers. Section 5 and Section 6 present numerical illustrations for regular and new representations of Trapezoidal Intuitionistic Fuzzy Numbers, respectively. Finally, a comparative study of existing models and the proposed model is presented in Section 7.

II. Preliminaries

In this section some basic definitions related to Intuitionistic Fuzzy set and Intuitionistic numbers are revived

Basic Definitions

This section deals various basic definitions(Nehi, H.M 2010) are discussed

Definition 1 : Fuzzy set

Let X be the universal set and the membership function, non membership functions defined on X by $\mu_{\tilde{A}} : X \to [0,1]$, $\nu_{\tilde{A}} : X \to [0,1]$ .Degree of membership function and non membership functions are $\mu_{\tilde{A}}(x)$, $\nu_{\tilde{A}}(x)$ which always satisfies the condition $\mu_{\tilde{A}}(x)+\nu_{\tilde{A}}(x)\leq 1 \ \ \forall x \in X$ then the set $\tilde{A}=\{x,\mu_{\tilde{A}}(x),\nu_{\tilde{A}}(x)/x\in X\}$ is an IFS.

Definition 2 : $\alpha-$cut

An Intuitionistic fuzzy set (IFS) $\tilde{A}$ and $\alpha \in [0,1]$, then the set

$\tilde{A}_{\alpha} = \{x\in X/\mu_{\tilde{A}}(x)\geq\alpha,\nu_{\tilde{A}}(x)\leq(1-\alpha)/x\in X\}$ is said to be $\alpha-$cut of $\tilde{A}$.

Definition 3 : Intuitionistic fuzzy normal

An Intuitionist fuzzy set (IFS) $\tilde{A}_{\alpha}$ is Intuitionist fuzzy normal if there

Exist points $x_0, x_1 \in X$ such that $\mu_{\tilde{A}}(x_0)=1, \nu_{\tilde{A}}(x_1)=1$.

Definition 4 : Intuitionistic fuzzy convex

An Intuitionist fuzzy set (IFS) $\tilde{A}=\{(x,\mu_{\tilde{A}},\nu_{\tilde{A}}, x \in X)\}$ is Intuitionist fuzzy convex if there exist points $x_0, x_1 \in X$, $\lambda \in [0,1]$ such that

$$\mu_{\tilde{A}}(\lambda x_0+(1-\lambda)x_1)\geq \mu_{\tilde{A}}(x_0)\wedge \mu_{\tilde{A}}(x_1)$$

$$\nu_{\tilde{A}}(\lambda x_0+(1-\lambda)x_1)\geq \nu_{\tilde{A}}(x_0)\wedge \nu_{\tilde{A}}(x_1)$$

Definition 5 : Intuitionist fuzzy number

An Intuitionist fuzzy set (IFS) $\tilde{A}=\{(x,\mu_{\tilde{A}}(x),\nu_{\tilde{A}}(x))/x\in X\}$ is Intuitionist fuzzy number if

(i) $\tilde{A}$ is Intuitionistic fuzzy normal

(ii) $\tilde{A}$ is Intuitionistic fuzzy convex

(iii) $\mu_{\tilde{A}}, \nu_{\tilde{A}}(x)$ are upper and lower semi continuous, the set $\tilde{A}=\{x\in X/\nu_{\tilde{A}}(x)<1\}$ is bounded

Representation Trapezoidal of Intuitionistic fuzzy number

In this section different novel representations of Trapezoidal Intuitionistic fuzzy numbers are presented.

Existing Representation Trapezoidal of Intuitionistic fuzzy number(TIF)

Trapezoidal intuitionistic fuzzy number represented $\tilde{A}=(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$

Definition 6: Generalized Trapezoidal(TIF) Intuitionist fuzzy number

A trapezoidal intuitionistic fuzzy number with parameters $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ is defined as the membership and non membership functions as

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & x<a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1\leq x<a_2 \\ 1 & a_2\leq x\leq a_3 \\ \dfrac{x-a_4}{a_3-a_4} & a_3\leq x<a_4 \\ 0 & a_4<x \end{cases} \qquad \nu_{\tilde{A}}(x)=\begin{cases} 0 & x<b_1 \\ \dfrac{x-b_1}{b_2-b_1} & b_1\leq x<b_2 \\ 1 & b_2\leq x<b_3 \\ \dfrac{x-b_4}{b_3-b_4} & b_3\leq x<b_4 \\ 0 & b_4<x \end{cases}$$

In the above definition if $b_1 \leq a_1 \leq (b_2=a_2=a_3=b_3) \leq a_4 \leq b_4$ then the trapezoidal(TIF) become Triangular(TrIF) Intuitionist fuzzy number is denoted by $\tilde{A}=(b_1,a_1,b_2,a_4,b_4)$

Definition 7: New representation of Trapezoidal Intuitionist fuzzy number (TIF)

New representation of trapezoidal intuitionistic fuzzy number $(x, y, \alpha, \beta, x', y', \alpha', \beta')$ where $x = b_1, y = b_4, \alpha = a_1 - b_1, \beta = b_2 - a_1, x' = a_2 - b_2, y' = a_3 - a_2, \alpha' = a_4 - a_3, \beta' = b_4 - a_4$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & t < y \\ \dfrac{t-y}{\beta - y} & y \le t < \beta \\ 1 & \beta \le t \le x' \\ \dfrac{t-\alpha'}{x'-\alpha'} & x' \le t \le \alpha' \\ 0 & \alpha' < t \end{cases} \qquad \nu_{\tilde{A}}(x) = \begin{cases} 0 & t < x \\ \dfrac{t-x}{\alpha - x} & x \le t < \alpha \\ 1 & \alpha \le t < y' \\ \dfrac{t-\beta'}{y'-\beta'} & y' \le t < \beta' \\ 0 & \beta' < t \end{cases}$$

Arithmetic operations between TIF numbers

Let R be the universal set of real numbers the arithmetic operation between TIF numbers are presented.

TIF numbers addition

Two TIF $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4 : w_1, w_2)$ and

$\tilde{B} = (b_1', a_1', b_2', a_2', a_3', b_3', a_4', b_4' : w_1', w_2')$ defined on universal set R then

$$\tilde{A} \oplus \tilde{B} = \begin{pmatrix} b_1 + b_1', a_1 + a_1', b_2 + b_2', a_2 + a_2', a_3 + a_3', b_3 + b_3', a_4 + a_4', b_4 + b_4'; \max\{w_1, w_2, w_1', w_2'\} \\ 0 < w_1, w_2, w_1', w_2' \le 1 \end{pmatrix}$$

TIF numbers multiplication

Two TIF $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4 : w_1, w_2)$ and

$\tilde{B} = (b_1', a_1', b_2', a_2', a_3', b_3', a_4', b_4' : w_1', w_2')$ defined on universal set R then

$$\tilde{A} \otimes \tilde{B} = (b_1 b_1', a_1 a_1', b_2 b_2', a_2 a_2', a_3 a_3', b_3 b_3', a_4 a_4', b_4 b_4'; 0 < w_1 w_2 w_1' w_2' \le 1)$$

TIF numbers multiplied by constant λ

Two TIF $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4 : w_1, w_2)$ and defined on universal set R then

$$\lambda \tilde{A} = (\lambda b_{11}, \lambda a_1, \lambda b_2, \lambda a_2, \lambda a_3, \lambda b_3, \lambda a_4, \lambda b_4, 0 < w_1, w_2 \le 1, \lambda > 0)$$

The Ranking Functions for Trapezoidal intuitionistic fuzzy numbers

The Ranking Function for trapezoidal intuitionistic fuzzy number $\tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ using membership function and non -membership is

$$R(\tilde{A}) = \left(\frac{a_1 + b_1 + 2(a_2 + b_3) + 5(a_3 + b_2) + (a_4 + b_4)}{18} \right) \left(\frac{4w_1 + 5w_2}{18} \right)$$

The Ranking Function for trapezoidal intuitionistic fuzzy number $(x, y, \alpha, \beta, x', y', \alpha', \beta')$ where $x = b_1, y = b_4, \alpha = a_1 - b_1, \beta = b_2 - a_1, x' = a_2 - b_2, y' = a_3 - a_2, \alpha' = a_4 - a_3, \beta' = b_4 - a_4$ representation of Trapezoidal intuitionistic fuzzy number using membership function and non -membership is

$$R(\tilde{A}) = \frac{x + y + \alpha + \beta}{4} + \frac{(\beta' + \alpha') - (x' + y')}{8}$$

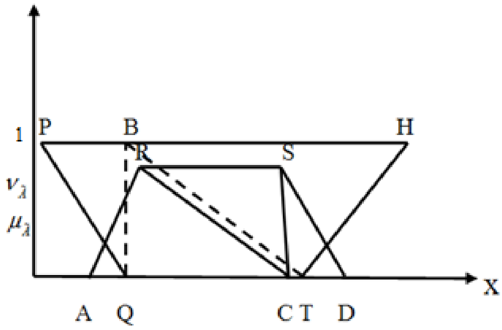

Figure 1:Trapezoidal Intuitionistic fuzzy number

III. Linear Programming Formulation of Crisp Critical Path and Fuzzy Critical Path Problems

The Crisp Critical Path Method (CPM) is a widely used technique in project management to determine the critical path of a project, which is the sequence of activities that must be completed on time to ensure that the project is completed within the specified time frame. The CPM method involves constructing a network diagram of the project activities, identifying their durations, and determining the critical path using forward and backward pass calculations.The critical path of a project is the longest path through the network and determines the minimum amount of time required to complete the project. The Linear Programming (LP) formulation of the Crisp CPM problem involves defining a set of decision variables and constraints that reflect the network structure and the project requirements.Let the project network be represented by a directed acyclic graph (DAG) G = (V, E), where V is the set of vertices or nodes representing the activities, and E is the set of edges representing the precedence relationships between the activities. Let Ti be the duration of activity i, and let Xi be a binary decision variable that takes the value 1 if activity i is included in the critical path, and 0 otherwise.The LP formulation of the Crisp CPM problem can be expressed as follows:
Minimize

$$z = \sum T_i X_i$$

Subject to:
Xi + Xj ≥ 1, for all (i, j) ∈ E Xi ∈ {0, 1}, for all i ∈ V

The objective function minimizes the total duration of the project, which is the sum of the durations of the activities included in the critical path. The first set of constraints ensures that each activity is included in the critical path if and only if all of its predecessors are also included. The second set of constraints enforces the binary nature of the decision variables.

Linear Formulation of Fuzzy Critical Path Problems:

The Fuzzy CPM method extends the Crisp CPM method by allowing for imprecise activity durations and precedence relationships. In the Fuzzy CPM approach, each activity is described by a fuzzy number that represents its duration, and each precedence relationship is described by a fuzzy logic relationship that reflects the degree of certainty or uncertainty about the relationship.

The LP formulation of the Fuzzy CPM problem is similar to that of the Crisp CPM problem, but with the addition of fuzzy variables that reflect the uncertainty in the activity durations and precedence relationships. Let Di be a fuzzy number that represents the duration of activity i, and let Rij be a fuzzy logic relationship that reflects the degree of certainty or uncertainty about the precedence relationship between activities i and j.

The LP formulation of the Fuzzy CPM problem can be expressed as follows:

Minimize Z = ∑DiXi

Subject to:

Xi + Xj ≥ Rij, for all (i, j) ∈ E Xi ∈ {0, 1}, for all i ∈ V

The objective function minimizes the total duration of the project, which is the sum of the durations of the activities included in the critical path. The first set of constraints ensures that each activity is included in the critical path if and only if the degree of certainty or uncertainty about its predecessors is satisfied. The second set of constraints enforces the binary nature of the decision variables.

The fuzzy LP formulation of the CPM problem can be solved using standard LP techniques, such as the simplex method. The resulting solution provides a fuzzy critical path that reflects the degree of uncertainty about the project schedule.

Linear Programming of Crisp Critical Path Problems

The linear programming model discussed in the book written by Taha(2003) is reviewed in this section.

Consider a project network G=(V,E,T) consisting of a finite set V={1,2,…,n} of n nodes and E is the set of activities (i,j) , T is a function from E to **R** and $t_{ij} \in R$ is the time period of activity (i,j). The Linear Programming formulation of Crisp Critical Path Problem is defined as follows:

Maximize $\sum_{(i,j)\in E} t_{ij} x_{ij}$

subject to the constraints

$$\sum_{(i,j)\in E} x_{1j} = 1,$$

$$\sum_{(i,j)\in E} x_{ij} = \sum_{(j,k)\in E} x_{jk}, \; i \neq 1, k \neq n,$$

$$\sum_{(i,n)\in E} x_{in} = 1, \; x_{ij} \text{ is a non-negative real number } \forall (i,j) \in E.$$

Linear Programming Formulation of Fuzzy Critical Path Problems

Suppose t_{ij} and x_{ij} , $\forall (i,j) \in E$ are vague and are represented by fuzzy numbers $\tilde{t}_{ij}$ and $\tilde{x}_{ij}$, $\forall (i,j) \in E$ respectively. Then the Fuzzy Critical Path Problems may be formulated into the following fuzzy linear programming problem:

Maximize $\sum_{(i,j)\in E} \tilde{t}_{ij} \otimes \tilde{x}_{ij}$

subject to

$$\sum_{(i,j)\in E} \tilde{x}_{1j} = \tilde{1},$$

$$\sum_{(i,j)\in E} \tilde{x}_{ij} = \sum_{(j,k)\in E} \tilde{x}_{jk}, \; i \neq 1, k \neq n,$$

$$\sum_{(i,n)\in A} \tilde{x}_{in} = \tilde{1}, \; (\tilde{1} = (1,1,1,1)),$$

$\tilde{x}_{ij}$ is a non-negative real number $\forall (i,j) \in E$.

IV. Method to find fuzzy critical path and fuzzy completion time of a fuzzy project

The steps of the approach are as follows(Kumar , A and Kaur P (2010))

Step 1: Represent all the parameters of linear programming formulation of Fuzzy Critical Path problems by a particular type of trapezoidal intuitionistic fuzzy number and formulate the given problem, as proposed in the section 3.2.

Step 2: Convert the fuzzy objective function and fuzzy constraints into the crisp objective function form by its ranking function.

Step 3: Find the critical path and completion fuzzy time for the obtained crisp linear Programming problem by using Tora software.

Step 4: Convert crisp solution to fuzzy solution using solution obtained from Step 3.

Step 5: Find the critical path and corresponding maximum total completion time in fuzzy sense from Step4.

Proposed method with $(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$ representation of Trapezoidal intuitionistic fuzzy number

If all the parameters of linear programming formulation of Fuzzy Critical Path problems are represented by $(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$ type Trapezoidal intuitionistic fuzzy number then the steps of the proposed method are as follows:

Step1: Suppose all the parameters $\tilde{t}_{ij}$ and $\tilde{x}_{ij}$ are represented by $(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$ type Trapezoidal intuitionistic fuzzy number and $(x_{ij},y_{ij},z_{ij},\gamma_{ij},x'_{ij},y'_{ij},z'_{ij},\gamma'_{ij})$ respectively then the Linear Programming formulation of Fuzzy Critical Path problems, proposed in the section 3.2., this can written as:

Maximize $\sum_{(i,j)\in E}(t_{ij},t'_{ij},t''_{ij},t'''_{ij},t^{iv}_{ij},t^{v}_{ij},t^{vi}_{ij},t^{vii}_{ij})\otimes(x_{ij},y_{ij},z_{ij},\gamma_{ij},x'_{ij},y'_{ij},z'_{ij},\gamma'_{ij})$

subject to

$$\sum_{(i,j)\in E}(x_{1j},y_{1j},z_{1j},\gamma_{1j},x'_{1j},y'_{1j},z'_{1j},\gamma'_{1j})=(1,1,1,1,1,1,1)$$

$$\sum_{(i,j)\in E}(x_{ij},y_{ij},z_{ij},\gamma_{ij},x'_{ij},y'_{ij},z'_{ij},\gamma'_{ij})=\sum_{(j,k)\in E}(x_{jk},y_{jk},z_{jk},\gamma_{jk},x'_{jk},y'_{jk},z'_{jk},\gamma'_{jk}),\ i\neq 1,k\neq n,$$

$$\sum_{(i,n)\in E}(x_{in},y_{in},z_{in},\gamma_{in},x'_{in},y'_{in},z'_{in},\gamma'_{in})=(1,1,1,1,1,1,1,1),$$

$(x_{in},y_{in},z_{in},\gamma_{in},x'_{in},y'_{in},z'_{in},\gamma'_{in})$ is a non-negative trapezoidal fuzzy number $\forall(i,j)\in E$.

Step2: The fuzzy linear programming of Fuzzy Critical Path problems may be written as:

Maximize $\Re\left[\sum_{(i,j)\in E}(t_{ij},t'_{ij},t''_{ij},t'''_{ij},t^{iv}_{ij},t^{v}_{ij},t^{vi}_{ij},t^{vii}_{ij})\otimes(x_{ij},y_{ij},z_{ij},\gamma_{ij},x'_{ij},y'_{ij},z'_{ij},\gamma'_{ij})\right]$

subject to the constraints

$$\sum_{(i,j)\in E}(x_{1j},y_{1j},z_{1j},\gamma_{1j},x'_{1j},y'_{1j},z'_{1j},\gamma'_{1j})=(1,1,1,1,1,1,1)$$

$$\sum_{(i,j)\in E}(x_{ij},y_{ij},z_{ij},\gamma_{ij},x'_{ij},y'_{ij},z'_{ij},\gamma'_{ij})=\sum_{(j,k)\in E}(x_{jk},y_{jk},z_{jk},\gamma_{jk},x'_{jk},y'_{jk},z'_{jk},\gamma'_{jk}),\ i\neq 1,k\neq n,$$

$$\sum_{(i,n)\in E}(x_{in},y_{in},z_{in},\gamma_{in},x'_{in},y'_{in},z'_{in},\gamma'_{in})=(1,1,1,1,1,1,1,1),$$

$(x_{in},y_{in},z_{in},\gamma_{in},x'_{in},y'_{in},z'_{in},\gamma'_{in})$ is a non-negative trapezoidal fuzzy number $\forall(i,j)\in E$.

Now, the linear programming problem becomes :

Maximize $\Re\left[\sum_{(i,j)\in E}(t_{ij},t'_{ij},t''_{ij},t'''_{ij},t^{iv}_{ij},t^{v}_{ij},t^{vi}_{ij},t^{vii}_{ij})\otimes(x_{ij},y_{ij},z_{ij},\gamma_{ij},x'_{ij},y'_{ij},z'_{ij},\gamma'_{ij})\right]$

subject to the constraints

$$\sum_{(i,j)\in E}x_{1j}=1,\ \sum_{j:(i,j)\in E}y_{1j}=1,\ \sum_{j:(i,j)\in E}z_{1j}=1,\ \sum_{j:(i,j)\in E}\gamma_{1j}=1,\ \sum_{(i,j)\in E}x'_{1j}=1,\ \sum_{j:(i,j)\in E}y'_{1j}=1,\ \sum_{j:(i,j)\in E}z'_{1j}=1,$$

$$\sum_{j:(i,j)\in E}\gamma'_{1j}=1,\quad \sum_{i:(i,j)\in E}x_{ij}=\sum_{j:(j,k)\in E}x_{jk},\ i\neq 1,k\neq n,\quad \sum_{i:(i,j)\in E}y_{ij}=\sum_{j:(j,k)\in E}y_{jk},\ i\neq 1,k\neq n,$$

$$\sum_{i:(i,j)\in E}z_{ij}=\sum_{j:(j,k)\in E}z_{jk},\ i\neq 1,k\neq n,\ \sum_{i:(i,j)\in E}\gamma_{ij}=\sum_{j:(j,k)\in E}\gamma_{jk},\ i\neq 1,k\neq n,$$

$$\sum_{i:(i,j)\in E}x'_{ij}=\sum_{j:(j,k)\in E}x'_{jk},\ i\neq 1,k\neq n,\ \sum_{i:(i,j)\in E}y'_{ij}=\sum_{j:(j,k)\in E}y'_{jk},\ i\neq 1,k\neq n,$$

$$\sum_{i:(i,j)\in E}z'_{ij}=\sum_{j:(j,k)\in E}z'_{jk},\ i\neq 1,k\neq n,\ \sum_{i:(i,j)\in E}\gamma'_{ij}=\sum_{j:(j,k)\in E}\gamma'_{jk},\ i\neq 1,k\neq n,$$

$$\sum_{i:(i,n)\in E}x_{in}=1,\ \sum_{i:(i,n)\in E}y_{in}=1,\ \sum_{i:(i,n)\in E}z_{in}=1,\ \sum_{i:(i,n)\in E}\gamma_{in}=1,$$

$\sum_{i:(i,n)\in E} x'_{in} = 1$, $\sum_{i:(i,n)\in E} y'_{in} = 1$, $\sum_{i:(i,n)\in E} z'_{in} = 1$, $\sum_{i:(i,n)\in E} \gamma'_{in} = 1$

$y_{ij} - x_{ij} \geq 0$, $z_{ij} - y_{ij} \geq 0$, $\gamma_{ij} - z_{ij} \geq 0$, $x'_{ij} - \gamma_{ij} \geq 0$, $y'_{ij} - x'_{ij} \geq 0$, $z'_{ij} - y'_{ij} \geq 0$, $\gamma'_{ij} - z'_{ij} \geq 0$

x_{ij}, y_{ij}, z_{ij}, γ_{ij}, x'_{ij}, y'_{ij}, z'_{ij}, γ'_{ij} $\forall (i, j) \in E$.

Step 3: Find the solution x_{ij}, y_{ij}, z_{ij}, γ_{ij}, x'_{ij}, y'_{ij}, z'_{ij}, γ'_{ij} γ_{ij} by solving the Crisp Linear Programming problem, which is obtained in Step2.

Step 4: Find the fuzzy solution $\tilde{x}_{ij}$ by putting the values of x_{ij}, y_{ij}, z_{ij}, γ_{ij}, x'_{ij}, y'_{ij}, z'_{ij}, γ'_{ij} in $\tilde{x}_{ij}$ = $(x_{ij}, y_{ij}, z_{ij}, \gamma_{ij}, x'_{ij}, y'_{ij}, z'_{ij}, \gamma'_{ij})$ and also calculate the maximum total completion fuzzy time by putting the values of $\tilde{x}_{ij}$ in $\sum_{(i,j)\in E} \tilde{t}_{ij} \otimes \tilde{x}_{ij}$.

Step 5: Find the fuzzy critical path by combining all the activities (i, j) such that $\tilde{x}_{ij} = (1,1,1,1,1,1,1,1)$.

Another Proposed method with $(x, y, \alpha, \beta, x', y', \alpha', \beta')$ representation of Trapezoidal intuitionistic fuzzy number

If all the parameters of linear programming formulation of Fuzzy Critical Path problems are represented by $(b_1, b_4, a_1 - b_1, b_2 - a_1, a_2 - b_2, a_3 - a_2, a_4 - a_3, b_4 - a_4)$ type Trapezoidal intuitionistic fuzzy number where $(x, y, \alpha, \beta, x', y', \alpha', \beta')$ where $x = b_1, y = b_4, \alpha = a_1 - b_1, \beta = b_2 - a_1, x' = a_2 - b_2, y' = a_3 - a_2, \alpha' = a_4 - a_3, \beta' = b_4 - a_4)$ then the steps of the proposed method are as follows:

Step1: Suppose all the parameters $\tilde{t}_{ij}$ and $\tilde{x}_{ij}$ are represented by $(t_{ij}, t'_{ij}, t''_{ij}, t'''_{ij}, t^{iv}_{ij}, t^{v}_{ij}, t^{vi}_{ij}, t^{vii}_{ij})$ type Trapezoidal intuitionistic fuzzy number k and $(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}, x'_{ij}, y'_{ij}, \alpha'_{ij}, \beta'_{ij})$ respectively then the Linear Programming formulation of Fuzzy Critical Path problems, proposed in the section 3.2., this can written as:

Maximize $\sum_{(i,j)\in E} (t_{ij}, t'_{ij}, t''_{ij}, t'''_{ij}, t^{iv}_{ij}, t^{v}_{ij}, t^{vi}_{ij}, t^{vii}_{ij}) \otimes (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}, x'_{ij}, y'_{ij}, \alpha'_{ij}, \beta'_{ij})$

subject to

$$\sum_{(1,j)\in E} (x_{1j}, y_{1j}, \alpha_{1j}, \beta_{1j}, x'_{1j}, y'_{1j}, \alpha'_{1j}, \beta'_{1j}) = (1,1,0,0,0,0,0)$$

$$\sum_{(i,j)\in E} (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}, x'_{ij}, y'_{ij}, \alpha'_{ij}, \beta'_{ij}) = \sum_{(i,j)\in E} (x_{jk}, y_{jk}, \alpha_{jk}, \beta_{jk}, x'_{jk}, y'_{jk}, \alpha'_{jk}, \beta'_{jk}),\ i \neq 1, k \neq n,$$

$$\sum_{(i,n)\in E} (x_{in}, y_{in}, \alpha_{in}, \beta_{in}, x'_{in}, y'_{in}, \alpha'_{in}, \beta'_{in}) = (1,1,0,0,0,0,0,0),$$

$(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}, x'_{ij}, y'_{ij}, \alpha'_{ij}, \beta'_{ij})$ is a non-negative trapezoidal fuzzy number $\forall (i, j) \in E$.

Step2: The fuzzy linear programming of Fuzzy Critical Path problems may be written as:

Maximize $\Re \sum_{(i,j)\in E} (t_{ij}, t'_{ij}, t''_{ij}, t'''_{ij}, t^{iv}_{ij}, t^{v}_{ij}, t^{vi}_{ij}, t^{vii}_{ij}) \otimes (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}, x'_{ij}, y'_{ij}, \alpha'_{ij}, \beta'_{ij})$

subject to the constraints

$$\sum_{(1,j)\in E} (x_{1j}, y_{1j}, \alpha_{1j}, \beta_{1j}, x'_{1j}, y'_{1j}, \alpha'_{1j}, \beta'_{1j}) = (1,1,0,0,0,0,0)$$

$$\sum_{(i,j)\in E} (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}, x'_{ij}, y'_{ij}, \alpha'_{ij}, \beta'_{ij}) = \sum_{(i,j)\in E} (x_{jk}, y_{jk}, \alpha_{jk}, \beta_{jk}, x'_{jk}, y'_{jk}, \alpha'_{jk}, \beta'_{jk}),\ i \neq 1, k \neq n,$$

$$\sum_{(i,n)\in E} (x_{in}, y_{in}, \alpha_{in}, \beta_{in}, x'_{in}, y'_{in}, \alpha'_{in}, \beta'_{in}) = (1,1,0,0,0,0,0,0),$$

$(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}, x'_{ij}, y'_{ij}, \alpha'_{ij}, \beta'_{ij})$ is a non-negative trapezoidal fuzzy number $\forall (i,j) \in E$.

Now, the crisp linear programming problem becomes :

Maximize $\Re \sum_{(i,j)\in E} (t_{ij}, t'_{ij}, t''_{ij}, t'''_{ij}, t^{iv}_{ij}, t^{v}_{ij}, t^{vi}_{ij}, t^{vii}_{ij}) \otimes (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}, x'_{ij}, y'_{ij}, \alpha'_{ij}, \beta'_{ij})$

subject to the constraints

$$\sum_{(i,j)\in E} x_{1j} = 1 \;,\; \sum_{j:(i,j)\in E} y_{1j} = 1 \;,\; \sum_{j:(i,j)\in E} \alpha_{1j} = 0 \;,\; \sum_{j:(i,j)\in E} \beta_{1j} = 0 \;,\; \sum_{(i,j)\in E} x'_{1j} = 0 \;,\; \sum_{j:(i,j)\in E} y'_{1j} = 0 \;,$$

$$\sum_{j:(i,j)\in E} \alpha'_{1j} = 0 \;,\; \sum_{j:(i,j)\in E} \beta'_{1j} = 0 \;,\; \sum_{i:(i,j)\in E} x_{ij} = \sum_{j:(j,k)\in E} x_{jk},\ i \neq 1, k \neq n,$$

$$\sum_{i:(i,j)\in E} y_{ij} = \sum_{j:(j,k)\in E} y_{jk},\ i \neq 1, k \neq n,$$

$$\sum_{i:(i,j)\in E} \alpha_{ij} = \sum_{j:(j,k)\in E} \alpha_{jk},\ i \neq 1, k \neq n,\ \sum_{i:(i,j)\in E} \beta_{ij} = \sum_{j:(j,k)\in E} \beta_{jk},\ i \neq 1, k \neq n,$$

$$\sum_{i:(i,j)\in E} x'_{ij} = \sum_{j:(j,k)\in E} x'_{jk},\ i \neq 1, k \neq n,\ \sum_{i:(i,j)\in E} y'_{ij} = \sum_{j:(j,k)\in E} y'_{jk},\ i \neq 1, k \neq n,$$

$$\sum_{i:(i,j)\in E} \alpha'_{ij} = \sum_{j:(j,k)\in E} \alpha'_{jk},\ i \neq 1, k \neq n,\ \sum_{i:(i,j)\in E} \beta'_{ij} = \sum_{j:(j,k)\in E} \beta'_{jk},\ i \neq 1, k \neq n,$$

$$\sum_{i:(i,n)\in E} x_{in} = 1,\ \sum_{i:(i,n)\in E} y_{in} = 1,\ \sum_{i:(i,n)\in E} \alpha_{in} = 0,\ \sum_{i:(i,n)\in E} \beta_{in} = 0,$$

$$\sum_{i:(i,n)\in E} x'_{in} = 0,\ \sum_{i:(i,n)\in E} y'_{in} = 0,\ \sum_{i:(i,n)\in E} \alpha'_{in} = 0,\ \sum_{i:(i,n)\in E} \beta'_{in} = 0$$

$$y_{ij} - x_{ij} \geq 0,\ \alpha_{ij} - y_{ij} \geq 0,\ \beta_{ij} - \alpha_{ij} \geq 0,\ x'_{ij} - \beta_{ij} \geq 0,\ y'_{ij} - x'_{ij} \geq 0,\ \alpha'_{ij} - y'_{ij} \geq 0,\ \beta'_{ij} - \alpha'_{ij} \geq 0$$

$$x_{ij},\ y_{ij}, \alpha_{ij}, \beta_{ij},\ x'_{ij},\ y'_{ij}, \alpha'_{ij}, \beta'_{ij}\ \forall (i,j) \in E.$$

Step 3: Find the solution $x_{ij},\ y_{ij}, \alpha_{ij}, \beta_{ij},\ x'_{ij},\ y'_{ij}, \alpha'_{ij}, \beta'_{ij}$ by solving the Crisp Linear Programming problem, which is obtained in Step2.

Step 4: Find the fuzzy solution $\tilde{x}_{ij}$ by putting the values of x_{ij}, y_{ij}, α_{ij}, β_{ij}, x'_{ij}, y'_{ij}, α'_{ij}, β'_{ij} in $\tilde{x}_{ij} = (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}, x'_{ij}, y'_{ij}, \alpha'_{ij}, \beta'_{ij})$ and also calculate the maximum total completion fuzzy time by putting the values of $\tilde{x}_{ij}$ in $\sum_{(i,j)\in E} \tilde{t}_{ij} \otimes \tilde{x}_{ij}$.

Step 5: Find the fuzzy critical path by combining all the activities (i,j) such that $\tilde{x}_{ij} = (1,1,0,0,0,0,0,0)$.

V. Numarical Example for Trapezoidal intuitionistic fuzzy number

The problem is to find the fuzzy critical path and maximum total completion fuzzy time of the project network, shown in Fig.2, in which the fuzzy time duration of each activity is represented by the following (a,b,c,d) type trapezoidal intuitionistic fuzzy numbers.

$\tilde{t}_{12} = (1,2,2,3,3,4,4,4)$, $\tilde{t}_{13} = (1,2,3,3,3,4,6,8)$, $\tilde{t}_{15} = (1,2,3,3,3,5,5,7)$, $\tilde{t}_{24} = (1,2,2,2,4,5,5,7)$
$\tilde{t}_{25} = (1,1,1,2,2,3,5,7)$, $\tilde{t}_{34} = (1,2,2,3,3,3,5,5)$ $\tilde{t}_{36} = (1,2,3,3,4,4,5,7)$, $\tilde{t}_{45} = (1,1,2,3,4,5,7,8)$,
$\tilde{t}_{46} = (1,2,2,3,3,5,6,8)$ and $\tilde{t}_{56} = (1,2,3,3,4,5,6,9)$

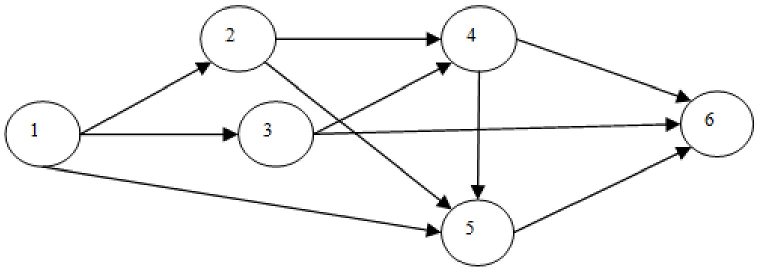

Figure 2:Project Network

Fuzzy optimal solution using $(b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ representation of Trapezoidal intuitionistic fuzzy number

Step-1

Maximize

$$\left\{\begin{array}{l}(1,2,2,3,3,4,4,4)\otimes(x_{12},y_{12},z_{12},\gamma_{12},x'_{12},y'_{12},z'_{12},\gamma'_{12})\oplus(1,2,3,3,3,4,6,8)\otimes(x_{13},y_{13},z_{13},\gamma_{13},x'_{13},y'_{13},z'_{13},\gamma'_{13})\oplus\\(1,2,3,3,3,5,5,7)\otimes(x_{15},y_{15},z_{15},\gamma_{15},x'_{15},y'_{15},z'_{15},\gamma'_{15})\oplus(1,2,2,2,4,5,5,7)\otimes(x_{24},y_{24},z_{24},\gamma_{24},x'_{24},y'_{24},z'_{24},\gamma'_{24})\oplus\\(1,1,1,2,2,3,5,7)\otimes(x_{25},y_{25},z_{25},\gamma_{25},x'_{25},y'_{25},z'_{25},\gamma'_{25})\oplus(1,2,2,3,3,3,5,5)\otimes(x_{34},y_{34},z_{34},\gamma_{34},x'_{34},y'_{34},z'_{34},\gamma'_{34})\\\oplus(1,2,3,3,4,4,5,7)\otimes(x_{36},y_{36},z_{36},\gamma_{36},x'_{36},y'_{36},z'_{36},\gamma'_{36})\oplus(1,1,2,3,4,5,7,8)\otimes(x_{45},y_{45},z_{45},\gamma_{45},x'_{45},y'_{45},z'_{45},\gamma'_{45})\\\oplus(1,2,2,3,3,5,6,8)\otimes(x_{46},y_{46},z_{46},\gamma_{46},x'_{46},y'_{46},z'_{46},\gamma'_{46})\oplus(1,2,3,3,4,5,6,9)\otimes(x_{56},y_{56},z_{56},\gamma_{56},x'_{56},y'_{56},z'_{56},\gamma'_{56})\end{array}\right\}$$

Subject to the constrains:

$$(x_{12},y_{12},z_{12},\gamma_{12},x'_{12},y'_{12},z'_{12},\gamma'_{12})\oplus(x_{13},y_{13},z_{13},\gamma_{13},x'_{13},y'_{13},z'_{13},\gamma'_{13})$$
$$\oplus(x_{15},y_{15},z_{15},\gamma_{15},x'_{15},y'_{15},z'_{15},\gamma'_{15})=(1,1,1,1,1,1,1,1)$$
$$(x_{24},y_{24},z_{24},\gamma_{24},x'_{24},y'_{24},z'_{24},\gamma'_{24})\oplus(x_{25},y_{25},z_{25},\gamma_{25},x'_{25},y'_{25},z'_{25},\gamma'_{25})=(x_{12},y_{12},z_{12},\gamma_{12},x'_{12},y'_{12},z'_{12},\gamma'_{12})$$
$$(x_{34},y_{34},z_{34},\gamma_{34},x'_{34},y'_{34},z'_{34},\gamma'_{34})\oplus(x_{36},y_{36},z_{36},\gamma_{36},x'_{36},y'_{36},z'_{36},\gamma'_{36})=(x_{13},y_{13},z_{13},\gamma_{13},x'_{13},y'_{13},z'_{13},\gamma'_{13})$$
$$(x_{45},y_{45},z_{45},\gamma_{45},x'_{45},y'_{45},z'_{45},\gamma'_{45})\oplus(x_{46},y_{46},z_{46},\gamma_{46},x'_{46},y'_{46},z'_{46},\gamma'_{46})$$
$$=(x_{24},y_{24},z_{24},\gamma_{24},x'_{24},y'_{24},z'_{24},\gamma'_{24})\oplus(x_{34},y_{34},z_{34},\gamma_{34},x'_{34},y'_{34},z'_{34},\gamma'_{34})$$
$$(x_{15},y_{15},z_{15},\gamma_{15},x'_{15},y'_{15},z'_{15},\gamma'_{15})\oplus(x_{25},y_{25},z_{25},\gamma_{25},x'_{25},y'_{25},z'_{25},\gamma'_{25})\oplus(x_{45},y_{45},z_{45},\gamma_{45},x'_{45},y'_{45},z'_{45},\gamma'_{45})$$
$$=(x_{56},y_{56},z_{56},\gamma_{56},x'_{56},y'_{56},z'_{56},\gamma'_{56})$$
$$(x_{36},y_{36},z_{36},\gamma_{36},x'_{36},y'_{36},z'_{36},\gamma'_{36})\oplus(x_{46},y_{46},z_{46},\gamma_{46},x'_{46},y'_{46},z'_{46},\gamma'_{46})\oplus(x_{56},y_{56},z_{56},\gamma_{56},x'_{56},y'_{56},z'_{56},\gamma'_{56})$$
$$=(1,1,1,1,1,1,1,1)$$

Where $(x_{12},y_{12},z_{12},\gamma_{12},x'_{12},y'_{12},z'_{12},\gamma'_{12}),(x_{13},y_{13},z_{13},\gamma_{13},x'_{13},y'_{13},z'_{13},\gamma'_{13})$etc. are non negative Trapezoidal intuitionistic fuzzy number integers.

Step-2

Maximize

$$\Re\{(1,2,2,3,3,4,4,4)\otimes(x_{12},y_{12},z_{12},\gamma_{12},x'_{12},y'_{12},z'_{12},\gamma'_{12})\oplus(1,2,3,3,3,4,6,8)\otimes(x_{13},y_{13},z_{13},\gamma_{13},x'_{13},y'_{13},z'_{13},\gamma'_{13})\oplus$$
$$(1,2,3,3,3,5,5,7)\otimes(x_{15},y_{15},z_{15},\gamma_{15},x'_{15},y'_{15},z'_{15},\gamma'_{15})\oplus(1,2,2,2,4,5,5,7)\otimes(x_{24},y_{24},z_{24},\gamma_{24},x'_{24},y'_{24},z'_{24},\gamma'_{24})\oplus$$
$$(1,1,1,2,2,3,5,7)\otimes(x_{25},y_{25},z_{25},\gamma_{25},x'_{25},y'_{25},z'_{25},\gamma'_{25})\oplus(1,2,2,3,3,3,5,5)\otimes(x_{34},y_{34},z_{34},\gamma_{34},x'_{34},y'_{34},z'_{34},\gamma'_{34})$$
$$\oplus(1,2,3,3,4,4,5,7)\otimes(x_{36},y_{36},z_{36},\gamma_{36},x'_{36},y'_{36},z'_{36},\gamma'_{36})\oplus(1,1,2,3,4,5,7,8)\otimes(x_{45},y_{45},z_{45},\gamma_{45},x'_{45},y'_{45},z'_{45},\gamma'_{45})$$
$$\oplus(1,2,2,3,3,5,6,8)\otimes(x_{46},y_{46},z_{46},\gamma_{46},x'_{46},y'_{46},z'_{46},\gamma'_{46})\oplus(1,2,3,3,4,5,6,9)\otimes(x_{56},y_{56},z_{56},\gamma_{56},x'_{56},y'_{56},z'_{56},\gamma'_{56})\}$$

Subject to the constrains:

$$(x_{12},y_{12},z_{12},\gamma_{12},x'_{12},y'_{12},z'_{12},\gamma'_{12})\oplus(x_{13},y_{13},z_{13},\gamma_{13},x'_{13},y'_{13},z'_{13},\gamma'_{13})$$
$$\oplus(x_{15},y_{15},z_{15},\gamma_{15},x'_{15},y'_{15},z'_{15},\gamma'_{15})=(1,1,1,1,1,1,1,1)$$
$$(x_{24},y_{24},z_{24},\gamma_{24},x'_{24},y'_{24},z'_{24},\gamma'_{24})\oplus(x_{25},y_{25},z_{25},\gamma_{25},x'_{25},y'_{25},z'_{25},\gamma'_{25})=(x_{12},y_{12},z_{12},\gamma_{12},x'_{12},y'_{12},z'_{12},\gamma'_{12})$$

$(x_{34}, y_{34}, z_{34}, \gamma_{34}, x'_{34}, y'_{34}, z'_{34}, \gamma'_{34}) \oplus (x_{36}, y_{36}, z_{36}, \gamma_{36}, x'_{36}, y'_{36}, z'_{36}, \gamma'_{36}) = (x_{13}, y_{13}, z_{13}, \gamma_{13}, x'_{13}, y'_{13}, z'_{13}, \gamma'_{13})$

$(x_{45}, y_{45}, z_{45}, \gamma_{45}, x'_{45}, y'_{45}, z'_{45}, \gamma'_{45}) \oplus (x_{46}, y_{46}, z_{46}, \gamma_{46}, x'_{46}, y'_{46}, z'_{46}, \gamma'_{46})$
$= (x_{24}, y_{24}, z_{24}, \gamma_{24}, x'_{24}, y'_{24}, z'_{24}, \gamma'_{24}) \oplus (x_{34}, y_{34}, z_{34}, \gamma_{34}, x'_{34}, y'_{34}, z'_{34}, \gamma'_{34})$

$(x_{15}, y_{15}, z_{15}, \gamma_{15}, x'_{15}, y'_{15}, z'_{15}, \gamma'_{15}) \oplus (x_{25}, y_{25}, z_{25}, \gamma_{25}, x'_{25}, y'_{25}, z'_{25}, \gamma'_{25}) \oplus (x_{45}, y_{45}, z_{45}, \gamma_{45}, x'_{45}, y'_{45}, z'_{45}, \gamma'_{45})$
$= (x_{56}, y_{56}, z_{56}, \gamma_{56}, x'_{56}, y'_{56}, z'_{56}, \gamma'_{56})$

$(x_{36}, y_{36}, z_{36}, \gamma_{36}, x'_{36}, y'_{36}, z'_{36}, \gamma'_{36}) \oplus (x_{46}, y_{46}, z_{46}, \gamma_{46}, x'_{46}, y'_{46}, z'_{46}, \gamma'_{46}) \oplus (x_{56}, y_{56}, z_{56}, \gamma_{56}, x'_{56}, y'_{56}, z'_{56}, \gamma'_{56})$
$= (1,1,1,1,1,1,1,1)$

Where $(x_{12}, y_{12}, z_{12}, \gamma_{12}, x'_{12}, y'_{12}, z'_{12}, \gamma'_{12}), (x_{13}, y_{13}, z_{13}, \gamma_{13}, x'_{13}, y'_{13}, z'_{13}, \gamma'_{13})$etc. are non negative Trapezoidal intuitionistic fuzzy number integers.The Crisp Linear Programming problem becomes
Maximize

$$0.125x_{12} + 0.25y_{12} + 0.25z_{12} + 0.375\gamma_{12} + 0.375x'_{12} + 0.5y'_{12} + 0.5z'_{12} + 0.5\gamma'_{12} +$$
$$0.125x_{13} + 0.25y_{13} + 0.375z_{13} + 0.375\gamma_{13} + 0.375x'_{13} + 0.5y'_{13} + 0.75z'_{13} + 1\gamma'_{13} +$$
$$0.125x_{15} + 0.25y_{15} + 0.375z_{15} + 0.375\gamma_{15} + 0.375x'_{15} + 0.625y'_{15} + 0.625z'_{15} + 0.875\gamma'_{15} +$$
$$0.125x_{24} + 0.25y_{24} + 0.25z_{24} + 0.25\gamma_{24} + 0.5x'_{24} + 0.625y'_{24} + 0.625z'_{24} + 0.875\gamma'_{24} +$$
$$0.125x_{25} + 0.125y_{25} + 0.125z_{25} + 0.25\gamma_{25} + 0.25x'_{25} + 0.375y'_{25} + 0.625z'_{25} + 0.875\gamma'_{25} +$$
$$0.125x_{34} + 0.25y_{34} + 0.25z_{34} + 0.375\gamma_{34} + 0.375x'_{34} + 0.375y'_{34} + 0.625z'_{34} + 0.625\gamma'_{34} +$$
$$0.125x_{36} + 0.25y_{36} + 0.375z_{36} + 0.375\gamma_{36} + 0.5x'_{36} + 0.5y'_{36} + 0.625z'_{36} + 0.875\gamma'_{36} +$$
$$0.125x_{45} + 0.125y_{45} + 0.25z_{45} + 0.375\gamma_{45} + 0.5x'_{45} + 0.625y'_{45} + 0.875z'_{4+5} + 1\gamma'_{45} +$$
$$0.125x_{46} + 0.25y_{46} + 0.25z_{46} +, 0.375\gamma_{46} + 0.375x'_{46} + 0.625y'_{46} + 0.75z'_{46} + 1\gamma'_{46} +$$
$$0.125x_{56} + 0.25y_{56} + 0.375z_{56} + 0.375\gamma_{56} + 0.5x'_{56} + 0.625y'_{56} + 0.75z'_{56} + 1.125\gamma'_{56}.$$

Subject to the constrains:

$x_{12} + x_{13} + x_{15} = 1, y_{12} + y_{13} + y_{15} = 1, z_{12} + z_{13} + z_{15} = 1, \gamma_{12} + \gamma_{13} + \gamma_{15} = 1$

$x'_{12} + x'_{13} + x'_{15} = 1, y'_{12} + y'_{13} + y'_{15} = 1, z'_{12} + z'_{13} + z'_{15} = 1, \gamma'_{12} + \gamma'_{13} + \gamma'_{15} = 1$

$x_{24} + x_{25} = x_{12}, y_{24} + y_{25} = y_{12}, z_{24} + z_{25} = z_{12}, \gamma_{24} + \gamma_{25} = \gamma_{12}$

$x'_{24} + x'_{25} = x'_{12}, y'_{24} + y'_{25} = y'_{12}, z'_{24} + z'_{25} = z'_{12}, \gamma'_{24} + \gamma'_{25} = \gamma'_{12}$

$x_{34} + x_{36} = x_{13}, y_{34} + y_{36} = y_{13}, z_{34} + z_{36} = z_{13}, \gamma_{34} + \gamma_{36} = \gamma_{13}$

$x'_{34} + x'_{36} = x'_{13}, y'_{34} + y'_{36} = y'_{13}, z'_{34} + z'_{36} = z'_{13}, \gamma'_{34} + \gamma'_{36} = \gamma'_{13}$

$x_{45} + x_{46} = x_{24} + x_{34}, y_{45} + y_{46} = y_{24} + y_{34}, z_{45} + z_{46} = z_{24} + z_{34}, \gamma_{45} + \gamma_{46} = \gamma_{24} + \gamma_{34}$

$x'_{45} + x'_{46} = x'_{24} + x'_{34}, y'_{45} + y'_{46} = y'_{24} + y'_{34}, z'_{45} + z'_{46} = z'_{24} + z'_{34}, \gamma'_{45} + \gamma'_{46} = \gamma'_{24} + \gamma'_{34}$

$x_{15} + x_{25} + x_{45} = x_{56}, y_{15} + y_{25} + y_{45} = y_{56}, z_{15} + z_{25} + z_{45} = z_{56}\ \gamma_{15} + \gamma_{25} + \gamma_{45} = \gamma_{56}$

$x'_{15} + x'_{25} + x'_{45} = x'_{56}, y'_{15} + y'_{25} + y'_{45} = y'_{56}, z'_{15} + z'_{25} + z'_{45} = z'_{56}\ \gamma'_{15} + \gamma'_{25} + \gamma'_{45} = \gamma'_{56}$

$x_{36} + x_{46} + x_{56} = 1, y_{36} + y_{46} + y_{56} = 1, z_{36} + z_{46} + z_{56} = 1, \gamma_{36} + \gamma_{46} + \gamma_{56} = 1$

$x'_{36} + x'_{46} + x'_{56} = 1, y'_{36} + y'_{46} + y'_{56} = 1, z'_{36} + z'_{46} + z'_{56} = 1, \gamma'_{36} + \gamma'_{46} + \gamma'_{56} = 1$

$y_{12} - x_{12} \geq 0, z_{12} - y_{12} \geq 0, \gamma_{12} - z_{12} \geq 0, y'_{12} - x'_{12} \geq 0, z'_{12} - y'_{12} \geq 0, \gamma'_{12} - z'_{12} \geq 0$

$y_{13} - x_{13} \geq 0, z_{13} - y_{13} \geq 0, \gamma_{13} - z_{13} \geq 0, y'_{13} - x'_{13} \geq 0, z'_{13} - y'_{13} \geq 0, \gamma'_{13} - z'_{13} \geq 0$

$y_{15} - x_{15} \geq 0, z_{15} - y_{15} \geq 0, \gamma_{15} - z_{15} \geq 0, y'_{15} - x'_{15} \geq 0, z'_{15} - y'_{15} \geq 0, \gamma'_{15} - z'_{15} \geq 0$

$y_{24} - x_{24} \geq 0, z_{24} - y_{24} \geq 0, \gamma_{24} - z_{24} \geq 0, y'_{24} - x'_{24} \geq 0, z'_{24} - y'_{24} \geq 0, \gamma'_{24} - z'_{24} \geq 0$

$y_{25} - x_{25} \geq 0, z_{25} - y_{25} \geq 0, \gamma_{25} - z_{25} \geq 0, y'_{25} - x'_{25} \geq 0, z'_{25} - y'_{25} \geq 0, \gamma'_{25} - z'_{25} \geq 0$

$y_{34} - x_{34} \geq 0, z_{34} - y_{34} \geq 0, \gamma_{34} - z_{34} \geq 0, y'_{34} - x'_{34} \geq 0, z'_{34} - y'_{34} \geq 0, \gamma'_{34} - z'_{34} \geq 0$

$y_{36}-x_{36}\ge 0, z_{36}-y_{36}\ge 0, \gamma_{36}-z_{36}\ge 0, y'_{36}-x'_{36}\ge 0, z'_{36}-y'_{36}\ge 0, \gamma'_{36}-z'_{36}\ge 0$
$y_{45}-x_{45}\ge 0, z_{45}-y_{45}\ge 0, \gamma_{45}-z_{45}\ge 0, y'_{45}-x'_{45}\ge 0, z'_{45}-y'_{45}\ge 0, \gamma'_{45}-z'_{45}\ge 0$
$y_{46}-x_{46}\ge 0, z_{46}-y_{46}\ge 0, \gamma_{46}-z_{46}\ge 0, y'_{46}-x'_{46}\ge 0, z'_{46}-y'_{46}\ge 0, \gamma'_{46}-z'_{46}\ge 0$
$y_{56}-x_{56}\ge 0, z_{56}-y_{56}\ge 0, \gamma_{56}-z_{56}\ge 0, y'_{56}-x'_{56}\ge 0, z'_{56}-y'_{56}\ge 0, \gamma'_{56}-z'_{56}\ge 0$

Step 3: On solving Crisp Linear Programming Using TORA System, obtained in Step 2, solution is
$x_{12}=y_{12}=z_{12}=\gamma_{12}=x'_{12}=y'_{12}=z'_{12}=\gamma'_{12}=x_{24}=y_{24}=z_{24}=\gamma_{24}=x'_{24}=y'_{24}=z'_{24}=\gamma'_{24}=$
$x_{45}=y_{45}=z_{45}=\gamma_{45}=x'_{45}=y'_{45}=z'_{45}=\gamma'_{45}=x_{56}=y_{56}=z_{56}=\gamma_{56}=x'_{56}=y'_{56}=z'_{56}=\gamma'_{56}=1$
remaining all values
$x_{13}=y_{13}=z_{13}=\gamma_{13}=x'_{13}=y'_{13}=z'_{13}=\gamma'_{13}=x_{15}=y_{15}=z_{15}=\gamma_{15}=x'_{15}=y'_{15}=z'_{15}=\gamma'_{15}=$
$x_{25}=y_{25}=z_{25}=\gamma_{25}=x'_{25}=y'_{25}=z'_{25}=\gamma'_{25}=x_{34}=y_{34}=z_{34}=\gamma_{34}=x'_{34}=y'_{34}=z'_{34}=\gamma'_{34}=$
$x_{36}=y_{36}=z_{36}=\gamma_{36}=x'_{36}=y'_{36}=z'_{36}=\gamma'_{36}=x_{45}=y_{45}=z_{45}=\gamma_{45}=x'_{45}=y'_{45}=z'_{45}=\gamma'_{45}=$
$x_{46}=y_{46}=z_{46}=\gamma_{46}=x'_{46}=y'_{46}=z'_{46}=\gamma'_{46}=x_{56}=y_{56}=z_{56}=\gamma_{56}=x'_{56}=y'_{56}=z'_{56}=\gamma'_{56}$
are zero.

Step 4: Putting the values of $x_{ij}, y_{ij}, z_{ij}, \gamma_{ij}, x'_{ij}, y'_{ij}, z'_{ij}, \gamma'_{ij}$ in $\tilde{x}_{ij}=\left(x_{ij}, y_{ij}, z_{ij}, \gamma_{ij}, x'_{ij}, y'_{ij}, z'_{ij}, \gamma'_{ij}\right)$. The solution is
$\tilde{x}_{12}=(1,1,1,1,1,1,1,1)$ $\tilde{x}_{24}=(1,1,1,1,1,1,1,1)$ $\tilde{x}_{45}=(1,1,1,1,1,1,1,1)$ $\tilde{x}_{56}=(1,1,1,1,1,1,1,1)$
$\tilde{x}_{13}=(0,0,0,0,0,0,0,0)$ $\tilde{x}_{15}=(0,0,0,0,0,0,0,0)$ $\tilde{x}_{25}=(0,0,0,0,0,0,0,0)$
$\tilde{x}_{34}=(0,0,0,0,0,0,0,0)$ $\tilde{x}_{36}=(0,0,0,0,0,0,0,0)$ $\tilde{x}_{46}=(0,0,0,0,0,0,0,0)$

Step 5: Using the fuzzy solution, the fuzzy critical path is 1-2-4-5-6. Replacing the values of $x_{ij}, y_{ij}, z_{ij}, \gamma_{ij}, x'_{ij}, y'_{ij}, z'_{ij}, \gamma'_{ij}$ in Step1, the maximum total completion fuzzy time is $(4,7,9,11,15,19,22,28)$
Hence, in this problem, the fuzzy critical path is 1-2-4-5-6 and the corresponding maximum total completion fuzzy time is $(4,7,9,11,15,19,22,28)$ respectively.

VI. Numerical example for New Trapezoidal intuitionistic fuzzy number

To show the advantages of New representations over existing representation of Trapezoidal intuitionistic fuzzy numbers, the same numerical example is solved by using all two representations of fuzzy numbers. The problem is to find the fuzzy critical path and maximum total completion fuzzy time of the project network, shown in Fig.2, in which the fuzzy time duration of each activity is represented by the following New Trapezoidal intuitionistic fuzzy number $(x, y, \alpha, \beta, x', y', \alpha', \beta')$ type trapezoidal fuzzy numbers.

$\tilde{t}_{12}=(1,4,1,0,1,0,1,0)$, $\tilde{t}_{13}=(1,8,1,1,0,0,1,2)$, $\tilde{t}_{15}=(1,7,1,1,0,0,2,2)$, $\tilde{t}_{24}=(1,7,1,0,0,2,1,2)$
$\tilde{t}_{25}=(1,7,0,0,1,0,1,2)$, $\tilde{t}_{34}=(1,5,1,0,1,0,0,0)$ $\tilde{t}_{36}=(1,7,1,1,0,1,0,2)$ $\tilde{t}_{45}=(1,8,0,1,1,1,1,1)$
$\tilde{t}_{46}=(1,8,1,0,1,0,2,2)$, $\tilde{t}_{56}=(1,9,1,1,0,1,1,3)$

Fuzzy optimal solution using $\left(x, y, \alpha, \beta, x', y', \alpha', \beta'\right)$ representation of Trapezoidal intuitionistic fuzzy number

Step-1

Maximize

$$\left\{\begin{array}{l}\left(1,4,1,0,1,0,1,0\right)\otimes(x_{12},y_{12},\alpha_{12},\beta_{12},x'_{12},y'_{12},\alpha'_{12},\beta'_{12})\oplus\left(1,8,1,1,0,0,1,2\right)\otimes(x_{13},y_{13},\alpha_{13},\beta_{13},x'_{13},y'_{13},\alpha'_{13},\beta'_{13})\oplus\\ \left(1,7,1,1,0,0,2,2\right)\otimes(x_{15},y_{15},\alpha_{15},\beta_{15},x'_{15},y'_{15},\alpha'_{15},\beta'_{15})\oplus\left(1,7,1,0,0,2,1,2\right)\otimes(x_{24},y_{24},\alpha_{24},\beta_{24},x'_{24},y'_{24},\alpha'_{24},\beta'_{24})\oplus\\ \left(1,7,0,0,1,0,1,2\right)\otimes(x_{25},y_{25},\alpha_{25},\beta_{25},x'_{25},y'_{25},\alpha'_{25},\beta'_{25})\oplus\left(1,5,1,0,1,0,0,0\right)\otimes(x_{34},y_{34},\alpha_{34},\beta_{34},x'_{34},y'_{34},\alpha'_{34},\beta'_{34})\\ \oplus\left(1,7,1,1,0,1,0,2\right)\otimes(x_{36},y_{36},\alpha_{36},\beta_{36},x'_{36},y'_{36},\alpha'_{36},\beta'_{36})\oplus\left(1,8,0,1,1,1,1,1\right)\otimes(x_{45},y_{45},\alpha_{45},\beta_{45},x'_{45},y'_{45},\alpha'_{45},\beta'_{45})\\ \oplus\left(1,8,1,0,1,0,2,2\right)\otimes(x_{46},y_{46},\alpha_{46},\beta_{46},x'_{46},y'_{46},\alpha'_{46},\beta'_{46})\oplus\left(1,9,1,1,0,1,1,3\right)\otimes(x_{56},y_{56},\alpha_{56},\beta_{56},x'_{56},y'_{56},\alpha'_{56},\beta'_{56})\end{array}\right\}$$

Subject to the constrains:

$$(x_{12},y_{12},\alpha_{12},\beta_{12},x'_{12},y'_{12},\alpha'_{12},\beta'_{12})\oplus(x_{13},y_{13},\alpha_{13},\beta_{13},x'_{13},y'_{13},\alpha'_{13},\beta'_{13})$$
$$\oplus(x_{15},y_{15},\alpha_{15},\beta_{15},x'_{15},y'_{15},\alpha'_{15},\beta'_{15})=\left(1,1,0,0,0,0,0,0\right)$$
$$(x_{24},y_{24},\alpha_{24},\beta_{24},x'_{24},y'_{24},\alpha'_{24},\beta'_{24})\oplus(x_{25},y_{25},\alpha_{25},\beta_{25},x'_{25},y'_{25},\alpha'_{25},\beta'_{25})=(x_{12},y_{12},\alpha_{12},\beta_{12},x'_{12},y'_{12},\alpha'_{12},\beta'_{12})$$
$$(x_{34},y_{34},\alpha_{34},\beta_{34},x'_{34},y'_{34},\alpha'_{34},\beta'_{34})\oplus(x_{36},y_{36},\alpha_{36},\beta_{36},x'_{36},y'_{36},\alpha'_{36},\beta'_{36})=(x_{13},y_{13},\alpha_{13},\beta_{13},x'_{13},y'_{13},\alpha'_{13},\beta'_{13})$$
$$(x_{45},y_{45},\alpha_{45},\beta_{45},x'_{45},y'_{45},\alpha'_{45},\beta'_{45})\oplus(x_{46},y_{46},\alpha_{46},\beta_{46},x'_{46},y'_{46},\alpha'_{46},\beta'_{46})$$
$$=(x_{24},y_{24},\alpha_{24},\beta_{24},x'_{24},y'_{24},\alpha'_{24},\beta'_{24})\oplus(x_{34},y_{34},\alpha_{34},\beta_{34},x'_{34},y'_{34},\alpha'_{34},\beta'_{34})$$
$$(x_{15},y_{15},\alpha_{15},\beta_{15},x'_{15},y'_{15},\alpha'_{15},\beta'_{15})\oplus(x_{25},y_{25},\alpha_{25},\beta_{25},x'_{25},y'_{25},\alpha'_{25},\beta'_{25})\oplus(x_{45},y_{45},\alpha_{45},\beta_{45},x'_{45},y'_{45},\alpha'_{45},\beta'_{45})$$
$$=(x_{56},y_{56},\alpha_{56},\beta_{56},x'_{56},y'_{56},\alpha'_{56},\beta'_{56})$$
$$(x_{36},y_{36},\alpha_{36},\beta_{36},x'_{36},y'_{36},\alpha'_{36},\beta'_{36})\oplus(x_{46},y_{46},\alpha_{46},\beta_{46},x'_{46},y'_{46},\alpha'_{46},\beta'_{46})\oplus(x_{56},y_{56},\alpha_{56},\beta_{56},x'_{56},y'_{56},\alpha'_{56},\beta'_{56})$$
$$=(1,1,0,0,0,0,0,0)$$

Where $(x_{12},y_{12},\alpha_{12},\beta_{12},x'_{12},y'_{12},\alpha'_{12},\beta'_{12}),(x_{13},y_{13},\alpha_{13},\beta_{13},x'_{13},y'_{13},\alpha'_{13},\beta'_{13})$etc. are non negative Trapezoidal intuitionistic fuzzy number integers.

Step-2

Maximize

$$\Re\left\{\left(1,4,1,0,1,0,1,0\right)\otimes(x_{12},y_{12},\alpha_{12},\beta_{12},x'_{12},y'_{12},\alpha'_{12},\beta'_{12})\oplus\left(1,8,1,1,0,0,1,2\right)\otimes(x_{13},y_{13},\alpha_{13},\beta_{13},x'_{13},y'_{13},\alpha'_{13},\beta'_{13})\oplus\right.$$
$$\left(1,7,1,1,0,0,2,2\right)\otimes(x_{15},y_{15},\alpha_{15},\beta_{15},x'_{15},y'_{15},\alpha'_{15},\beta'_{15})\oplus\left(1,7,1,0,0,2,1,2\right)\otimes(x_{24},y_{24},\alpha_{24},\beta_{24},x'_{24},y'_{24},\alpha'_{24},\beta'_{24})\oplus$$
$$\left(1,7,0,0,1,0,1,2\right)\otimes(x_{25},y_{25},\alpha_{25},\beta_{25},x'_{25},y'_{25},\alpha'_{25},\beta'_{25})\oplus\left(1,5,1,0,1,0,0,0\right)\otimes(x_{34},y_{34},\alpha_{34},\beta_{34},x'_{34},y'_{34},\alpha'_{34},\beta'_{34})$$
$$\oplus\left(1,7,1,1,0,1,0,2\right)\otimes(x_{36},y_{36},\alpha_{36},\beta_{36},x'_{36},y'_{36},\alpha'_{36},\beta'_{36})\oplus\left(1,8,0,1,1,1,1,1\right)\otimes(x_{45},y_{45},\alpha_{45},\beta_{45},x'_{45},y'_{45},\alpha'_{45},\beta'_{45})$$
$$\left.\oplus\left(1,8,1,0,1,0,2,2\right)\otimes(x_{46},y_{46},\alpha_{46},\beta_{46},x'_{46},y'_{46},\alpha'_{46},\beta'_{46})\oplus\left(1,9,1,1,0,1,1,3\right)\otimes(x_{56},y_{56},\alpha_{56},\beta_{56},x'_{56},y'_{56},\alpha'_{56},\beta'_{56})\right\}$$

Subject to the constrains:

$$(x_{12},y_{12},\alpha_{12},\beta_{12},x'_{12},y'_{12},\alpha'_{12},\beta'_{12})\oplus(x_{13},y_{13},\alpha_{13},\beta_{13},x'_{13},y'_{13},\alpha'_{13},\beta'_{13})$$
$$\oplus(x_{15},y_{15},\alpha_{15},\beta_{15},x'_{15},y'_{15},\alpha'_{15},\beta'_{15})=\left(1,1,0,0,0,0,0,0\right)$$
$$(x_{24},y_{24},\alpha_{24},\beta_{24},x'_{24},y'_{24},\alpha'_{24},\beta'_{24})\oplus(x_{25},y_{25},\alpha_{25},\beta_{25},x'_{25},y'_{25},\alpha'_{25},\beta'_{25})=(x_{12},y_{12},\alpha_{12},\beta_{12},x'_{12},y'_{12},\alpha'_{12},\beta'_{12})$$
$$(x_{34},y_{34},\alpha_{34},\beta_{34},x'_{34},y'_{34},\alpha'_{34},\beta'_{34})\oplus(x_{36},y_{36},\alpha_{36},\beta_{36},x'_{36},y'_{36},\alpha'_{36},\beta'_{36})=(x_{13},y_{13},\alpha_{13},\beta_{13},x'_{13},y'_{13},\alpha'_{13},\beta'_{13})$$
$$(x_{45},y_{45},\alpha_{45},\beta_{45},x'_{45},y'_{45},\alpha'_{45},\beta'_{45})\oplus(x_{46},y_{46},\alpha_{46},\beta_{46},x'_{46},y'_{46},\alpha'_{46},\beta'_{46})$$
$$=(x_{24},y_{24},\alpha_{24},\beta_{24},x'_{24},y'_{24},\alpha'_{24},\beta'_{24})\oplus(x_{34},y_{34},\alpha_{34},\beta_{34},x'_{34},y'_{34},\alpha'_{34},\beta'_{34})$$
$$(x_{15},y_{15},\alpha_{15},\beta_{15},x'_{15},y'_{15},\alpha'_{15},\beta'_{15})\oplus(x_{25},y_{25},\alpha_{25},\beta_{25},x'_{25},y'_{25},\alpha'_{25},\beta'_{25})\oplus(x_{45},y_{45},\alpha_{45},\beta_{45},x'_{45},y'_{45},\alpha'_{45},\beta'_{45})$$
$$=(x_{56},y_{56},\alpha_{56},\beta_{56},x'_{56},y'_{56},\alpha'_{56},\beta'_{56})$$
$$(x_{36},y_{36},\alpha_{36},\beta_{36},x'_{36},y'_{36},\alpha'_{36},\beta'_{36})\oplus(x_{46},y_{46},\alpha_{46},\beta_{46},x'_{46},y'_{46},\alpha'_{46},\beta'_{46})\oplus(x_{56},y_{56},\alpha_{56},\beta_{56},x'_{56},y'_{56},\alpha'_{56},\beta'_{56})$$
$$=(1,1,0,0,0,0,0,0)$$

Where $(x_{12},y_{12},\alpha_{12},\beta_{12},x'_{12},y'_{12},\alpha'_{12},\beta'_{12}),(x_{13},y_{13},\alpha_{13},\beta_{13},x'_{13},y'_{13},\alpha'_{13},\beta'_{13})$etc. are non negative Trapezoidal intuitionistic fuzzy number integers.The Crisp Linear Programming problem becomes

Maximize

$$0.125x_{12}+0.25y_{12}+0.25\alpha_{12}+0.375\beta_{12}+0.375x'_{12}+0.5y'_{12}+0.5\alpha'_{12}+0.5\beta'_{12}+$$
$$0.125x_{13}+0.25y_{13}+0.375\alpha_{13}+0.375\beta_{13}+0.375x'_{13}+0.5y'_{13}+0.75\alpha'_{13}+1\beta'_{13}+$$
$$0.125x_{15}+0.25y_{15}+0.375\alpha_{15}+0.375\beta_{15}+0.375x'_{15}+0.625y'_{15}+0.625\alpha'_{15}+0.875\beta'_{15}+$$
$$0.125x_{24}+0.25y_{24}+0.25\alpha_{24}+0.25\beta_{24}+0.5x'_{24}+0.625y'_{24}+0.625\alpha'_{24}+0.875\beta'_{24}+$$
$$0.125x_{25}+0.125y_{25}+0.125\alpha_{25}+0.25\beta_{25}+0.25x'_{25}+0.375y'_{25}+0.625\alpha'_{25}+0.875\beta'_{25}+$$
$$0.125x_{34}+0.25y_{34}+0.25\alpha_{34}+0.375\beta_{34}+0.375x'_{34}+0.375y'_{34}+0.625\alpha'_{34}+0.625\beta'_{34}+$$
$$0.125x_{36}+0.25y_{36}+0.375\alpha_{36}+0.375\beta_{36}+0.5x'_{36}+0.5y'_{36}+0.625\alpha'_{36}+0.875\beta'_{36}+$$
$$0.125x_{45}+0.125y_{45}+0.25\alpha_{45}+0.375\beta_{45}+0.5x'_{45}+0.625y'_{45}+0.875\alpha'_{4+5}+1\beta'_{45}+$$
$$0.125x_{46}+0.25y_{46}+0.25\alpha_{46}+,0.375\beta_{46}+0.375x'_{46}+0.625y'_{46}+0.75\alpha'_{46}+1\beta'_{46}+$$
$$0.125x_{56}+0.25y_{56}+0.375\alpha_{56}+0.375\beta_{56}+0.5x'_{56}+0.625y'_{56}+0.75\alpha'_{56}+1.125\beta'_{56}.$$

Subject to the constrains:

$$x_{12}+x_{13}+x_{15}=1, y_{12}+y_{13}+y_{15}=1, \alpha_{12}+\alpha_{13}+\alpha_{15}=0, \beta_{12}+\beta_{13}+\beta_{15}=0$$
$$x'_{12}+x'_{13}+x'_{15}=0, y'_{12}+y'_{13}+y'_{15}=0, \alpha'_{12}+\alpha'_{13}+\alpha'_{15}=0, \beta'_{12}+\beta'_{13}+\beta'_{15}=0$$
$$x_{24}+x_{25}=x_{12}, y_{24}+y_{25}=y_{12}, \alpha_{24}+\alpha_{25}=\alpha_{12}, \beta_{24}+\beta_{25}=\beta_{12}$$
$$x'_{24}+x'_{25}=x'_{12}, y'_{24}+y'_{25}=y'_{12}, \alpha'_{24}+\alpha'_{25}=\alpha'_{12}, \beta'_{24}+\beta'_{25}=\beta'_{12}$$
$$x_{34}+x_{36}=x_{13}, y_{34}+y_{36}=y_{13}, \alpha_{34}+\alpha_{36}=\alpha_{13}, \beta_{34}+\beta_{36}=\beta_{13}$$
$$x'_{34}+x'_{36}=x'_{13}, y'_{34}+y'_{36}=y'_{13}, \alpha'_{34}+\alpha'_{36}=\alpha'_{13}, \beta'_{34}+\beta'_{36}=\beta'_{13}$$
$$x_{45}+x_{46}=x_{24}+x_{34}, y_{45}+y_{46}=y_{24}+y_{34}, \alpha_{45}+\alpha_{46}=\alpha_{24}+\alpha_{34}, \beta_{45}+\beta_{46}=\beta_{24}+\beta_{34}$$
$$x'_{45}+x'_{46}=x'_{24}+x'_{34}, y'_{45}+y'_{46}=y'_{24}+y'_{34}, \alpha'_{45}+\alpha'_{46}=\alpha'_{24}+\alpha'_{34}, \beta'_{45}+\beta'_{46}=\beta'_{24}+\beta'_{34}$$
$$x_{15}+x_{25}+x_{45}=x_{56}, y_{15}+y_{25}+y_{45}=y_{56}, \alpha_{15}+\alpha_{25}+\alpha_{45}=\alpha_{56}\ \beta_{15}+\beta_{25}+\beta_{45}=\beta_{56}$$
$$x'_{15}+x'_{25}+x'_{45}=x'_{56}, y'_{15}+y'_{25}+y'_{45}=y'_{56}, \alpha'_{15}+\alpha'_{25}+\alpha'_{45}=\alpha'_{56}\ \beta'_{15}+\beta'_{25}+\beta'_{45}=\beta'_{56}$$
$$x_{36}+x_{46}+x_{56}=1, y_{36}+y_{46}+y_{56}=1, \alpha_{36}+\alpha_{46}+\alpha_{56}=0, \beta_{36}+\beta_{46}+\beta_{56}=0$$
$$x'_{36}+x'_{46}+x'_{56}=0, y'_{36}+y'_{46}+y'_{56}=0, \alpha'_{36}+\alpha'_{46}+\alpha'_{56}=0, \beta'_{36}+\beta'_{46}+\beta'_{56}=0$$

Step 3: On solving Crisp Linear Programming Using TORA System, obtained in Step 2, solution is $x_{12}=y_{12}=1, z_{12}=\gamma_{12}=x'_{12}=y'_{12}=z'_{12}=\gamma'_{12}=0$,

$$x_{24}=y_{24}=1, z_{24}=\gamma_{24}=x'_{24}=y'_{24}=z'_{24}=\gamma'_{24}=0$$
$$x_{45}=y_{45}=1, z_{45}=\gamma_{45}=x'_{45}=y'_{45}=z'_{45}=\gamma'_{45}=0, x_{56}=y_{56}=1, z_{56}=\gamma_{56}=x'_{56}=y'_{56}=z'_{56}=\gamma'_{56}=0$$

remaining all values

$$x_{13}=y_{13}=z_{13}=\gamma_{13}=x'_{13}=y'_{13}=z'_{13}=\gamma'_{13}=x_{15}=y_{15}=z_{15}=\gamma_{15}=x'_{15}=y'_{15}=z'_{15}=\gamma'_{15}=$$
$$x_{25}=y_{25}=z_{25}=\gamma_{25}=x'_{25}=y'_{25}=z'_{25}=\gamma'_{25}=x_{34}=y_{34}=z_{34}=\gamma_{34}=x'_{34}=y'_{34}=z'_{34}=\gamma'_{34}=$$
$$x_{36}=y_{36}=z_{36}=\gamma_{36}=x'_{36}=y'_{36}=z'_{36}=\gamma'_{36}=x_{45}=y_{45}=z_{45}=\gamma_{45}=x'_{45}=y'_{45}=z'_{45}=\gamma'_{45}=$$
$$x_{46}=y_{46}=z_{46}=\gamma_{46}=x'_{46}=y'_{46}=z'_{46}=\gamma'_{46}=x_{56}=y_{56}=z_{56}=\gamma_{56}=x'_{56}=y'_{56}=z'_{56}=\gamma'_{56}$$

are zero.

Step 4: Putting the values of $x_{ij}, y_{ij}, z_{ij}, \gamma_{ij}, x'_{ij}, y'_{ij}, z'_{ij}, \gamma'_{ij}$ in $\tilde{x}_{ij}=\left(x_{ij}, y_{ij}, z_{ij}, \gamma_{ij}, x'_{ij}, y'_{ij}, z'_{ij}, \gamma'_{ij}\right)$. The solution is

$\tilde{x}_{12}=(1,1,0,0,0,0,0,0)$ $\tilde{x}_{24}=(1,1,0,0,0,0,0,0)$ $\tilde{x}_{45}=(1,1,0,0,0,0,0,0)$ $\tilde{x}_{56}=(1,1,0,0,0,0,0,0)$

$\tilde{x}_{13}=(0,0,0,0,0,0,0,0)$ $\tilde{x}_{15}=(0,0,0,0,0,0,0,0)$ $\tilde{x}_{25}=(0,0,0,0,0,0,0,0)$

$\tilde{x}_{34}=(0,0,0,0,0,0,0,0)$ $\tilde{x}_{36}=(0,0,0,0,0,0,0,0)$ $\tilde{x}_{46}=(0,0,0,0,0,0,0,0)$

Step 5: Using the fuzzy solution, the fuzzy critical path is 1-2-4-5-6. Replacing the values of $x_{ij}, y_{ij}, z_{ij}, \gamma_{ij}, x'_{ij}, y'_{ij}, z'_{ij}, \gamma'_{ij}$ in Step1, the maximum total completion fuzzy time is $(4, 28, 3, 2, 2, 4, 4, 4)$
Hence, in this problem, the fuzzy critical path is 1-2-4-5-6 and the corresponding maximum total completion fuzzy time is $(4, 28, 3, 2, 2, 4, 4, 4)$ respectively.

VII. Comparative Study

The results of the numerical example obtained from the sections 5 , 6 are presented in table 3. From table 3, we can easily seen that number of constraints in crisp linear programming problem represented by new representation of trapezoidal intuitionistic fuzzy numbers are less compare to crisp linear programming problem represented by existing representations of trapezoidal fuzzy numbers. Hence, it is better to use crisp linear programming problem represented by new representation of trapezoidal intuitionistic fuzzy numbers.

Trapezoidal intuitionistic fuzzy number Representation	Number of constraints in Fuzzy Linear programming problem	Number of constraints in Crisp Linear Programming problem	Fuzzy critical path	Maximum total completion fuzzy time
$(b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$	6	108	1→2→4→5→6	$(4, 7, 9, 11, 15, 19, 22, 28)$
$(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}, x'_{ij}, y'_{ij}, \alpha'_{ij}, \beta'_{ij})$	6	48	1→2→4→5→6	$(4, 28, 3, 2, 2, 4, 4, 4)$

Table 3: Results for existing method and proposed representation of trapezoidal intuitionistic fuzzy numbers

VIII. Conclusion

A new method has been proposed to find the fuzzy critical path and fuzzy completion time of a fuzzy project. Also a new representation of trapezoidal intuitionistic fuzzy number is proposed and showing better to use the proposed representation of trapezoidal intuitionistic fuzzy numbers instead of existing representations to find the fuzzy critical path and fuzzy completion time of fuzzy critical path problems.

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