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# An Independent Demand Pattern Inventory System

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***Abstract:*** *This paper is an improvement of the analysis undertaken in the paper of Deng, Lin, and Chu that was published in the European Journal of Operational Research. It shows that the lengthy computations can be replaced by a more elegant one that yields the generalization of the targeted system. We generalize their model from ramp-type demand to arbitrary demand while theoretically discovering a fascinating phenomenon: the optimal solution is actually independent of the demand. This study provides a justification that a form of abstract generalization sometimes has the capability of solving the inventory model in a much more efficient manner.*

***Key Word****: Inventory model; Deteriorating item.*

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## I. Introduction

Since Hill [4] studied the inventory system with ramp-type demands there has been a trend toward examining this kind of inventory model both in depth and breadth. Some examples include Mandal and Pal [5] with deterioration items; Wu et al. [7] assuming that the backlogging rate is proportional to the waiting time; Wu and Ouyang [8] with two different strategies beginning with stock or shortage; Wu [9] with the Weibull distributed deterioration; Giri et al. [3] with a more generalized Weibull distributed deterioration; Deng [1] to revise the work of Wu et al. [7]; Manna and Chaudhuri [6] to extend the inventory model a with time-dependent deterioration rate;Deng et al. [2] who modify Mandal and Pal [5], Wu and Ouyang [8].

However, their work is always restricted to ramp-type demands. This paper points out that there is a general property that is shared for every demand without being limited to the ramp type or any type of demand. Our findings show that the lengthy discussion and derivation of Deng et al. [2] in the consideration of two different expressions of ramp-type demand are in fact unnecessary.

This paper attempts to investigate an inventory model with stock in the first phase and then shortages in the second phase. Of particular interest is that when all targeted stock items reach zero simultaneously, and all subsequent demands are intentionally backlogged until the end of the inventory cycle, then one-shot replenishment takes place. In so setting, the total inventory cost including holding cost, deteriorating cost, and shortage cost will achieve its minimum. As a matter of fact, this kind of design of an inventory system has been prevalent in various OEM Original Equipment Manufacturing) and ODM (Original Design Manufacturing) industries such as the electronic industry, food industry, construction industry, and so on. For instance, three major passive components: resistance, capacitance, and inductance in the IC industry, the memory chip sets, or IO (Input-Output) control devices in IC sub-industries all refer to common materials. For years, most common materials have been standardized in specifications so as to be widely used in manufacturing electronic components, semi-products, or finished goods. A standardized common material accounts for the following facts:

a. It can be used in manufacturing several or even a number of different components, semi-, or finishedproducts.
b. Owing to the property of a., if the stock of one of the common materials conspicuously outnumbers the others, the mechanism of consuming more of that particular item will be initiated so as to rapidly bring down its stock level.
c. The replaceable function is possible among some common materials. For instance, for passive components, one type-A resistance can be replaced with two type-B resistances. For the construction industry, some round-cap nails can be replaced with square-cap nails, or one large-size nail can be replaced with two small-size nails. Moreover, the substitution among some particular bricks frequently takes place on construction sites.
d. The characteristic of make-to-order (MTO) in the OEM or ODM industry indicates that the production quantity in the near future is highly explicit. It implies that the stock and consumption of common materials can be well-planned and well-controlled.

In other words, under planned production and inventory control, the consumption rate for each of the common materials can be accurately tracked, mastered, and modulated respectively. In this way, the stock levels for those well-scheduled common materials will reduce to zero at the same time and hereupon the reordering processes and replenishment activities are automatically initiated.

## II. Assumptions And Notation

We try to generalize the inventory model of Mandal and Pal [5], Wu and Ouyang [8], and Deng et al. [2] with the following assumptions and notations for the deterministic inventory replenishment policy with general demand.

(1) The replenishment rate is infinite; thus, replenishments are instantaneous.
(2) The lead time is zero.
(3) $T$ is the finite time horizon under consideration. Thus, it is a constant.
(4) $C_h$ is the inventory holding cost per unit, per unit of time.
(5) $C_s$ is the shortage cost per unit, per unit of time.
(6) $C_d$ is the cost of each deteriorated item.
(7) $\theta$ is the constant fraction of the on-hand inventory deterioration per unit of time.
(8) $I(t)$ is the on-hand inventory level at the time $t$ of the ordering cycle $[0,T]$.
(9) The shortage is allowed and fully back-ordered.
(10) The demand rate $R(t)$ is assumed to be any positive function for $t > 0$.
(11) $t_1$ is the time when the inventory level reaches zero.
(12) $f(t_1)$ is an auxiliary function defined as $\left(C_d + \frac{C_h}{\theta}\right)\left(e^{\theta t_1} - 1\right) - C_s\left(T - t_1\right)$.
(13) $C(t_1)$ is the total cost that consists of holding cost, deterioration cost, and shortage cost.

## III. Review Of Previous Results

In Deng et al. [2], they only considered the ramp type demand where demand $R(t)$ is assumed as follows:

$$R(t) = \begin{cases} D_0 t\,, & t < \mu \\ D_0 \mu\,, & t \ge \mu \end{cases} \tag{1}$$

so that $\mu$ is the changing point from linear demand to constant demand. Depending on the relation between $\mu$ and $t_1$, they developed two inventory models: when $\mu \ge t_1$, the total cost, $TC_0(t_1)$, is defined for $0 \le t_1 \le \mu$ and for $\mu < t_1$, $TC_1(t_1)$ is defined for $\mu \le t_1 \le T$. They studied the properties of $TC_1(t_1)$ in their Theorem 1, and the properties of $TC_0(t_1)$ in their Theorem 2. Based on their findings of Theorems 1 and 2, they developed their Theorem 3, for the ramp type demand, that if $\Delta_1 < 0$, the minimum occurs at $TC_1\left(t_1^{\#}\right)$. On the other hand, if $\Delta_1 \ge 0$, then the minimum occurs at $TC_0\left(t_1^{\#}\right)$ where $t_1^{\#}$ is the unique solution of $f(t_1) = 0$ for $0 \le t_1 \le T$ with

$$f(t_1) = \left(C_d + \frac{C_h}{\theta}\right)\left(e^{\theta t_1} - 1\right) - C_s\left(T - t_1\right) \tag{2}$$

and $\Delta_1 = f(\mu)$.

## IV. Our Proposed Inventory Model

We consider an inventory model that starts with stock. This model was first proposed by Hill [4], and then further investigated by Mandal and Pal [5], Wu and Ouyang [8], and Deng et al. [2]. Replenishment occurs at the time $t = 0$ when the inventory level attains its maximum, $S$. From $t = 0$ to $t_1$, the inventory level reduces due to both demand, $R(t)$, and deterioration. At $t_1$, the inventory level achieves zero, after which shortages are allowed during the time interval $(t_1, T)$, and all of the demand during the shortage period $(t_1, T)$ is completely backlogged. The inventory level $I(t)$ of the model is described by the following equations:

$$\frac{d}{dt} I(t) + \theta\, I(t) = -R(t),\ 0 < t < t_1 \qquad (3)$$

and

$$\frac{d}{dt} I(t) = -R(t),\ t_1 < t < T\,. \qquad (4)$$

We directly solve Equations (3) and (4) to imply that

$$I(t) = e^{-\theta t} \int_t^{t_1} R(x) e^{\theta x} dx \text{, for } 0 \le t \le t_1\,, \qquad (5)$$

and

$$I(t) = \int_t^{t_1} R(x)\, dx \text{, for } t_1 \le t \le T\,. \qquad (6)$$

The amount of deteriorated items during the time interval, $[0, t_1]$, is evaluated

$$I(0) - \int_0^{t_1} R(x) dx = \int_0^{t_1} R(x)\left(e^{\theta x} - 1\right) dx\,. \qquad (7)$$

Using integration by part, the holding cost during the time interval, $[0, t_1]$, is evaluated

$$C_h \int_0^{t_1} I(t)\, dt = C_h \int_0^{t_1} R(x) \frac{e^{\theta x} - 1}{\theta} dx\,. \qquad (8)$$

The shortage cost during the time interval, $[t_1, T]$, is evaluated through integration by part

$$C_s \int_{t_1}^{T} -I(t) dt = C_s \int_{t_1}^{T} (T - x) R(x) dx\,. \qquad (9)$$

Therefore, the total cost is expressed as

$$C(t_1) = C_d \int_0^{t_1} R(x)\left(e^{\theta x} - 1\right) dx + C_h \int_0^{t_1} R(x) \frac{e^{\theta x} - 1}{\theta} dx + C_s \int_{t_1}^{T} (T - x) R(x) dx\,. \qquad (10)$$

From Equation (10), it follows that

$$C'(t_1) = R(t_1)\left[\left(C_d + \frac{C_h}{\theta}\right)\left(e^{\theta t_1} - 1\right) - C_s\left(T - t_1\right)\right]. \qquad (11)$$

Motivated by Equation (11), we assume an auxiliary function, say $f(t_1)$, with

$$f(t_1) = \left(C_d + \frac{C_h}{\theta}\right)\left(e^{\theta t_1} - 1\right) - C_s\left(T - t_1\right). \qquad (12)$$

As a matter of fact, it is nothing but the same one proposed by Deng et al. [2]. By taking the derivative of $f(t_1)$, i.e., $f'(t_1) = (\theta C_d + C_h) e^{\theta t_1} + C_s > 0$, it is easy to find that $f(t_1)$ increases from $f(0) = -C_s T < 0$ to $f(T) = \left(C_d + \frac{C_h}{\theta}\right)\left(e^{\theta T} - 1\right) > 0$. Hence, obviously, there exists a unique point, say $t_1^{\#}$, that satisfies $f(t_1^{\#}) = 0$ and the following equation holds.

$$\left(C_d + \frac{C_h}{\theta}\right)\left(e^{\theta t_1^{\#}} - 1\right) = C_s\left(T - t_1^{\#}\right). \qquad (13)$$

Since $C'(t_1) \le 0$ for $0 \le t_1 \le t_1^{\#}$ and $C'(t_1) \ge 0$ for $t_1^{\#} \le t_1 \le T$ such that $t_1^{\#}$ is the minimum solution for $C(t_1)$. We summarize our findings in the next theorem.

**Theorem 1.**For the inventory model beginning with stock, the minimum solution satisfies $f(t_1^{\#}) = 0$ and is independent of the demand.

## V. Different Views OfOur Findings

We intend to provide a marginal (cost) analysis to discuss our findings. If there is an item with a demand quantity $R(t)dt$ where $dt$ is a small (infinitesimal) time interval, then there are two replenishment policies: (a) fulfill the demand from the stock, or (b) satisfy the demand from backorder. If we decide to fulfill the demand from the stock, we need to store $R(t)e^{\theta t}dt$ at time $t = 0$. Note that the solution of $\frac{d}{dt} I(t) + \theta\, I(t) = 0$ is $I(t) = I(0)e^{-\theta t}$ so that the beginning stock is $I(0) = R(t)e^{\theta t}dt$. It follows that after deteriorated items are removed, the remaining stock $R(t)dt$ is just enough to meet the demand. The amount of deteriorated items is $R(t)(e^{\theta t} - 1)dt$. From the inventory level $R(t)e^{\theta t}e^{-\theta x}dt$ for $x \in [0, t]$, the holding cost can be calculated by

$$C_h \int_0^t R(t)dte^{\theta t}e^{-\theta x}dx = C_h R(t)dt \frac{e^{\theta t} - 1}{\theta}. \tag{14}$$

Hence, the total cost for demand $R(t)dt$ fulfilled from the stock is $\left(\frac{C_h}{\theta} + C_d\right)R(t)dt\left(e^{\theta t} - 1\right)$. On the other hand, if the policy of backlog is adopted then the shortage cost is $R(t)dtC_s(T - t)$. We find that the better policy is to hold demand as backlog until the replenishment takes place. Recall from Equation (12), we have shown that $f(t) = \left(C_d + \frac{C_h}{\theta}\right)\left(e^{\theta t} - 1\right) - C_s(T - t)$ has a unique root, say $t_1^{\#}$. It means that for those demands occurring during $\left[0, t_1^{\#}\right)$, they should be fulfilled from the stock And for those demands occurring during $\left(t_1^{\#}, T\right]$, they should be replenished by backorder.

Our microscope approach comes up with the same results as our previous presentation in Section 4.

## VI. Conclusion

In the paper, we discover an interesting phenomenon for the finite time horizon inventory model to show that the optimal solution is independent of the demand. The generalized form of the inventory model is developed and the corresponding optimal solution is derived. We believe that our findings provide an essential benchmark for those researchers who have the motive of pursuing different optimal inventory systems along with the changes in demand types. This study gives solid evidence that the optimal solution is independent of the demand no matter the ramp type, trapezoid type, fixed type, or anyother kind that is targeted.

## References


[1]. Deng, P.S., (2005) Improved inventory models with ramp type demand and Weibull deterioration.International Journal of Information and Management Sciences, 16 (4), 79-86.

[2]. Deng, P.S., Lin, R., and Peter Chu, P. (2007) A note on the inventory models for deteriorating items with ramp type demand rate. European Journal of Operational Research, 178, 112-120.

[3]. Giri, B.C., Jalan, A.K., and Chaudhuri, K.S. (2003) Economic Order Quantity model with Weibull deterioration distribution, shortage and ramp-type demand.International Journal of Systems Science 34 (4), 237-243.

[4]. Hill, R.M., (1995) Inventory models for increasing demand followed by level demand.*Journal of the Operational Research Society*, 46 (10), 1250-1259.

[5]. Mandal, B. and Pal, A.K. (1998) Order level inventory system with ramp type demand rate for deteriorating items. Journal of Interdisciplinary Mathematics, 1, 49-66.

[6]. Manna, S.K., Chaudhuri, K.S., (2006) An EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages. European Journal of Operational Research 171 (2), 557-566.

[7]. Wu, J.W., Lin, C., Tan, B., and Lee, W.C. (1999) An EOQ model with ramp type demand rate for items with Weibull deterioration. International Journal of Information and Management Sciences, 10, 41-51.

[8]. Wu, K.S., and Ouyang, L.Y. (2000) A replenishment policy for deteriorating items with ramp type demand rate, Proceeding of National Science Council ROC (A), 24, 279-286.
[9]. Wu, K.S. (2001) An EOQ inventory model for items with Weibull distribution deterioration, ramp type demand rate and partial backlogging, Production Planning & Control, 12, 787-793.

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